ANOTHER PROOF OF FUZZY POSYNOMIAL GEOMETRIC PROGRAMMING DUAL THEOREM

Cao Bing－yuan

(Changsha University of Electric Power, 410077, P. R. CHINA)

Abstract Based on the theorem of type fuzzy Kuhn－Tucker of fuzzy convex programming, a dual theorem is obtained for fuzzy posynomial geometric programming. Another proof is given in this paper before the algorithm is obtained for a dual programming.

Keywords Fuzzy posynomial, Geometric programming, Dual theorem, Proof.

Let a prime fuzzy posynomial geometric programming be

$$\begin{aligned} &\min \ \tilde{g}_0(x), \\ &\text{s.t.} \ \tilde{g}_m(x) \lesssim 1, \ (m=1, 2, \cdots, M) \\ &\quad \tilde{x} > 0 \end{aligned} \tag{1}$$

and a nonnegative T－vector be given tallying with $\sum_{t=1}^{T_m} \varepsilon_{mt} = 1$ $(m=0, 1, \cdots, M)$, $\varepsilon = (\varepsilon_{01}, \cdots, \varepsilon_{0T_0}, \cdots, \varepsilon_{M1}, \cdots, \varepsilon_{MT_M})^T \geqslant 0$. Let

$$\bar{g}_m(x) = \sum_{t=1}^{T_m} \left(\frac{\tilde{v}_{mt}(x)}{\varepsilon_{mt}}\right)^{\varepsilon_{mt}} = \sum_{t=1}^{T_m} \left(\frac{\tilde{c}_{mt}}{\varepsilon_{mt}} \prod_{n=1}^{N} x_n^{\gamma_{mtn}}\right)^{\varepsilon_{mt}} = \bar{\tilde{c}}_m \prod_{n=1}^{N} x_n^{\bar{\tilde{\gamma}}_{mn}} \tag{2}$$

where $\bar{\tilde{c}}_m = \prod_{t=1}^{T_m} \left(\frac{\tilde{c}_{mt}}{\varepsilon_{mt}}\right)^{\varepsilon_{mt}}$; $\bar{\gamma}_{mn} = \sum_{t=1}^{T_m} \tilde{\gamma}_{mtn} \varepsilon_{mt}$.

If $\bar{\tilde{g}}_m(x)$ is used instead of $\tilde{g}_m(x)$ (m=0, 1, ⋯ , M), then a single fuzzy geometric programming is obtained as follows

$$(\tilde{P}_\varepsilon) \qquad \begin{aligned} &\min \ \bar{g}_0(x) \\ &\text{s.t.} \ \bar{\tilde{g}}_m(x) \leqslant 1, \ (m=1, 2, \cdots, M) \\ &\quad \tilde{x} > 0 \end{aligned}$$

while the dual form of (1) is

$$\begin{aligned} &\widetilde{\max} \ \tilde{d}(\omega) \\ &\text{s.t.} \ \tilde{\Gamma}^T\omega = 0, \ \alpha, \beta \in [0, 1] \\ &\quad \omega \geqslant 0 \end{aligned} \tag{3}$$

where $\tilde{d}(\omega) = (\tilde{a}_0/\omega_{00})^{\omega_{00}} \prod_{t=1}^{T_0} (\tilde{c}_{0t}\tilde{a}_{0t}/\omega_{0t})^{\omega_{0t}} \prod_{m=1}^{M} \prod_{t=1}^{T_m} (\tilde{c}_{mt}\tilde{a}_{mt}/\omega_{mt})^{\omega_{mt}} \omega_{m0}^{\omega_{m0}}$.

Definition 1 $\tilde{M}_P$ and $\tilde{M}_D$ denote the constraint least upper and greatest lower bounds of (1) and (3) respectively, i.e.

$\tilde{M}_P = \inf \ \tilde{g}_0(x)$ $\qquad$ $\tilde{M}_D = \sup \ \tilde{d}(\omega)$

s.t. $\tilde{g}_m(x) \leqslant 1$ $(m=1, \cdots, M)$, $\qquad$ s.t. $\omega_{00} = 1$, $\sum_{t=1}^{T_0} \tilde{\gamma}_{0tn} \omega_{0t} = \omega_{00}$ $(n=1, \cdots, N)$, Lemma 1 If for continuous nondecreasing functions (CMNF) $\tilde{B}$, $\tilde{c}$ and $\tilde{\gamma}$ with $\tilde{g}_0(x^*)=\tilde{d}(x^*)$, then x^* and ω^* must be an optimal solution of (1) and (3) respectively, and for any fuzzy feasible solution x in [3], we have

$$\tilde{g}_0(x^*)\prod_{m=0}^{M}\prod_{t=1}^{T_m}(\omega_{mt}^*)^{\omega_m}=\prod_{m=1}^{M}(\omega_{m0}^*)^{\omega_{m0}}\prod_{m=0}^{M}\tilde{c}_{mt}^{\omega_{mt}} \tag{4}$$

