AN INTRODUCTION TO INTUITIONISTIC FUZZY TOPOLOGICAL SPACES IN SOSTAK'S SENSE

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Abstract: In this paper we introduce intuitionistic fuzzy topological spaces in Sostak's sense and define the fuzzy compactness spectrum by means of right fuzzy inclusion.

Keywords: Intuitionistic fuzzy set; intuitionistic fuzzy pair; right fuzzy inclusion; intuitionistic fuzzy topology in Sostak's sense; intuitionistic fuzzy topological space in Sostak's sense; fuzzy continuity; fuzzy compactness spectrum.

1. Introduction

The concept of fuzzy set was introduced by Zadeh [15], and later Chang [4] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Sostak [13,14], used the idea of degree of openness. This type of generalization of a fts was later rephrased by Chattopadhyay, Hazra and Samanta in 1992 [5], and by Ramadan in 1992 [12] (cf. [10] also) (He generalized the same idea under the name of "smooth topological space" using lattices in the following manner: Let L and L′ be two lattices which will be copies of [0,1] and [0,1], respectively. Then, a smooth topological space is a pair (X,τ), where X is a nonempty set and $\tau : L \to L'$ is a mapping satisfying (T1), (T2) and (T3).)

In 1983, Atanassov introduced the concept of "intuitionistic fuzzy set" [1,2,3]. Using this type of generalized fuzzy set, Çoker [6,7] defined "intuitionistic fuzzy topological spaces".

2. Preliminaries

For the sake of completeness, first we give the concept of intuitionistic fuzzy set defined by Atanassov:

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Definition 2.1. [1,2,3] Let X be a nonempty fixed set. An intuitionistic fuzzy set (IFS for short) A is an object having the form

$$A=\{ \langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X \}$$

where the functions $\mu_A : X \to I$ and $\gamma : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element $x\in X$ to the set A, respectively, and $0\le\mu_A(x)+\gamma_A(x)\le 1$ for each $x\in X$.

Definition 2.2. [3,6,7] Let X be a nonempty set, and the IFS's A and B in X be in the form

$$A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle:x\in X\},\ B=\{\langle x,\mu_B(x),\gamma_B(x)\rangle:x\in X\} .$$

Furthermore, let $\{A_i:i\in J\}$ be an arbitrary family of IFS's in X. Then

(a) $A\subseteq B$ iff $\mu_A(x)\le\mu_B(x)$ and $\gamma_A(x)\ge\gamma_B(x)$ for all $x\in X$;

(b) $A=B$ iff $A\subseteq B$ and $B\subseteq A$;

(c) $\bar{A}=\{ \langle x,\gamma_A(x),\mu_A(x)\rangle : x\in X \}$;

(d) $\cap A_i=\{ \langle x,\wedge\mu_{A_i}(x),\vee\gamma_{A_i}(x)\rangle : x\in X \}$;

(e) $\cup A_i=\{ \langle x,\vee\mu_{A_i}(x),\wedge\gamma_{A_i}(x)\rangle : x\in X \}$;

(f) $[]A=\{ \langle x,\mu_A(x),1-\mu_A(x)\rangle : x\in X\}$;

(g) $\langle\rangle A=\{ \langle x,1-\gamma_A(x),\gamma_A(x)\rangle : x\in X\}$;

(h) $\underset{\sim}{0}=\{ \langle x,0,1\rangle : x\in X \}$ and $\underset{\sim}{1}=\{ \langle x,1,0\rangle : x\in X \}$.

