The Characterizations of Ultra-fuzzy Compact Spaces and Ultra - fuzzy Paracompactness

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Abstract In this paper, a conception of relative R-neighborhood family in L-fuzzy topological spaces is introduced, by which some characterizitions of ultra-fuzzy compactness are given; using the conception, we've proved Alexander Subbase Lemma of ultra- fuzzy compactness. Furthermore the ultra- fuzzy paracompactness defined by strong locally finite relative remote neighborhood family is proved to be an L-good extension.

Keywords Ultra-fuzzy compactness, Relative remote neighborhood family.

1. Introduction

The ultra-fuzzy compactness of fuzzy topological spaces was firstly introduced by Lowen in 1978 [1]. Since it was defined by the compactness of the introduced crisp topology based on the support sets of the fuzzy topological spaces, on the one hand, it seems to be very formulization; on the other hand, to study its characterizations about fuzzy topology characteristic is very difficult. So the further study on the ultra -fuzzy compactness is very less up to now. In this paper, the conception of relative remote neighborhood family in L- fuzzy topological spaces is introduced, by which, the ultra-fuzzy compact spaces is descripted by remote neighborhood and some characterizations of ultra- fuzzy compact spaces and some results about product are obtained. Furthermore, an ultra- fuzzy paracompactness based on the ultra-fuzzy compactness is discussed by means of the relative remote neighborhood family. It is proved that the ultra- fuzzy paracompactness is equivelant to the paracompactness of its crisp space in any weakly induced space.

Throughout this paper, X always denotes a nonempty crisp set, L denotes a fuzzy lattice, 0 and 1 denote the least element and the largest element of L or L^X respectively. $M(L)$ and $M^*(L^X)$ denote the sets consisting of all nonzero union irreducible elements of L and L^X respectively. (L^X, δ) always denotes an L-fuzzy topological space(briefly, L-fts). the rest notions and symbols in the paper are from [2].

2. The characterizations of ultra-fuzzy compact spaces

Let (L^X, δ) be an L-fts. To take $\Gamma(\delta) = \{l_r(A): A \in L, r$ is a prime element of L and $r \neq 1\}$ as a subbase produce a crisp topology on X [2], which is called induced crisp topology by δ *on* X and denoted as $l_L(\delta)$, where $l_r(A) = \{x \in X: A(x) \leq r\}$. If $(X, l_L(\delta))$ is a compact space, then we call (L^X, δ) an ultra-fuzzy compact space [1,2] (*UFCS* for short).

Definition 2.1 Let (L^X, δ) be an L-fts, $P \in \delta'$, $r \in M(L)$ and $x \in X$. If $P \in \eta(x_r)$, then the pair (P, r) is called a relative remote neighborhood (RR, for short) of x. The set $\{(P(i), r(i)): i \in I\} \subset \delta' \times M(L)$ is called a relative remote neighborhood family (*RRF* for short) of X, if for any $x \in X$, there exists $i \in I$ such that $(P(i), r(i))$ is a *RR* of x.

We can describe the ultra-fuzzy compactness of an L-fts with the *RRF*.

Proposition 2.1 *(L^X, δ) is an UFCS iff for any RRF Φ of X, there is a finite subfamily of Φ that is a RRF of X.*

Notice that $\Phi = \{(\mathrm{P}(\mathrm{i}), \mathrm{r}(\mathrm{i})): \mathrm{i} \in \mathrm{I}\}$ is a *RRF* of X iff $\{\mathrm{l}_{(\mathrm{r}(\mathrm{i}))'}(\mathrm{P}(\mathrm{i})'): \mathrm{i} \in \mathrm{I}\}$ is an open covering of $(X, l_L(\delta))$ consisted in $\Gamma(\delta)$. From Alexander Subbase Lemma in crisp topological spaces, Proposition 2.1 can be proved.

Now we give some characterizations of *UFCS* successively.

Definition 2.2 [5] Two fuzzy nets $S = \{s(n): n \in D\}$ and $T = \{t(n): n \in D\}$ in (L^X, δ) are called similar, if $s(n)$ and $t(n)$ possess the same support point for each $n \in D$. $x_\alpha \in M^*(L^X)$ is called a transitive α-cluster point of net S, if x_α is a cluster point of S and for each $c \in M(L)$ the constant c-net that similar to S clusters to x_c.