Proof For any fuzzy feasible solution x of (1), from Lemma 4.1 in [2], we have

$$\tilde{g}_0(x)\geqslant\tilde{d}(\omega^*)=\tilde{g}_0(x^*).$$

Therefore x^* must be a fuzzy optimal solution of (1). We can prove in the similar way that x^* and ω^* must tally with

$$\omega_{mt}=\begin{cases}\tilde{v}_{0t}(x)/\tilde{g}_0(x), & (m=0,\ t=1,\cdots,T_0),\\ \omega_{m0}v_{mt}(x), & (m\neq 0;\ t=1,\cdots,T_m).\end{cases}$$

Therefore $(\tilde{g}_0(x^*)\omega_{0t}^*)=(\tilde{v}_{0t}(x^*))^{\omega_{0t}}$, $(\omega_{mt}^*)^{\omega_{mt}}=(\omega_{m0}^*\,\tilde{v}_{mt}(x^*))^{\omega_{mt}}(m\neq 0)$, from $\omega_{00}=1$, we have

$$\tilde{g}_0(x^*)\prod_{m=0}^{M}\prod_{t=1}^{T_m}(\omega_{mt}^*)^{\omega_{mt}}=\prod_{m=1}^{M}(\omega_{m0}^*)^{\omega_{m0}}\prod_{m=0}^{M}\prod_{t=1}^{T_m}(\tilde{v}_m(x^*))^{\omega_{mt}}$$

$$=\prod_{m=1}^{M}(\omega_{m0}^*)^{\omega_{m0}}\prod_{m=0}^{M}\prod_{t=1}^{T_m}\tilde{c}_{mt}^{\omega_{mt}}\prod_{n=1}^{N}x_n^{\sum_{m=0}^{M}\sum_{t=1}^{T_m}\tilde{\gamma}_{mtn}\omega_{mt}}\left(\tilde{\Gamma}^T\omega=\sum_{m=0}^{M}\sum_{t=1}^{T_m}\tilde{\gamma}_{mtn}\omega_{mt}\right),$$

and from orthogonality condition, Eg. (4) is know to be true .

Lemma 2 For a CMNF $\tilde{B}$, $\tilde{c}$ and $\tilde{\gamma}$, when (1) and (3) are both fuzzy consistent, there must be

$$0<\tilde{M}_D\leqslant\tilde{M}_P<\infty .$$

Proof Since (3) is fuzzy consistent, $\tilde{M}_D>0$, and (1) fuzzy consistent, $\tilde{M}_P<\infty$

$$\tilde{g}_0(x)\geqslant\tilde{g}_0(x)\prod_{m=1}^{M}(\tilde{g}_m(x))^{\omega_m}\geqslant\tilde{d}(\omega).$$ [1] [2]

Therefore, the lemma holds.

Theorem 1 Let $\tilde{B}$, $\tilde{c}$ and $\tilde{\gamma}$ be a CMNF. If x is fuzzy feasible solution of (1), it must be that of a single posynomial fuzzy geometric programming $(\tilde{P}_s)$. If (1) is fuzzy consistent, so is $(\tilde{P}_s)$. Again $M_{\tilde{P}_s}\leqslant\tilde{M}_P$, here $M_{\tilde{P}_s}$ and $\tilde{M}_P$ denote the constrained greatest lower bound of (1) and $(\tilde{P}_s)$ respectively. Then, the dual form of $(\tilde{P}_s)$ is

$$(\tilde{D}_s)\qquad \max\,(\tilde{a}_0'/\omega_0')^{\omega_0'}\prod_{m=0}^{M}(\tilde{c}_m\,\tilde{a}_m/\omega_m)^{\omega_m}$$

$$\text{s.t. } \omega_0'=1,\ \bar{\gamma}_0^*\,\tilde{\omega}_0=\omega_0',\qquad \tilde{\bar{\gamma}}_0^*\ \text{is an index of}\ \tilde{g}_0(x).$$