Here are the basic properties of inclusion and complementation:

Corollary 2.3. [6,7] Let A, B, C be IFS's in X. Then

(a) $A\subseteq B$ and $B\subseteq C \Longrightarrow A\subseteq C$

(b) $A\subseteq B$ and $C\subseteq D \Longrightarrow A\cup C\subseteq B\cup D$ and $A\cap C\subseteq B\cap D$

(c) $A_i\subseteq B$ for each $i\in J \iff \cup A_i\subseteq B$

(d) $B\subseteq A_i$ for each $i\in J \iff B\subseteq\cap A_i$

(e) $\overline{\cup A_i}=\cap\overline{A_i}$

(f) $\overline{\cap A_i}= \cup\overline{A_i}$

(g) $A\subseteq B \iff \bar{B}\subseteq\bar{A}$

(h) $\overline{(\bar{A})}=A$

(i) $\overline{\underset{\sim}{1}}=\underset{\sim}{0}$

(j) $\overline{\underset{\sim}{0}}=\underset{\sim}{1}$

Here we define the preimages and images of IFS's under the function $f : X \to Y$:

Definition 2.4. [6,7]

(a) If $B=\{\langle y,\mu_B(y),\gamma_B(y)\rangle : y\in Y\}$ is an IFS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is the IFS in X defined by

$$f^{-1}(B) = \{ \langle x,f^{-1}(\mu_B)(x),f^{-1}(\gamma_B)(x)\rangle : x\in X \}.$$

(b) If $A=\{\langle x,\lambda_A(x),\vartheta_A(x)\rangle : x\in X\}$ is an IFS in X, then the image of A under f, denoted by f(A), is the IFS in Y defined by

$$f(A) = \{ \langle y,f(\lambda_A)(y),f_-(\vartheta_A)(y)\rangle : y\in Y \} ,$$

where $f_-(\vartheta_A)=1-f(1-\vartheta_A)$.

Corollary 2.5. [6,7] Let A, A_i ($i\in J$) be IFS's in X, B, B_j ($j\in K$) IFS's in Y and $f : X \to Y$ a function. Then

(a) $A_1\subseteq A_2 \Longrightarrow f(A_1)\subseteq f(A_2)$

(b) $B_1\subseteq B_2 \Longrightarrow f^{-1}(B_1)\subseteq f^{-1}(B_2)$

(c) $f^{-1}(\cup B_j)=\cup f^{-1}(B_j)$

(d) $f^{-1}(\cap B_j)=\cap f^{-1}(B_j)$

(e) $f^{-1}(\overline{B})=\overline{f^{-1}(B)}$

3. Fuzzy inclusion in the intuitionistic sense

Given the nonempty set X, we shall denote the family of all IFS's in X by the symbol $\mathcal{I}^X$. Now we shall present a supplementary tool, called "intuitionistic fuzzy pair" which is first introduced in [8]:

Definition 3.1. [8] Let a and b be two real numbers in [0,1] satisfying the inequality $a+b\leq 1$. Then the pair $\langle a,b\rangle$ is called an intuitionistic fuzzy pair.

Let $\langle a_1,b_1\rangle$, $\langle a_2,b_2\rangle$ be two intuitionistic fuzzy pairs. Then we define

(a) $\langle a_1,b_1\rangle\leq\langle a_2,b_2\rangle \iff a_1\leq a_2$ and $b_1\geq b_2$.

(b) $\langle a_1,b_1\rangle=\langle a_2,b_2\rangle \iff \langle a_1,b_1\rangle\leq\langle a_2,b_2\rangle$ and $\langle a_1,b_1\rangle\geq\langle a_2,b_2\rangle$.

(c) If $\{ \langle a_i,b_i\rangle : i\in J \}$ is a family of intuitionistic fuzzy pairs, then

$$\vee\langle a_i,b_i\rangle=\langle \vee a_i,\wedge b_i\rangle \text{ and } \wedge\langle a_i,b_i\rangle=\langle\wedge a_i,\vee b_i\rangle .$$

Definition 3.2. [8] The complement of an intuitionistic fuzzy pair $\langle a,b\rangle$ is the intuitionistic fuzzy pair defined by $\overline{\langle a,b\rangle}=\langle b,a\rangle$.

Definition 3.3. [8] $1^\sim = \langle 1,0\rangle$ and $0^\sim = \langle 0,1\rangle$.