Lemma 2.1 [4,5] *Suppose that $a \in L$ and B is the minimal set of a. Then $a \in M(L)$ iff B is direct.*

Theorem 2.1 *(L^X, δ) is an UFCS iff any α - net in L^X possesses a transitive α-cluster point for each $\alpha \in M(L)$.*

Proof Let (L^X, δ) be an *UFCS* and $S = \{x^n_{\lambda(n)}: n \in D\}$ be a α-net in L^X, where D is a directed set and $x^n = supp\ x^m_{\lambda(n)}$ for each $n \in D$. Taking $Y = \{x \in X: x^\alpha$ is a cluster point of $S\} \subset X$, then

For each $x \in X \setminus Y$, there exists $P(x) \in \eta^-(x_{r(x)})$ and $n(x) \in D$ such that

$$x^n_{\lambda(n)} \leqslant P(x) \text{ whenever } n \geqslant n(x) \tag{2.1}$$

In which $r(x) \in \beta^*(\alpha)$ such that $P(x) \in \eta(x_{r(x)})$ whenever $P(x) \in \eta(x_\alpha)$. Now asuume that for each $x \in Y$, x_α isn't a transitive cluster point of S. Then there exists $c(x) \in M(L)$ such that $x_{c(x)}$ *isn't a cluster point of the constant* $c(x)$-net $S_{c(x)} = \{x^n_{c(x)}: n \in D\}$ that is similar to S.i.e.

There is $c(x) \in M(L), Q(x) \in \eta^-(x_{c(x)})$ and $n(x) \in D$ such that $x^n_{c(x)} \leqslant Q(x)$

$$\text{whenener} \quad n \geqslant n(x) \tag{2.2}$$

Taking $\Phi = \{(P(x), r(x)): x \in X \setminus Y\} \cup \{(Q(x), c(x)): x \in Y\}$, then Φ is a *RRF* of X. Let Φ_0 is a finite subfamily of Φ which is a *RRF* of X. In this time, $\Phi_0 = \{(P(x_i), r(x_i)) : i = 1, \cdots, k\} \cup \{(Q(x_j), c(x_j)): j = k+1, \cdots, k+m\}$. By Lemma 2.1, we can take $r^* \in \beta^*(\alpha)$ such that $r^* \geqslant r(x_i)(i = 1, \cdots, k)$, then $\Phi_0{}^* = \{(P(x_i), r^*): i = 1, \cdots, k\} \cup \{(Q(x_j), c(x_j)) : j = k+1, \cdots, k+m\}$ is also a *RRF* of X. Notice that S is a α-net, therefore for $r^* \in \beta^*(\alpha)$, we obtain the following result:

There exists $n(r^*) \in D$, when $n \geqslant n(r^*)$, $r^* \leqslant V(x^n_{\lambda(n)}) = \lambda(n)$, i.e.

$$x^n_{r*} \leqslant x^n_{\lambda(n)} \tag{2.3}$$

Taking $N \in D$ such that $N \geqslant n(r^*)$, $N \geqslant n(x_t)(t = 1, \cdots, k+m)$, then for each $n \geqslant N$, from (2.1) ~ (2.3), $x^n_{r*} \leqslant x^n_{\lambda(n)} \leqslant P(x_i)(i = 1, \cdots, k)$ and $x^n_{c(x_j)} \leqslant Q(x_j)(j = k+1, \cdots, k+m)$. Thus when $n \geqslant N$, there is no *RR* of $x^n \in X$. This contradicts that $\Phi_0{}^*$ is a *RRF* of X. Hence S possesses a transitive α-cluster point.

Inversely, suppose that every α-net in L^X possesses a transitive α-cluter point for each $\alpha \in M(L)$ and $\Phi = \{(P(i), r(i)): i \in I\}$ is a *RRF* of x. Assume that all finite subfamily of Φ aren't *RRF* of X.i.e.