$$\sum_{m=1}^{M}\bar{\gamma}_{mn}\omega_m=0,\ (n=1,\cdots,N),\ \alpha,\ \beta\in[0,1],$$

$$\omega\geqslant 0.$$

Proof If we use $\tilde{\bar{g}}_m(x)$ instead of $\tilde{g}_m(x)$ $(m=0,1,\cdots,M)$, then (1) is turned into $(\tilde{P}_s)$. Owing to $\left(\sum_{t=1}^{T}\tilde{v}_t\right)^{\varepsilon_t}\geqslant\prod_{t=1}^{T}(\tilde{v}_t/\omega_t)^{\omega_t}\,\omega_0^{\omega_0}$ with $\omega_t/\omega_0=\varepsilon_{mt}$, we use ε_{mt} as weight vector, from

$$\Longrightarrow \prod_{t=1}^{T_m} (\underset{\sim}{v}_{mt}(x)/\varepsilon_{mt}\omega_0)^{\varepsilon_{mt}} = \prod_{t=1}^{T_m} (\underset{\sim}{v}_{mt}(x)/\varepsilon_{mt})^{\varepsilon_{mt}} = \underset{\sim}{\bar{c}}_m \prod_{n=1}^{N} x_n^{\bar{\gamma}_{mn}} \leqslant \sum_{t=1}^{T_m} \underset{\sim}{v}_{mt}(x).$$

Obviously, $\underset{\sim}{\bar{g}}_m(x) \leqslant \underset{\sim}{g}_m(x)$, such that a fuzzy feasible solution is that of $(\underset{\sim}{P}_t)$.

$$(\underset{\sim}{P}_t) \Longleftrightarrow \begin{array}{l} \widetilde{\min}\ \underset{\sim}{\bar{c}}_0 \prod_{n=1}^{N} x_n^{\bar{\gamma}_{0n}} \\ \text{s.t. } \underset{\sim}{\bar{c}}_m \prod_{n=1}^{N} x_n^{\bar{\gamma}_{mn}} \underset{\sim}{\leq} 1 \\ x>0 \end{array} \Longleftrightarrow \begin{array}{l} \max\ ((1-\alpha)/\bar{\omega}_0')^{\bar{\omega}_0'} \prod_{m=0}^{N} (\underset{\sim}{c}_m^{-1}(\beta)\underset{\sim}{B}_m^{-1}(\alpha)/\bar{\omega}_m)^{\bar{\omega}_m} \\ \text{s.t. } \bar{\omega}_0' = 1,\ \underset{\sim}{\bar{\gamma}}_0^{*-1}(\beta)\bar{\omega}_0 = \bar{\omega}_0',\ \alpha, \beta \in [0, 1] \\ \sum_{m=0}^{M} \underset{\sim}{\bar{\gamma}}_{mn}^{-1}(\beta)\bar{\omega}_m = 0,\ \bar{\omega} \geqslant 0 (n=1, \cdots, N) \end{array} \Longleftrightarrow (\underset{\sim}{\bar{D}}_t)$$

where $\bar{\omega} = (\bar{\omega}_0, \bar{\omega}_1, \cdots, \bar{\omega}_M)^T$ is (m+1)−dimension dual parameter vector.

Theorem 2 Let $\underset{\sim}{B}_m, \underset{\sim}{B}_m^{-1}$; $\underset{\sim}{c}_m, \underset{\sim}{c}_m^{-1}$ and $\underset{\sim}{\gamma}_m, \underset{\sim}{\gamma}_m^{-1}$ be continuous and strictly monotonous. If $(\underset{\sim}{P}_t)$ is fuzzy consistent with $M_{\underset{\sim}{P}_t} > 0$, then there must exist dual feasible solution ω', such that weight vector ε,

$$\varepsilon_{mt}\omega'_{m0} = \omega'_{mt}\ (m=0, 1, \cdots, M;\ t=1, 2, \cdots, T_m) \tag{5}$$

with $M_{\underset{\sim}{P}_t} = \underset{\sim}{d}(\omega')$.

Conversely, if dual fuzzy feasible solution ω' makes (5) hold, $M_{\underset{\sim}{P}_t} > 0$ must exist.