Here comes a corollary stating the relations between intuitionistic fuzzy pairs:

Corollary 3.4. [8]

(1) $\langle a,b\rangle \le \langle c,d\rangle$ and $\langle c,d\rangle \le \langle e,f\rangle \implies \langle a,b\rangle \le \langle e,f\rangle$;

(2) $\langle a,b\rangle \le \langle c_i,d_i\rangle$ for each $i\in J \implies \langle a,b\rangle \le \bigwedge \langle c_i,d_i\rangle$;

(3) $\langle c_i,d_i\rangle \le \langle a,b\rangle$ for each $i\in J \implies \bigvee \langle c_i,d_i\rangle \le \langle a,b\rangle$;

(4) $\overline{\langle a,b\rangle} \le \overline{\langle c,d\rangle} \iff \langle a,b\rangle \ge \langle c,d\rangle$;

(5) $\overline{\bigvee \langle c_i,d_i\rangle} = \bigwedge \overline{\langle c_i,d_i\rangle}$; (6) $\overline{\bigwedge \langle c_i,d_i\rangle} = \bigvee \overline{\langle c_i,d_i\rangle}$.

Definition 3.5. [8] Let X be a nonempty set. Then the right fuzzy inclusion, denoted by $\succsim$, is the IFS on $\mathcal{I}^X \times \mathcal{I}^X$ defined by

$$\mu_{\succsim}(A,B)=\inf\{ (\gamma_A \vee \mu_B)(x) : x\in X \} \text{ and}$$

$$\gamma_{\succsim}(A,B)=\sup\{ (\mu_A \wedge \gamma_B)(x) : x\in X \},$$

for each $A,B\in \mathcal{I}^X$. Here $\mu_{\succsim}(A,B)$ denotes the degree of inclusion of A in B, while $\gamma_{\succsim}(A,B)$ denotes the degree of noninclusion of A in B.

It is easy to show that the pair $\langle \mu_{\succsim}(A,B), \gamma_{\succsim}(A,B)\rangle$ is indeed an intuitionistic fuzzy pair:

Definition 3.6. [8] For any two IFS's $A,B\in \mathcal{I}^X$, the intuitionistic fuzzy pair $\langle \mu_{\succsim}(A,B), \gamma_{\succsim}(A,B)\rangle$ will be denoted by $[A\succsim B]$, i.e.

$$[A\succsim B] = \langle \mu_{\succsim}(A,B), \gamma_{\succsim}(A,B)\rangle .$$

Here we list some of the basic properties of right fuzzy inclusion:

Proposition 3.7. [8] Assume that A,B,C,D are IFS's in X. Then the following properties hold:

(1) If $A\subseteq B$ and $C\supseteq D$, then $[A\succsim C]\ge[B\succsim D]$.

(2) $[\bar{A}\succsim\bar{B}]=[B\succsim A]$ (3) $[A\cup B\succsim C\cap D]\le[A\succsim C]\wedge[B\succsim D]$

(4) $[A\succsim C]\vee[B\succsim D]\le[A\cap B\succsim C\cup D]$ (5) $[A\cap\bar{B}\succsim \underset{\sim}{0}]=[A\succsim B]=[\underset{\sim}{1}\succsim\bar{A}\cup B]$

(6) If $\{B_i : i\in J\}\subseteq \mathcal{I}^X$, then

$$\bigwedge[A\precsim B_i]=[A\precsim\bigcap B_i] \text{ and } \bigwedge[B_i\precsim A]=[\bigcup B_i\precsim A] .$$

Now we present the properties of fuzzy inclusion related to images and preimages:

Proposition 3.8. [8] Assume that A, B are IFS's in X and C, D are IFS's in Y. If $f : X \to Y$ is a function, then the following properties hold:

(1) $[A\precsim B]\le[f(A)\precsim f(B)]$. If, furthermore, f is injective, then $[A\precsim B]=[f(A)\precsim f(B)]$.