For each $\Psi \in 2^{(\Phi)}$, there is $x^\Psi \in X$ such that $x^\Psi_{r(i)} \leqslant P(i)$ for any

$$(P(i), r(i)) \in \Psi \tag{2.4}$$

where $2^{(\Phi)} = \{\Psi \subset \Phi: \Psi \text{ is finite}\}$. Taking $S = \{x^\Psi_\alpha: \Psi \in 2^{(\Phi)}\}$, in which the directed set is $2^{(\Phi)}$, the pair order relation is the containing relation of sets. Then S is a constant α-net in L^X. By the condition, S possesses a transitive α-cluster point y_α. Then there exists $i_0 \in I$ such that $P(i_0) \in \eta^-(y_{r(i_0)})$ from that Φ is a *RRF* of X. On the other hand, for each $\Psi \in 2^{(\Phi)}$, when $\Psi \geqslant \{(P(i_0), r(i_0))\} \in 2^{(\Phi)}$, there exists $x^\Psi \in X$ such that $x^\Psi_{r(i_0)} \leqslant P(i_0)$ by (2.4) since $(P(i_0), r(i_0)) \in \Psi$. But from that y_α is a transitive cluster point of S, $y_{r(i_0)}$ should be a cluster point of constant $r(i_0)$-net $S_{r(i_0)} = \{x^\Psi_{r(i_0)}: \Psi \in 2^{(\Phi)}\}$ that is similar to S. This contradicts that $P(i_0) \in \eta(y_{r(i_0)})$ and $x^\Psi_{r(i_0)} \leqslant P(i_0)$ when $\Psi \geqslant \{(P(i_0), r(i_0))\}$. So there is a finite subfamily of Φ to be a *RRF* of X. Hence (L^X, δ) is an *UFCS* by Proposition 2.1.

Definition 2.3 [4] Let F be a nonempty subsets family in L^X. If (1) $0 \notin F$; (2) $F_1, F_2 \in F$ implies $F_1 \wedge F_2 \in F$, then F is called a filter base in L^X. If F also satisfies that (3) $E \geqslant F \in F$ implies $E \in F$, then F is called a filter in L^X. Let $\alpha \in M(L)$, call filter F in L^X a α-filter, if $\vee_{x \in X} F(x) \geqslant \alpha$ for each $F \in F$; call x_α a α-cluster point of F if $F \not\leqslant P$ for each $F \in F$ and each $P \in \eta(x_\alpha)$.

Let $F = \{F_i: i \in I\}$ be a filter in L^X. Expressing F as $F = \bigcup_{j \in J}\{F_{ij}: i \in I(j)\}$, in which

$\bigcup_{j\in J}I(j) = I$ and supp $F_{ij} = A(j) \subset X(j\in J)$ for each $i\in I(j)$. Then $A_c = \{c\chi_{A(j)} : j\in J\}$ is a filter base in L^X for any $c\in M(L)$. The filter induced by A_c is denoted as F_c.

Definition 2.4 Let F be a filter L^X, $x\in X$ and $\alpha\in M(L)$. Call x_α a transitive α-cluster point of F, if x_α is a α-cluster point of F and F_c cluster to x_c for any $c\in M(L)$.

From the relation between net and filter, it is easy to obtain the characterization of ultra-fuzzy compactness by filters as following.

Theorem 2.2 *(L^X, δ) is an UFCS iff any α - filter in L^X possesses a transitive α-cluster point for each $\alpha\in M(L)$.*

Let (L^X, δ) be an L-fts, then to take $\delta\cup\{[\lambda]:\lambda\in L\}$ as a subbase may product an L-fuzzy topology on X [6] denoted by δ_c and call δ_c the fully stratification of δ, where $[\lambda]$ denotes the LF set with constant value λ on X.

Theorem 2.3 *(L^X, δ) is an UFCS iff (L^X, δ_c) is an UFCS.*

Proof It needs to prove the necessity only. Suppose that (L^X, δ) is an *UFCS* and Φ is a *RRF* in (L^X, δ_c), i.e. for any $y\in X$, there exists $(P(y), r(y))\in\Phi$ such that $r(y)\not< P(y)(y)$, where $P(y)\in\delta'_c$ and $r(y)\in M(L)$. Since δ_c is the fully stratification of δ, δ_c is an L-fuzzy topology on X producted by taking $\{P\vee[\lambda]: P\in\delta'$ and $\lambda\in L\}$ as a closed base. Therefore $P(y) = \bigwedge_{t\in T(y)}(P(t)\vee[\lambda_t])$ in which $P(t)\in\delta'$ and $\lambda_t\in L$ for any $t\in T(y)$. By $r(y)\not< P(y)(y) = \bigwedge_{t\in T(y)}(P(t)\vee[\lambda_t])(y)$, there is $t(y)\in T(y)$ such that $r(y)\not< (P(t(y))\vee[\lambda_{t(y)}])(y)$. Whence $r(y)\not< P(t(y))(y)$ and $r(y)\not<[\lambda_{t(y)}](y)$. Furthermore we have that

$$r(y)\not< P(t(y))(y) \text{ and } r(y)\not<[\lambda_{t(y)}](x) \text{ for each } x\in X \tag{2.5}$$