Proof Owing to $(\underset{\sim}{P}_t) \Longleftrightarrow$

$$\begin{array}{l} \max (1-\alpha) \\ \text{s.t. } \underset{\sim}{B}_m(\underset{\sim}{\bar{c}}_m^{-1}(\beta)\prod_{n=1}^{N} x_n^{\bar{\gamma}_{mn}^{-1}(\beta)}) \geqslant \alpha,\ x>0, \\ \alpha, \beta \in [0, 1],\ m=0, 1, \cdots, M \end{array} \Longleftrightarrow \begin{array}{l} \max (1-\alpha) \\ \text{s.t. } \underset{\sim}{\bar{c}}_m^{-1}(\beta)\, \underset{\sim}{B}_m^{-1}(\alpha)\prod_{n=1}^{N} x_n^{\bar{\gamma}_{mn}^{-1}(\beta)} \leqslant 1,\ x>0, \\ \alpha, \beta \in [0, 1],\ m=0, 1, \cdots, M \end{array} \quad (\bar{P}_t)$$

$$\overset{Z_n = \log x_n}{\Longleftrightarrow} \begin{array}{l} \max\ (1-\alpha) \\ \text{s.t. } \sum_{n=1}^{N} \bar{\gamma}_{mn}^{-1}(\beta)\ z_n \leqslant \log(\underset{\sim}{\bar{c}}_m^{-1}\ \underset{\sim}{B}_m^{-1}(\alpha)) \\ (m=0, 1, \cdots, M),\ \alpha, \beta \in [0, 1]. \end{array}$$

If Let $M_{\bar{P}_t} > 0$ as the proof in Theorem 1.7.2 in [3], then its linear programming $(\bar{P})$ has the greatest lower bound $\log M_{\bar{P}_t}$, hence, from dual theory of an ordinary linear programming, a linear one corresponding to $(\underset{\sim}{D}_t)$:

$$\begin{array}{l} \max\{\bar{\omega}_0' \log(1-\alpha) + \sum_{m=0}^{M} \bar{\omega}_m \log \underset{\sim}{\bar{c}}_m^{-1}(\beta)\underset{\sim}{B}_m^{-1}(\alpha)\} \\ \text{s.t. } \bar{\omega}_0' = 1,\ \underset{\sim}{\bar{\gamma}}_0^{-1}(\beta)\bar{\omega}_0' = 1 \\ \sum_{m=0}^{M} \underset{\sim}{\bar{\gamma}}_{mn}^{-1}(\beta)\bar{\omega}_m = 0,\ \alpha, \beta \in [0, 1],\ (n=1, 2, \cdots, N) \end{array} \tag{6}$$

has the greatest value $\log M_{\bar{P}_t}$. Moreover $(\underset{\sim}{\bar{D}}_t)$ has fuzzy biggest value $\underset{\sim}{M}_{P_t}$. Now let $\bar{\omega}^{*\prime}$ be an optimal solution of $(\underset{\sim}{\bar{D}}_t)$ and (6), then

$$M_{\bar{P}_t} = ((1-\alpha)/\bar{\varepsilon}_{0t})^{\bar{\varepsilon}_{0t}\bar{\omega}_0'} \prod_{m=0}^{M}\prod_{t=1}^{T_m} (c_{mt}^{-1}(\beta)\underset{\sim}{B}_m^{-1}(\alpha)/\bar{\varepsilon}_{mt})^{\bar{\varepsilon}_{mt}\bar{\omega}_m'}$$

and from

$$\bar{\varepsilon}_{0t} = \bar{\omega}_{0t}^{*\prime} / \bar{\omega}_0',\quad \bar{\varepsilon}_{mt} = \bar{\omega}_{mt}^{*} / \bar{\omega}_m',\quad \bar{\omega}_{m0}^{*} = \sum_{t=1}^{T_m} \bar{\omega}_{mt}^{*} = \sum_{t=1}^{T_m} \bar{\varepsilon}_{mt}\bar{\omega}_m' = \bar{\omega}_m' \tag{7}$$

Therefore $M_{\overline{P}_s}=((1-\alpha)\overline{\omega}^*_{00}/\overline{\omega}'_{0})^{\overline{\omega}'_0}\prod_{m=0}^{M}\prod_{t=1}^{T_m}(\underset{\sim}{c}^{-1}_{mt}(\beta)B^{-1}_m(\alpha)\,\overline{\omega}^*_{m0}/\overline{\omega}^*_{mt})^{\overline{\omega}^*_{mt}}=d(\overline{\omega}^*)$ (8)