(2) $[C\precsim D]\le[f^{-1}(C)\precsim f^{-1}(D)]$. If, furthermore, f is surjective, then $[C\precsim D]=[f^{-1}(C)\precsim f^{-1}(D)]$.

(3) $[A\precsim f^{-1}(f(A))]\le[f(A)\precsim f(A)]$, $[f^{-1}(f(A))\precsim A]\le[A\precsim A]$, $[f(f^{-1}(C))\precsim C]\le[f^{-1}(C)\precsim f^{-1}(C)]$, $[C\precsim f(f^{-1}(C))]\le[C\precsim C]$.

(4) $[f(A)\precsim C]=[A\precsim f^{-1}(C)]$.

(5) If $\{C_i:i\in J\}\le \mathscr{I}^Y$, then

$$[f(A)\precsim\bigcup C_i]=[A\precsim\bigcup f^{-1}(C_i)] .$$

(6) If $\{A_i:i\in J\}\le\mathscr{I}^X$, then

$$[f(\bigcap A_i)\precsim C\ ]=[\bigcap A_i\precsim f^{-1}(C\ )] .$$

In order to obtain intuitionistic fuzzy topological spaces we need to define the concept "intuitionistic fuzzy family":

Definition 3.9. [8] An IFS $\mathscr{F}$ on the set $\mathscr{I}^X$ is called an intuitionistic fuzzy family (IFF for short) on X. In symbols, we shall denote such an IFF in the form $\mathscr{F}=\langle\mu_{\mathscr{F}},\gamma_{\mathscr{F}}\rangle$.

Definition 3.10. [8] Let $\mathscr{F}$ be an IFF on X. Then the IFF of complemented IFS's on X is defined by $\mathscr{F}^*=\langle\mu_{\mathscr{F}^*},\gamma_{\mathscr{F}^*}\rangle$, where

$$\mu_{\mathscr{F}^*}(A)=\mu_{\mathscr{F}}(\bar{A}) \quad\text{and}\quad \gamma_{\mathscr{F}^*}(A)=\gamma_{\mathscr{F}}(\bar{A})$$

for each $A\in\mathscr{I}^X$.

4. Intuitionistic fuzzy topological spaces in Sostak's sense

Now we give the basic definitions and properties of intuitionistic fuzzy topological spaces in Sostak's sense, which is a generalized form of "fuzzy topological spaces" developed by Sostak [13,14]. For the sake of brevity, we shall use the following notation: If τ is an IFF on X, then, for any $A\in\mathscr{I}^X$, we can construct the intuitionistic fuzzy pair $\langle\mu_\tau(A),\gamma_\tau(A)\rangle$, and use the symbol

$$\tau(A)=\langle\mu_\tau(A),\gamma_\tau(A)\rangle .$$

Definition 4.1. An intuitionistic fuzzy topology in Sostak's sense (So-IFT for short) on a nonempty set X is an IFF τ on X satisfying the following axioms:

(T1) $\tau(\underset{\sim}{0})=\tilde{1}$ and $\tau(\underset{\sim}{1})=\tilde{1}$;

(T2) $\tau(A_1\cap A_2) \geq \tau(A_1) \wedge \tau(A_2)$ for any $A_1,A_2\in\mathscr{I}^X$;

(T3) $\tau(\cup A_i) \geq \bigwedge\tau(A_i)$ for any $\{A_i:i\in J\}\subseteq\mathscr{I}^X$.

In this case the pair (X,τ) is called an intuitionistic fuzzy topological space in Sostak's sense (So-IFTS for short). For any $A\in\mathscr{I}^X$, the number $\mu_\tau(A)$ is called the openness degree of A, while $\gamma_\tau(A)$ is called the nonopenness degree of A.

Definition 4.2. Let (X,τ_1), (X,τ_2) be two So-IFTS's on X. Then τ_1 is said to be contained in τ_2 (in symbols, $\tau_1\leq\tau_2$) if $\tau_1(A)\leq\tau_2(A)$ for each $A\in\mathscr{I}^X$. In this case, we also say that τ_1 is coarser than τ_2, or τ_2 is finer than τ_1.