Let $\Psi = \{(P(t(y)), r(y)) : y\in X\}$, then Ψ is a *RRF* in (L^X, δ) . By the condition, there exists $\Psi_0 = \{(P(t(y_j), r(y_j)) : j = 1,\cdots,n\}\in 2^{(\Psi)}$ to be a *RRF* in (L^X, δ). i. e. for each $x\in X$, there is $j\leqslant n$ such that $r(y_j)\not< P(t(y_j))(x)$. From (2.5), $r(y_j)\not<[\lambda_{t(y_j)}](x)$. Notice that $r(y_j)\in M(L)$. So $r(y_j)\not< P(t(y_j))\vee[\lambda_{t(y_j)}])(x)$, thus $r(y_j)\not<\bigwedge_{t\in T(y)}(P(t)\vee[\lambda_t])(x) = P(y_j)(x)$. Let $\Phi_0 = \{(P(y_j), r(y_j)) : j = 1,\cdots,n\}$, then $\Phi_0\in 2^{(\Phi)}$ is a *RRF* in (L^X, δ_c). So (L^X, δ_c) is an *UFCS* .

3. The product of UFCS

Theorem 3.1 (*Alexander Subbase Lemma*) *Suppose that (L^X, δ) is an L-fts, γ is the subbase of δ and $\Delta = \{(P(s), r(S)) : s\in S\}$ is an arbitrary RRF of X, in which $P(s)\in\gamma'$ for each $s\in S$. If there is a finite subfamily Δ_0 of Δ to be a RRF of X, then (L^X, δ) is an UFCS.*

Proof Let Φ be arbitrary *RRF* of X (it needn't that $\Phi\subset\gamma'\times M(L)$). Assume that for each $\Psi\in 2^{(\Phi)}$, Ψ isn't a *RRF* of X. Taking $H = \{\Omega : \Phi\subset\Omega\subset\delta'$, for each $\Sigma\in 2^{(\Omega)}$, Σ isn't a *RRF* of $X\}$, then $\Phi\in H$. i. e. $H\neq\varphi$. It is obvious that every full order subset in H has an upper bound according to the containing relation of sets. By Zorn Lemma, there is a maximal element Ω_0 in H. It can be obtained that the following results.

(1) Ω_0 is a *RRF* of X.

(2) If $(P,r)\in\Omega_0$ and $P\leqslant Q$ then $(Q,r)\in\Omega_0$.

(3) If $(P\vee Q,r)\in\Omega_0$, then $(P,r)\in\Omega_0$ or $(Q,r)\in\Omega_0$.

where $P,Q\in\delta'$, $r\in M(L)$. From (2) and (3), we have that

If $(P,r)\in\Omega_0$, $P_i\in\delta'(i=1,\cdots,n)$ and $P\leqslant\vee_{i=1}^{n}P_i$, then there exists $i\leqslant n$ such that

$$(P_i,r)\in\Omega_0 \tag{3.1}$$

Now let $\Omega_0=\{(P(t),r(t)):t\in T\}$ and take a subfamily $\Delta=\{(P(s),r(s)):s\in S\}\subset\Omega_0$, where for each $t\in T$, if $P(t)\in\gamma'$ then $(P(t),r(t))\in\Delta$. From the condition of the lemma and $\Omega_0\in H$, we see that Δ is surely not a *RRF* of X. Thus

there exists $x\in X$, such that $x_r(s)\leqslant P(s)$ for each $s\in$ S (3.2)