Since $\overline{\omega}^*$ is a dual feasible solution of $(\overline{D}_s)$, and from Eq. (2) and (7), we have

$$\sum_{m=0}^{M}\sum_{t=1}^{T_m}\underset{\sim}{\gamma}^{-1}_{mtn}(\beta)\,\overline{\omega}^*_{mt}=\sum_{m=0}^{M}\sum_{t=1}^{T_m}\underset{\sim}{\gamma}^{-1}_{mtn}(\beta)\,\overline{\varepsilon}_{mt}\overline{\omega}'_m=\sum_{m=0}^{M}\underset{\sim}{\gamma}^{-1}_{mt}(\beta)\,\overline{\omega}'_m=0.$$

Again since $\overline{\omega}'\geqslant 0$, $\overline{\varepsilon}\geqslant 0$, then $\overline{\omega}^*\geqslant 0$ with $\overline{\omega}^*_{00}=\overline{\omega}'_0=1$. Hence $\overline{\omega}^*$ is certainly a feasible solution of $(\overline{D}_s)$, moreover ω^* is a fuzzy feasible one of (3).

Conversely, let $\overline{\omega}^*$ be a dual feasible solution which makes (7) hold, and define $\overline{\omega}'_m=\overline{\omega}^*_{m0}$ $(m=0, 1, \cdots, M)$, we have $\overline{\omega}'_0=1$ and $\overline{\omega}'_0\geqslant 0$ with

$$\sum_{m=0}^{M}\gamma^{-1}_{mn}(\beta)\,\overline{\omega}'_m=\sum_{m=0}^{M}\sum_{t=1}^{T_m}\underset{\sim}{\gamma}^{-1}_{mtn}(\beta)\,\overline{\varepsilon}_{mt}\,\overline{\omega}'_m=\sum_{m=0}^{M}\sum_{t=1}^{T_m}\underset{\sim}{\gamma}^{-1}_{mt}(\beta)\,\overline{\omega}^*_{mt}.$$

Therefore, $\overline{\omega}'$ is a feasible solution of (D_s) from Theorem 1.7.2 in [3], and $M_{P_s}>0$.

Since $(\underset{\sim}{P}_s)$ consistence $\Longleftrightarrow$ $(\overline{P}_s)$ consistence $\Longleftrightarrow$ $(\overline{D}_s)$ consistence. While

$$(7)\ \Longleftrightarrow\ \underset{\sim}{B}(\overline{\varepsilon}_{mt}\,\overline{\omega}^*_{m0})=\underset{\sim}{B}(\overline{\omega}^*_{mt}),\ \text{i.e. } \varepsilon_{mt}\omega^*_{m0}=\omega^*_{mt},$$

$$(8)\ \Longleftrightarrow\ \underset{\sim}{B}(M_{\overline{P}_s})=\underset{\sim}{B}(d(\overline{\omega}^*_{mt})),\ \text{i.e. } M_{\underset{\sim}{P}_s}=\underset{\sim}{d}(\omega^*),$$

$$(m=0, 1, \cdots, M;\ t=1, 2, \cdots, T_m).$$

$$M_{\overline{P}_s}>0\ \Longleftrightarrow\ M_{\underset{\sim}{P}_s}>0.$$

Therefore, the theorem holds.

Theorem 3 Suppose (1) is fuzzy super consistent with a mininal point x^*, when $\underset{\sim}{B}$, $\underset{\sim}{c}$ and $\underset{\sim}{\gamma}$ are continuous and strictly monotonous, x^* is a minimal one of single posynomial fuzzy geometric programming $(\underset{\sim}{P}_s)$ corresponding to the weight

$$\varepsilon_{mt}=\underset{\sim}{v}_{mt}(x^*)/\underset{\sim}{g}_m(x^*)(m=0, 1, \cdots, M;\ t=1, 2, \cdots, T_m) \tag{9}$$

and $M_{\underset{\sim}{P}_s}=M_{\underset{\sim}{P}}$.