Proposition 4.3. Let $\{\tau_i:i\in J\}$ be a family of So-IFT's on X. Then the IFS $\bigwedge\tau_i$ on $\mathscr{I}^X$ defined by

$$(\bigwedge\tau_i)(A)=\bigwedge\{\tau_i(A) : i\in J\},$$

where $A\in\mathscr{I}^X$, is a So-IFT on X. Furthermore, $\bigwedge\tau_i$ is the coarsest So-IFT on X containing all τ_i's.

Definition 4.4. Let (X,τ) be a So-IFTS on X. Then the IFF's

(a) $[\,]\tau$,   (b) $\langle\rangle\tau$

defined on X by

$$([\,]\tau)(A)=\langle\mu_\tau(A),1-\mu_\tau(A)\rangle \quad \text{and} \quad (\langle\rangle\tau)(A)=\langle 1-\gamma_\tau(A),\gamma_\tau(A)\rangle$$

are also So-IFT's on X.

Proposition 4.5. If (X,τ) is a So-IFTS on X, then we have

$$[\,]\tau \leq \tau \leq \langle\rangle\tau .$$

A So-IFTS (X,τ) is, of course, in the sense of Chang. Now we can obtain the definition of a So-IFTS in the sense of Lowen [11], too:

Definition 4.6. A So-IFTS in the sense of Lowen is a pair (X,τ) where (X,τ) is a So-IFTS and for each IFS in the form

$$c_{\alpha,\beta}=\{ \langle x,\alpha,\beta\rangle : x\in X \},$$

where $\alpha,\beta\in I$ are arbitrary $(\alpha+\beta\le 1)$, we have $\tau(c_{\alpha,\beta})=\tilde{1}$.

Definition 4.7. Let (X,τ) be a So-IFTS on X. Then the IFF τ^* of complemented IFS's on X is defined by $\tau^*(A)=\tau(\bar{A})$. The number $\mu_{\tau^*}(A)=\mu_{\tau}(\bar{A})$ is called the closedness degree of A, while $\gamma_{\tau^*}(A)=\gamma_{\tau}(\bar{A})$ is called the nonclosedness degree of A.

Proposition 4.8. The IFF τ^* on X satisfies the following properties:

(C1) $\tau^*(\underset{\sim}{0})=\tilde{1}$ and $\tau^*(\underset{\sim}{1})=\tilde{1}$;

(C2) $\tau^*(A_1\cup A_2) \ge \tau^*(A_1) \wedge \tau^*(A_2)$ for any $A_1,A_2\in\mathcal{I}^X$;

(C3) $\tau^*(\cap A_i) \ge \bigwedge\tau^*(A_i)$ for any $\{A_i:i\in J\}\subseteq\mathcal{I}^X$.

If (X,τ) is a So-IFTS, then for each $\alpha\in(0,1]$ and $\beta\in[0,1)$ with $\alpha+\beta\le 1$, the family $\tau_{\alpha,\beta}$ defined by

$$\tau_{\alpha,\beta} = \{A\in\mathcal{I}^X : \tau(A)\ge c_{\alpha,\beta}\}$$

is an IFTS [6]. $\tau_{\alpha,\beta}$ is called the (α,β)-level IFTS on X, and in this case the family of all intuitionistic fuzzy closed sets in this IFTS can be written as

$$\tau^*_{\alpha,\beta}=\{A\in\mathcal{I}^X : \tau^*(A)\ge c_{\alpha,\beta}\} .$$

Now one can obtain the closure and interior operators in the IFTS $(X,\tau_{\alpha,\beta})$ for each $\alpha\in(0,1]$, $\beta\in[0,1)$ with $\alpha+\beta\le 1$ as in [6]:

$$cl_{\alpha,\beta}(A)=\cap\{K\in\mathcal{I}^X: A\subseteq K,\ K\in\tau^*_{\alpha,\beta}\} \text{ and}$$

$$int_{\alpha,\beta}(A)=\cup\{G\in\mathcal{I}^X: G\subseteq A,\ G\in\tau_{\alpha,\beta}\}$$

for each $A\in\mathcal{I}^X$. Notice that we have $\tau^*_{\alpha,\beta}(cl_{\alpha,\beta}(A))\ge c_{\alpha,\beta}$ and $\tau_{\alpha,\beta}(int_{\alpha,\beta}(A))\ge c_{\alpha,\beta}$.

