Some convergence theorem for sequences of integrals of B-function on the fuzzy set

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Abstract: In this paper, some convergence theorems for sequences of integrals of real-valued B-functions with respect to real-valued fuzzy measure on the fuzzy set are disscussed.

Keywords: Fuzzy set, fuzzy measure, B-function, m-integral.

1. Introduction

Dan [1-3] introduced some particularly additive operations with fuzzy sets,given the integral of the real-valued B-function with respect to real-valued fuzzy measure on the fuzzy set,and obtained the monotone increasing convergence,the Fatou's lemma, and the control convergence theorem for the sequence of intergrals of B-functions on the fuzzy set. Zhang [4-5] introduced the fuzzy number-valued measure and the fuzzy numbere-valued fuzzy intergral on the fuzzy set, given a series of convergence theorems for the sequence of fuzzy number-valued integrals on the fuzzy set.

In this paper,we go right to discuss convergencity for the sequence of integrals of B-funotions on the basis of the Dan [1- 3] , obtain the decreasing convergence theorem,Fatou's lemma,the everywhere convergence theorem,the almost everywhere convergence and so on. The theory of integrals of B-functions on the fuzzy set is perfected.

The paper is divided into three sections.In Section 2,we recall some elementary definitions in [1-3].In Section 3 , we show a series of important convergence theorems for sequences of B- functions on the fuzzy set.

Throughout this paper,let X be a nonempty set,$F(X)=\{A;A:X\to[0,1]\}$ be a the class of fuzzy sets.All concepts and signs are not explained this paper may be founed in [1-3].We also make the convention $0\cdot\infty=0$.

2. Preliminaries

Definition2.1 Let A and B be twe fuzzy sets:

(a) The sum of A and B is the fuzzy set $A\oplus B$ defined by

$(A\oplus B)(x)=\min(1, A(x)+B(x))\quad (x\in X)$.

(b) The difference of A and B is the fuzzy set $A\ominus B$ defined by

$(A\ominus B)(x)=\mathrm{man}(0, A(x)-B(x))\quad (x\in X)$.

(c) The conjunction of A and B is the fuzzy set A&B defined by

$(A\&B)(x)=\max(0, A(x)+B(x)-1)\quad (x\in X)$.

(D) The product of A and B is the fuzzy set $A\cdot B$ defined by

$(A\cdot B)(x)=A(x)\cdot B(x)\quad (x\in X)$.

Definition2.2 Let $F\subset F(X)$ be a σ-additive class of fuzzy sets, a fuzzy measure on F is a set funotion $m: F\rightarrow\bar{R}_+$ with the properties:

(1) $m(X)=0$;

(2) If $(A_n)\subset F$ is a disjoint sequence, then

$m(\bigoplus_{n=1}^{\infty}A_n)=\sum_{n=1}^{\infty}m(A_n)$.

Definition2.3 We say that f is a B-function on A iff there exists a sequence $(s_n)_{n\in N}$ of nonnegative dominions of f on A so that

$s_{n+1}\geqslant_A s_n (n\in N)$ and $\lim_{n\to\infty} s_n(x)\cdot A(x)=f(x)\cdot A(x)\ (\forall x\in X)$. (*)

If f is a B-function on X, we say that f is a B-function.

Definition2.4 Let $f: X\rightarrow\bar{R}_+$ be a B-function on $A\in F$. If $(s_n)_{n\in N}\subset B_+(f, A)$ so that (*) holds, then we denote

$$\int_A f\ dm=\lim_{n\to\infty}\int_A s_n\ dm$$

and we call it m-intergal of f on A. If $\int_A f\ dm<+\infty$, then we say that f is m-integrable on A.

Definition2.5 We say that f is a B-function on $A\in F$ iff f_+ and f_- are B-function on A. If f+ and f_- are m-integrable B-functions on A, then f is said to be m-integrable on A. And the real number

$$\int_A f\ dm=\int_A f_+\ dm-\int_A f_-\ dm$$

is called m-integral of f on A.

Definition2.6 Let $A\in F$, P is a proposition and m be a fuzzy measure.

(a) If $X_{\mathrm{suppA}}\in F$, such that P is true on suppA, then we say "P is every where true on A".

(b) If there exists a $E \in F$, $E \subset A$ with $m(E)=0$, such that P is true on A $\ominus$E, then we say "P is almost everywhere true on A."

We denote 'almost everywhere' by 'a.e.'

3. Convergence theorems for sequence of integrals of B-function on the fuzzy set

Theorem3.1 Let $f_n: X \to \bar{R}_+ (n \in N)$ be B-functions, $A \in F$, $f_n \downarrow f (x \in X)$. If f is m-integral on A, then $\lim_{n\to\infty} \int_A f_n\, dm$ exists, and

$$\int_A f\, dm = \lim_{n\to\infty} \int_A f_n\, dm.$$

Theorem3.2 (Fatou's Lemma) Let $f_n: X \to \bar{R}_+ (n \in N)$ be B-functions, $A \in F$, $f = \lim_{n\to\infty} f_n (x \in X)$, $A \in F$. If f is m-integrable on A, then $\lim_{n\to\infty} \sup \int_A f_n\, dm$ exists, and

$$\int_A f\, dm \geqslant \lim_{n\to\infty} \sup \int_A f_n\, dm.$$

Theorem3.3 Let $f_n: X \to \bar{R} (n \in N)$ be B-funotions, $f = \lim_{n\to\infty} f_n (x \in X)$, $A \in F$. If f is m-integrable on A, then $\lim_{n\to\infty} \int_A f_n\, dm$ exists, and

$$\int_A f\, dm = \lim_{n\to\infty} \int_A f_n\, dm.$$

Lemma3.1 Let $f: X \to \bar{R}$ be a B-function. If f is m-integrable on $A \oplus B$, and $A \& B = \phi$, $A, B \in F$, then

$$\int_{A \oplus B} f\, dm = \int_A f\, dm + \int_B f\, dm.$$

Lemma3.2 Let $f, g: X \to \bar{R}$ be two B-functions, and $f=g$ a.e., $A \in F$, then

$$\int_A f\, dm = \int_A g\, dm.$$

Therem3.4 Let $f_n: X \to \bar{R} (n \in N)$ be B-functions, $f = \lim_{n\to\infty} f_n$ a.e. $(x \in X)$, $A \in F$. If f is m-integrable on A, then $\lim_{n\to\infty} \int_A f_n\, dm$ exists, and

$$\int_A f\, dm = \lim_{n\to\infty} \int_A f_n\, dm.$$

Definition3.1 (F-aean) Let $f_n: X \to \bar{R} (n \in N)$ be B-functions, $A \in F$, then $\{f_n\}$ is said to F-mean converge to an a.e. finite B-function f, if

$$\lim_{n\to\infty} \int_A |f_n|\, dm = 0.$$

Theorem3.5 Let $f_n: X \to \overline{R} (n \in N)$ be B-functions, $A \in F$. If $\{f_n\}$ F-mean converge to f, and f is m-integrable on A, then $\lim_{n\to\infty} \int_A f_n \, dm$ exists, and

$$\int_A f \, dm = \lim_{n\to\infty} \int_A f_n \, dm.$$

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