Proof Because of that and according to

$$(1)\ \Longleftrightarrow\ \begin{aligned}&\max(1-\alpha)\\ &\text{s.t.}\sum_{t=1}^{T_m}\underset{\sim}{c}^{-1}_{mt}(\beta)\underset{\sim}{B}^{-1}_m(\alpha)\prod_{n=1}^{N}x_n^{\gamma^{-1}_{mtn}(\beta)}\leqslant 1\\ &x>0,\ \alpha,\ \beta\in[0, 1]\ (m=0, 1, \cdots, M)\end{aligned} \tag{10}$$

we know super consistence (10) has a minimal point $\overline{x}^*$. And from the knowledge of Theorem 1.7.3 in [3], a minimal one $\overline{x}^*$ exists in single posynomial geometric programming $(\overline{P}_s)$ corresponding to weight $\varepsilon_{mt}=v_{mt}(\overline{x}^*)/g_m(\overline{x}^*)$ $(m=0, 1, \cdots, M;\ t=1, 2, \cdots, T_m)$, and $M_{\overline{P}_s}=M_{\overline{P}}$, such that

$$\varepsilon_{mt}=\underset{\sim}{v}_{mt}(x^*)/\underset{\sim}{g}_m(x^*),\ M_{\underset{\sim}{P}_s}=M_{\underset{\sim}{P}}.$$

Hence the theorem is true.

Theorem 4 Suppose (1) is a fuzzy super consistence with a fuzzy optimal solution x^*, then there must exist Lagrange multiplier $\mu^*=(\mu^*_1, \mu^*_2, \cdots, \mu^*_M)^T\geqslant 0$ with continuous and strictly nondecreasing functions $\underset{\sim}{B}$, $\underset{\sim}{c}$ and $\underset{\sim}{\gamma}$, such that

$$\nabla g_0(x^*)+\sum^{M}\mu^*_m\ \nabla g_m(x^*)=0$$

while ω^* defined by

$$\omega^*_{mt}=\begin{cases}\tilde{v}_{0t}(x^*)/\tilde{g}_0(x^*) & (m=1,2,\cdots,M)\\ \mu^*_m \tilde{v}_{mt}(x^*)/\tilde{g}_0(x^*) & (m\neq 0,\ t=1,2,\cdots,T_m)\end{cases}$$

is a dual fuzzy optimal solution of (3). And

$$\tilde{g}_0(x^*)=\tilde{d}(\omega^*).$$

Proof First, from Eq. (9), and the knowledge of Theorem 3, a minimal point corresponding to $(\tilde{P}_t)$ is x^* with a minimal value being $\bar{\tilde{g}}_0(x^*)=\tilde{g}_0(x^*)>0$; but from Theorem 2, $\exists$: ω^* tallying with (5), such that

$$\tilde{d}(\omega^*)=\bar{\tilde{g}}_0(x^*)=\tilde{g}_0(x^*).$$

From Theorem 1, we know, ω^* is a fuzzy optimal solution of (3).

Lastly, from Eq. (5) and (9), then

$$\omega^*_{mt}=\varepsilon^*_{m0}=\tilde{v}_{mt}(x^*)\omega^*_{m0}/\tilde{g}_m(x^*),$$

when $m=0$, $\omega^*_{0t}=\tilde{v}_{0t}(x^*)/\tilde{g}_0(x^*)$ owing to $\omega^*_{00}=1$; when $m\neq 0$, owing to

$$\omega_{m0}=\sum_{t=1}^{T_m}\omega^*_{mt}=\mu^*_m\sum_{t=1}^{T_m}v_{mt}(x^*)/\tilde{g}_0(x^*)=\mu^*_m\,\tilde{g}_m(x^*)/\tilde{g}_0(x^*)\overset{\text{from(11)}}{=\!=\!=}\mu^*_m/\tilde{g}_0(x^*).$$

Hence

$$\omega^*_{mt}=\tilde{v}_{mt}(x^*)\mu^*_m/\tilde{g}_m(x^*)\tilde{g}_0(x^*)=\mu^*_m\tilde{v}_{mt}(x^*)/\tilde{g}_0(x^*).$$

Suming up, we have

$$\omega^*_{mt}=\begin{cases}\tilde{v}_{0t}(x^*)/\tilde{g}_0(x^*), & (m=0;\ t=1,\cdots,T_0),\\ \mu^*_m\tilde{v}_{mt}(x^*)/\tilde{g}_0(x^*), & (m\neq 0;\ t=1,\cdots,T_m).\end{cases}$$

References

[1] Cao Bingyuan, Study of fuzzy positive geometric programming dual form, *Proc. IFSA−89, U.S.A.*, 775−779.

[2] Cao Bingyuan, Fuzzy geometric programming(I), *Int. J. of Fuzzy Sets and Systems,* 53(1993), 135−154.

[3] Wu Fang, Yuan Yunyao, Geometric programming, *Math. in Practice and Theory,* 1.3(1982).