Proposition 4.9. The closure and interior operators satisfy the following properties:

(1) $cl_{\alpha,\beta}(A)\supseteq A$ (1′) $int_{\alpha,\beta}(A)\subseteq A$

(2) $A \subseteq B$ and $\langle\alpha,\beta\rangle \le \langle\gamma,\delta\rangle \Longrightarrow cl_{\alpha,\beta}(A) \subseteq cl_{\gamma,\delta}(B)$

(2′) $A \subseteq B$ and $\langle\alpha,\beta\rangle \le \langle\gamma,\delta\rangle \Longrightarrow int_{\gamma,\delta}(A) \subseteq int_{\alpha,\beta}(B)$

(3) $cl_{\alpha,\beta}(cl_{\alpha,\beta}(A)) = cl_{\alpha,\beta}(A)$

(3′) $int_{\alpha,\beta}(int_{\alpha,\beta}(A)) = int_{\alpha,\beta}(A)$

(4) $cl_{\alpha,\beta}(A \cup B) = cl_{\alpha,\beta}(A) \cup cl_{\alpha,\beta}(B)$

(4′) $int_{\alpha,\beta}(A \cap B) = int_{\alpha,\beta}(A) \cap int_{\alpha,\beta}(B)$

(5) $cl_{\alpha,\beta}(\underset{\sim}{0}) = \underset{\sim}{0}$ (5′) $int_{\alpha,\beta}(\underset{\sim}{1}) = \underset{\sim}{1}$

Proposition 4.10. For each $A \in \mathcal{I}^X$ and for each $\alpha \in (0,1]$, $\beta \in [0,1)$ with $\alpha+\beta \le 1$, we have

(1) $\overline{cl_{\alpha,\beta}(A)} = int_{\alpha,\beta}(\bar{A})$ (2) $\overline{int_{\alpha,\beta}(A)} = cl_{\alpha,\beta}(\bar{A})$

Fuzzy Continuity

Definition 4.11. Let (X,τ) and (Y,Φ) be two So-IFTS's and $f : X \to Y$ be a function. Then f is said to be fuzzy continuous iff

$$\tau(f^{-1}(B)) \ge \Phi(B)$$

for each $B \in \mathcal{I}^Y$ (cf. [13,14])

Proposition 4.12. The following properties are equivalent:

(a) $f : (X,\tau) \to (Y,\Phi)$ is fuzzy continuous.

(b) $\tau^*(f^{-1}(B)) \ge \Phi^*(B)$ for each $B \in \mathcal{I}^Y$.

Definition 4.13. Let (X,τ) and (Y,Φ) be two So-IFTS's and $f : X \to Y$ be a function. Then f is said to be fuzzy open iff

$$\Phi(f(A)) \ge \tau(A)$$

for each $A \in \mathcal{I}^X$. (cf. [13,14])

Fuzzy Compactness Spectrum

If $\mathcal{U}$ is a subfamily of $\mathcal{I}^X$, then we define the following intuitionistic fuzzy pairs:

$\tau(\mathcal{U}) = \bigwedge\{\tau(A) : A \in \mathcal{U}\}$ and similarly $\tau^*(\mathcal{U}) = \bigwedge\{\tau^*(A) : A \in \mathcal{U}\}$.

