Fuzzy Risk Analysis in Natural Hazard

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Abstract: If we use pure probabilistic methods to analyse the risk, the results must be unreliable because a great amount of information is necessary to do it. In applying the theory of fuzzy sets, in this article we give an overview over advanced method to calculate the risk of release, exposure and consequence assessment in natural hazard. We also present a example to show how to use the model.

Keywords: Natural hazard; fuzzy risk; risk source; exposures; consequences; earthquake; information distribution.

1. Introduction

The intellectual task of analyzing the risk present in any large undertaking is an endeavor that abounds both with inherent imprecision and with a scarcity of historical data.

In situations for which a great amount of data has been collected, probabilistic methods can be an extremely effective way of quantifying uncertainty about the risk. Unfortunately, the data base supporting natural hazard is particularly poor. For example, earthquake events is rare for a city, adequate data for meaningful statistical inference simply may not exist.

Professor Lance Hoffoman[1] and his associate Don Clements [2] were the first to explore the application of the theory of fuzzy sets to risk analysis, and Mr. Schucker[3] deserves to be complimented for presenting a coherent and self-contained account of a body of concepts and techniques which are of considerable relevance to risk analysis. In their works, they propose a new method for evaluation of computer security systems. The central ideal is the application of natural language as the vehicle for the expression of imprecise and sometimes subjective evaluations by a 'security rate' in the absence of objective measures of security performance. The overall structure of this conventional fuzzy risk analysis grows in that one can provide the input of natural language estimate as probability of failure, severity of loss and reliability of the estimate. In fact, the theory only tell us how to combine the fuzzy risk of subsystems to calculate the fuzzy risk of the entire system.

In natural hazard, to obtain the inputs is more important than to combine. In this paper, we will give an overview over advanced method to calculate the risk of natural hazard.

2. The concept of fuzzy risk

Definition 2.1 Let $Y = \{y\}$ be the universe of discourse natural hazard, and $P = \{p(\xi \geq y)|y \in Y\}$ be the probability distribution of exceeding magnitude y. P is called probabilistic risk.

In the following, $p(\xi \geq y)$ is denoted as $p(y)$ in short, moreover, $p(y)$ can be denoted as p.

In many cases, the risk may be related to a period of time. We call

$$P_T = \{p_T(y)|y \in Y\} \tag{2.1}$$

the probabilistic risk within period T years about Y.

P_T can be denoted as P simply when it doesn't cause confusion.

If we use P to support risk management, it implicates that we have gotten a better and clearer understanding of the statistical laws of the hazard patterns. However, it is not easy to do that economically and reliably.

In most cases, there exists a large gap between the cognition and the reality. The main reason lies in the fact that the birth and pattern of many kinds of natural hazard is unclear. When P is itself unreliable, it is dangerous to use P as a basis for decision-making.

P is unreliable, that is to say, there is possibility for $p(y)$ to move in a range. Namely, for given y, $p(y)$ may be a fuzzy number which can be expressed by a possibility distribution function $\pi(y,x)$, which means that the possibility value of $p(y) = x$ $(x \in [0,1])$ is $\pi(y,x)$.

Definition 2.2 Let $Y = \{y\}$ be the universe of discourse natural hazard, and $\pi(y,x)$ be the possibility distribution of that probability value of exceeding magnitude y is x. $\Pi = \{\pi(y,x)|y \in Y, x \in [0,1]\}$ is called fuzzy risk.

For convenience sake, $\pi(y,x)$ is also called fuzzy risk.

Suppose probability risk $p(y)$ is known, it can be turned to fuzzy risk $\pi(y,x)$. In fact,

$$\pi(y,x) = \begin{cases} 1, & \text{when } x = p(y) \\ 0, & \text{others} \end{cases} \tag{2.2}$$

That is to say, probability risk is a special case of the fuzzy risk.

3. Risk assessment of natural hazard

In general, a risk exists when three conditions are satisfied. First, a source of risk must be present–that is, a system, process, or activity must exist that can release or otherwise introduce a risk agent into the environment. For natural hazard, the ***risk source*** might be flood, earthquake, typhoon, or drought. Second, an ***exposure process*** must exist by which people or the things they value may be exposed to the released risk agent. Exposure might result from people building homes below a dam. Third, a ***causal process*** must exist by which exposures produce adverse health or environmental consequences. Adverse consequences may consist of property damage and drownings due to water released in a dam failure.