Now assume that for x stated above, there is $t\in T$ such that $(P(t),r(t))$ is a *RR* of x, i.e. $x_{r(t)}\not\leqslant P(t)$. Then there is $\{P_{ij}:i\in I,j\in J(i)\}\subset\gamma'$ such that $P(t)=\wedge_{i\in I}\vee_{j\in J(i)}P_{ij}$ (for each $i\in I$, $J(i)$ is finite) since $P(t)\in\delta'$ and γ is the subbase of δ. Hence there exists $i\in I$ such that $x_{r(t)}\not\leqslant\vee_{j\in J(i)}P_{ij}$ and $P(t)\leqslant\vee_{j\in J(i)}P_{ij}$. From (3.1), there is $j\in J(i)$ such that $(P_{ij},r(t))\in\Omega_0$. Thus $(P_{ij},r(t))\in\Delta$. But in this time, $x_{r(t)}\not\leqslant P_{ij}$. This contradicts the formula (3.2). So all elements in Ω_0 aren't *RR* of x. But this also contradicts (1). The contradiction makes clear that there is a finite subfamily of Φ to be a *RRF* of X. So (L^X,δ) is an *UFCS* by Proposition 2.1.

Let $\{(L^{X_i},\delta_i):i\in I\}$ be a family of L-fts and (L^X,δ) be the product space. It is easy to prove $l_L(\delta)=\prod_{i\in I}l_L(\delta_i)$, in which $\prod_{i\in I}l_L(\delta_i)$ denotes the product topology of $\{l_L(\delta_i):i\in I\}$. So we have the following result.

Theorem 3.2 *Let $\{(L^{X_i},\delta_i):i\in I\}$ be a family of L-fts, and (L^X,δ) be the product space* [2]. *If (L^{X_i},δ_i) is an UFCS for each $i\in I$, then (L^X,δ) is an UFCS.*

By the way, the proof of Theorem 2.2 is also obtained by Alexander Subbase Lemma and Proposition 2.1.

Theorem 3.3 *Let (L^X,δ) and (L^Y,μ) be both L-fts and $f:(L^X,\delta)\to(L^Y,\mu)$ be a continuous fully L-value Zadeh type function. If (L^X,δ) is an UFCS, then so is (L^Y,μ).*

Proof It is easy to prove by Proposition 2.1.

Theorem 3.4 *(T_{NXOHOB} Product Theorem) Let $\{(L^{X_i},\delta_i):i\in I\}$ be a family of L-fts, and (L^X,δ) be the product space. Then (L^X,δ) is an UFCS iff (L^{X_i},δ_i) is an UFCS for each $i\in I$.*

Proof It is trivial by Theorem 3.2, Theorem 3.3, and the result that projection mapping p_i: $(L^X,\delta)\to(L^{X_i},\delta_i)(i\in I)$ is a continuous full L-value Zadeh type function.

4. Strong locally finite RRF and the ultra-fuzzy paracompactness

In this section, an L-fuzzy ultra-fuzzy paracompactness based on the ultra -fuzzy compactness is introduced. The result make clear that the ultra- fuzzy paracompactness is an L-good extension.

Definition 4.1 [2,7] Let (L^X, δ) be an L-fts and $A = \{A_t : t \in T\} \subset L^X$. If for each $e \in M^*(L^X)$, there exist a crisp closed remote neighborhood P of e and a finite subfamily T_0 of T such that $A_t \leqslant P$ for each $t \in T \setminus T_0$, then call A a strong locally finite in X.

Definition 4.2 Let (L^X, δ) be an L-fts, $\Phi = \{(P(i), r(i)) : i \in I\}$ and $\Psi = \{(Q(j), s(j)) : j \in J\}$ be two *RRF* of X. Call Ψ a corefinement of Φ, if for each $(Q(j), s(j)) \in \Psi$ there exists $(P(i), r(i)) \in \Phi$ such that $Q_j \geqslant P_i$ and $s(j) = r(i)$. Call Ψ a strong locally finite *RRF* of X, if $\{Q(j)' : j \in J\}$ is strong locally finite in X.

Definition 4.3 Let (L^X, δ) be an L-fts. If for each *RRF* Φ of X, there exists a *RRF* Ψ such that Ψ is a corefinement of Φ and a strong locally finite *RRF* of X, then (L^X, δ) is called a Ⅱ-type [2] ultra- fuzzy paracompact space (*UFPCS* for short).