Right fuzzy inclusion plays an important role in developing the fuzzy compactness spectrum in So-IFTS's using a construction similar to [13,14,9]:

Definition 4.14. Let (X,τ) be a So-IFTS on X, $A\in\mathcal{I}^X$ and $\langle\alpha,\beta\rangle$ an intuitionistic fuzzy pair such that $\alpha\in(0,1]$, $\beta\in[0,1)$. The fuzzy compactness spectrum of A at a level $\langle\alpha,\beta\rangle$ is defined by

$$C_{\alpha,\beta}(A)=\{\langle\eta,\delta\rangle : \forall\mathcal{U}\,(\tau(\mathcal{U})\geq\langle\alpha,\beta\rangle)\,[A\precsim\vee\mathcal{U}]\geq\langle\eta,\delta\rangle \Longrightarrow \sup\{[A\precsim\vee\mathcal{U}_0] : \mathcal{U}_0\leq'\mathcal{U}\}\geq\langle\eta,\delta\rangle\}.$$

[Here the expression " $\mathcal{U}_0\leq'\mathcal{U}$ " means that $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$.]

Remark 4.15. Notice that we always have $0^\sim\in C_{\alpha,\beta}(A)$.

Proposition 4.16. (cf. [9]) Let $A_1, A_2\in\mathcal{I}^X$. Then

$$C_{\alpha,\beta}(A_1\cup A_2)\geq C_{\alpha,\beta}(A_1)\cap C_{\alpha,\beta}(A_2).$$

Proposition 4.17. (cf. [9]) Let (X,τ), (Y,Φ) be So-IFTS's and $f : X\rightarrow Y$ be a fuzzy continuous function. Then, for any $A\in\mathcal{I}^X$, we have

$$C_{\alpha,\beta}(f(A))\geq C_{\alpha,\beta}(A).$$

REFERENCES

[1] K. Atanassov, Intuitionistic fuzzy sets, *VII ITKR's Session*, Sofia (September, 1983) (in Bulgarian).

[2] K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems* **20** (1986) 87-96.

[3] K. Atanassov, Review and new results on intuitionistic fuzzy sets, *Preprint IM-MFAIS-88-1*, Sofia, pp. 1-8.

[4] C. Chang, Fuzzy topological spaces, *J.Math.Anal.Appl.* **24** (1968) 182-190.

[5] K. C. Chattopadhyay, R. N. Hazra and S. K. Samanta, Gradation of openness: Fuzzy topology, *Fuzzy Sets and Systems* **49** (1992) 237-242.

[6] D. Çoker, An introduction to intuitionistic fuzzy topological spaces, to appear in *Fuzzy Sets and Systems*.

[7] D. Çoker and A. H. Eş, On fuzzy compactness in intuitionistic fuzzy topological spaces, *J. Fuzzy Mathematics* **3-4** (1995) 899-909.

[8] D. Çoker and M. Demirci, On fuzzy inclusion in the intuitionistic sense, to appear in *J. Fuzzy Mathematics*.

[9] A. H. Eş and D. Çoker, On several types of degrees of fuzzy compactness in fuzzy topological spaces in Sostak's sense, *J. Fuzzy Mathematics* 3-3 (1995) 481-491.

[10] R. N. Hazra, S. K. Samanta and K. C. Chattopadhyay, Fuzzy topology redefined, *Fuzzy Sets and Systems* **45** (1991) 78-82.

[11] R. Lowen, Fuzzy topological spaces and fuzzy compactness, *J.Math.Anal.Appl.* **56** (1976) 621-633.

[12] A. A. Ramadan, Smooth topological spaces, *Fuzzy Sets and Systems* **48** (1992) 371-375.

[13] A. Sostak, On a fuzzy topological structure, *Supp. Rend. Circ. Mat. Palermo (Ser. II)* **11** (1985) 89-103.

[14] A. Sostak, On compactness and connectedness degrees of fuzzy sets in fuzzy topological spaces, in General Topology and its Relations to Modern Analysis and Algebra (Helderman Verlag, Berlin, 1988) 519-532.

[15] L. A. Zadeh, Fuzzy sets, *Information and Control,* **8** (1965) 338-353.