# Fuzzy Logic Explains the Golden Proportion

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**Abstract**

It is known that aesthetically pleasing works of art, music, etc. contain a ratio $\phi = (\sqrt{5}-1)/2$ known as the *golden proportion*. There have been many attempts to understand why this ratio is pleasing; however, there is no convincing and universally accepted explanation. In this paper, we provide an explanation based on fuzzy logic.

## 1 Introduction

Starting with the ancient Greeks, it has been known that one of the most aesthetically pleasing rectangles has a ratio of $(\sqrt{5}-1)/2$ ($\approx 0.618/1$) between its sides [7]. This ratio, called the *golden ratio* or a *golden proportion*, has been studied by the Pythagoreans [2] and used by many Greek architects and sculptors, including Phidias, the chief sculptor of the Parthenon. In honor of Phidias, this ratio is denoted in mathematics by $\phi$, the first letter of his name ([10], Chapter 1).

Mathematically, the golden ratio means that we divide a unit segment into two parts of lengths $x$ and $1-x$ so that the following two ratios coincide:

- the ratio $(1-x)/x$ of the smaller part to the larger one; and
- the ratio $x/1$ of the larger part to the entire segment.

The equality between these ratios is equivalent to

$$x^2 = 1 - x,$$

and this quadratic equation has only one root on the interval $[0, 1]$:

$$x = (\sqrt{5}-1)/2 = \phi - 1 \approx 0.618.$$

In the 13th century, interest in the golden ratio was revived by Fibonacci, who showed that this ratio appears in plants, leaves, mollusks, and other living creatures. With the Renaissance came a renewed interest in the golden ratio. In 1478, Piero della Francesca mentioned the golden ratio as one of the main techniques in painting. In 1509, Luca Pacioli published a book called *De divina proportione* (The Divine Proportion) about this ratio. This book was illustrated by Leonardo da Vinci (for a detailed history see [2, 7]).

The golden proportion was successfully used by architects, painters, musicians, cinematographers, etc. [7, 9]. Modern artists still consider the golden ratio to be aesthetically pleasing.

Many people have tried to explain why this is so, but there has not yet been a successful explanation. In this paper, we provide an explanation based on fuzzy logic.

# 2 The Golden Proportion and Computers

In several computer algorithms, the golden proportion shows up naturally in the most optimal cases of the algorithm. The major two examples are:

- The golden proportion is used as a basis of one of the most efficient *hashing* algorithms, when an item with a key equal to $k$ is placed in slot number $n \approx N(k\phi - \lfloor k\phi \rfloor)$ of an $N$–slot table ([4], Section 6.4; [6], Section 8.5).
- In the optimal algorithm for finding the *largest common factor* of two positive integers $m$ and $n$, the worst case computational complexity (number of iterations) is equal to $\lceil -\log_\phi(\sqrt{5}n) \rceil - 2$.

However, these results do not lead to a direct explanation of why exactly the golden proportion is aesthetically pleasing.

# 3 Explanation in Terms of Fuzzy Logic

## 3.1 What is Pleasing? Examples

When is something in life pleasing? Let's give a few examples:

- If you put a little bit of sugar in your coffee, it will taste better (at least to many coffee drinkers). If you put a little more sugar, it will taste even better. However, if you put too much sugar in your coffee, the result will be the opposite: your coffee will taste awful. The "optimal," the most pleasing, amount of sugar can be described as the amount that, if it is increased, will create an opposite effect.
- Another example is perfume. If someone puts a little bit of nice perfume on, it will increase that person's attractiveness. However, an overdose of perfume doesn't smell good at all.

- If someone with a headache takes a little bit of aspirin, he will feel better; but an overdose of aspirin can make a person even sicker.
- A certain amount of parental love makes the kids' lives better. However, too much parental love (e.g., worrying about the kids all of the time) makes the kids' lives a nightmare. It can even lead to psychological problems [1].

### 3.2 Summarizing the Examples

Summarizing these examples, we can formulate the condition for the degree $x$ in which something is most pleasing: when further increase in this degree leads to an opposite effect. Formally, this condition can be described as follows:

$$\text{"very" } x = \text{"not" } x.$$

### 3.3 Formalization in Fuzzy Logic

To formalize "very" and "not", it is natural to use fuzzy logic [3, 8], where:

- "very" $x$ is typically interpreted as $x^2$, and
- "not" $x$ is usually interpreted as $1 - x$.

*Historical comment.* The interpretation of "very" as $x^2$ was originally proposed by L.A. Zadeh in his pioneer paper [11]; the experimental results of [5] turned out to be consistent with this interpretation of "very".

If we use these interpretations in the above formula, we get the equation

$$x^2 = 1 - x.$$

This is exactly the *equation for the golden proportion.*

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