If $\{P(i) : i \in I\}$ is an arbitrary α-*RF*$(\alpha \in M(L))$ in (L^X, δ), then $\{(P(i), \alpha) : i \in I\}$ is a *RRF* of X. So it can be obtained by the definition of Ⅱ - type paracompact in [2] that

Corollary 4.1 *If (L^X, δ) is an UFPCS, then (L^X, δ) is Ⅱ-type paracompact.*

Theorem 4.1 *Being weakly induced space, (L^X, δ) is Ⅱ - type paracompact iff its support space $(X, [\delta])$*[2,6] *is crisp paracompact.*

Corollary 4.2 *In every weakly induced space (L^X, δ), if (L^X, δ) is an UFPCS, then $(X, [\delta])$ is crisp paracompact.*

Theorem 4.2 *Let (L^X, δ) be a weakly induced space. If $(X, [\delta])$ is crisp paracompact, then (L^X, δ) is an UFPCS.*

Proof Let $\Phi = \{(P(i), r(i)) : i \in I\}$ be every *RRF* in (L^X, δ). For each $i \in I$, taking $U(i) = \{x \in X : P(i)(x) \not\geqslant r(i)\}$. Then $U(i) \in l_L(\delta) = [\delta]$ by (L^X, δ) is weakly induced. So $U = \{U(i) : i \in I\}$ is an open covering of $(X, [\delta])$. From the condition, we can suppose that $V = \{V(j) : j \in J\}$ is an open refinement covering of U and locally finite in X. For each $j \in J$, taking $i = i(j) \in I$ such that $V(j) \subset U(i(j))$. Let $B(j) = P(i(j)) \vee \chi_{V(j)'}$. Then $B(j) \in \delta'$ and $B(j) \geqslant P(i(j))$ from $\chi_{V(j)'} \in \delta'$. Let $\Psi = \{(B(j), r(i(j))) : j \in J\}$. Next we shall prove that Ψ is a *RRF* of X which is a corefinement of Φ and strong locally finite in X.

Let $x \in X$ and take $j \in J$ such that $x \in V(j)$. Then $x \in U(i(j))$. Hence $P(i(j)) \not\geqslant r(i(j))$ and $B(j) = P(i(j)) \vee \chi_{V(j)'} \not\geqslant \chi_{r(i(j))}$. Therefore Ψ is a *RRF* of X. It is obvious that Ψ is a corefinement of Φ. Let again $x_\lambda \in M^*(L^X)$, then by V is locally finite in X, there is an open neighborhood W of x such that W and one of $\{V(j_1), \cdots, V(j_k)\} \subset V$ intersect. Let $Q = \chi_{W'}$, then Q is a crisp closed remote neighborhood of x_λ. Now let $j \in J \setminus \{j_1, \cdots, j_k\}$, then $V(j)$ and W don't intersect, i. e. $V(j) \subset W'$. So for each $j \in J \setminus \{j_1, \cdots, j_k\}$, $B'_j = P(i(j))' \wedge \chi_{V(j)} \leqslant \chi_{V(j)} \leqslant \chi_{W'} = Q$. This proved that Ψ is strong locally finite in X. So (L^X, δ) is an *UFPCS*.

Theorem 4.3 *Let (L^X, δ) be weakly induced, then (L^X, δ) is an UFPCS iff $(X, [\delta])$ is crisp paracompact.*

Proof It is obvious from Corollary 4.2 and Theorem 4.2.

Theorem 4.4 *The ultra-fuzzy paracompactness is an L-good extension.*

References

[1] R. Lowen, Acomparison of different compactness notions in fuzzy topological spaces, *J. Math. Anal. Appl.*, 64(1978): 446 ~ 454.

[2] G. J. Wang, Theory of *L*-fuzzy topological spaces, *Press of Shaanxi Normal University*, 1988 (in Chinese).

[3] G. J. Wang, A new fuzzy compactness defined by fuzzy nets, *J. Math. Anal. Appl.*, 94(1983): 1 ~ 23.

[4] D. S. Zhao, The *N*-compactness in *L*-fuzzy topological spaces, *ibid.*, 128(1987): 64 ~ 79.

[5] Y. W. Peng, The *N*-compactness in *L*-fuzzy topological spaces, *Acta. Math. Sinica*, 29(1986): 555 ~ 558 (in Chinese).

[6] X. Q. Xu, The graduated structure of *N*- compact sets and fuzzy Wallace Theorem, *Kexue Tongbao*, 14(1989): 1052 ~ 1054 (in Chinese).

[7] J. L. Fan, The paracompactness and strong paracompactness in *L*- fuzzy topological spaces, *J. Shaanxi Normal University*, 3(1989) (in Chinese).