Because the level of risk depends on the specific nature and characteristics of the ***risk source,*** the ***exposure process,*** and the ***consequence process,*** a comprehensive risk assessment must address each of these components comprehensively. Risk assessment must determine, characterize, and quantify the following factors: (1) the potential of the source to release a risk agent; (2) the intensity, frequency, and duration of exposure, and nature of the populations and other valued entities that might be exposed; and (3) the relationship between exposure and the resulting health or environmental consequences. finally, the combined influence of these factors on risk must be determined, characterized, and quantified. The final outputs of this process are estimates of the magnitudes of possible adverse heath or environmental consequences, including always a characterization of the probabilities, uncertainties, or degree of confidence associated with these estimates.

4. Release assessment

Let z be a risk agent which may be released by one or several risk sources. For example, constructive earthquake is the agent of active faults, and flood waters may be the agent of rainstorms.

Let m be measure of agent z. For example, when z is earthquake, m may be the Richter magnitude. If z is flood water, m may be the water level of a dam or a river.

Definition 4.1 Let $M = \{m\}$ be the universe of discourse z, and $\pi_z(m, x)$ be the possibility distribution of that probability value of exceeding m is x. We call

$$\Pi_z = \{\pi_z(m, x) | m \in M, x \in [0, 1]\} \tag{4.1}$$

fuzzy risk of agent z.

5. Exposure assessment

For many risk assessments, exposure assessment is the most difficult task. The reason for this is that exposure assessment often depends on factors that are hard to estimate and for which there are few data. Critical information on the conditions of exposure is often lacking.

Hundreds of exposure models have been developed for a diversity of risk source, risk agents, and routes of exposures. For example, models have been developed to represent the transmission of ground motion from the source of an earthquake to a given site, taking into account the magnitude of the earthquake, local soil conditions, and the distance from the epicenter to urban areas at risk. Although modeling methods provide a means for estimating exposures in the absence of comprehensive monitoring data, most uncertainties in modeling exposures are not caused by inherent deficiencies in modeling techniques. Instead, the uncertainties arise from lack of understanding and lack of data.

Assume that the attenuation relationship may be expressed as:

$$w = f(m, d) \tag{5.1}$$

where w is the site intensity, m is the magnitude at the source, and d is the shortest distance of the site from the source. The relationship can be improved by using a fuzzy relationship of M, D and W:

$$R_1 = R_{M,D,W} = \{r^{(1)}(m, d, w)\} \tag{5.2}$$

which can be obtained by the experts. Where M, D and W is the universe of discourse m, d and w respectively.

The site fuzzy intensity $\underset{\sim}{W}$ can be got by using:

$$\mu_W(w) = \sup_{m\in M, d\in D} \{r^{(1)}(m, d, w) \wedge \mu_{M,D}(m, d)\} \tag{5.3}$$

where $\mu_{M,D}(m, d)$ is the membership function of fuzzy magnitude and distance.

Let d_1, d_2 be the nearest and farthest distance from the site to the source respectively, The fuzzy distance $\underset{\sim}{D}$ can be expressed simply by using a bell function:

$$\mu_D(d) = \exp[-\frac{(\frac{d_2+d_1}{2} - d)^2}{\frac{(d_2-d_1)^2}{6}}] = exp[-1.5(\frac{d_2 + d_1 - 2d}{d_2 - d_1})^2] \tag{5.4}$$

If

$$\mu_M(m) = \pi_z(m, x) \tag{5.5}$$

we have

$$\mu_{M,D}(m, d) = \pi_z(m, x) \wedge \mu_D(d) \tag{5.6}$$

therefore, we can get the fuzzy risk of the site intensity as the following:

$$\pi_W(w, x) = \sup_{m\in M, d\in D} \{r^{(1)}(m, d, w) \wedge \pi_z(m, x) \wedge \mu_D(d)\} \tag{5.7}$$

6. Consequence assessment

The primary purpose of a *consequence assessment model* is to translate exposure to a specified risk agent into damage consequences. The principal type of consequence assessment model is the dose-response model. A *dose-response model* is a functional relationship between the dose (i.e., measure of exposure) and a adverse entity response (i.e., the measure of damage). Most dose-response models are derived from statistical data such as that from monitoring or testing. Examples are the linear dose-response models used to estimate human health effects and materials damage of buildings. Alternatively, dose-response models may be derived from theoretical considerations with little or no basis in empirical data.

Dose-respones models have many limitations, including the availability of the data or the knowledge and understanding needed to set their parameters and verify their accuracy. For the overwhelming majority of risk agents, knowledge is insufficient to permit confidence in the selection of a dose-response function.

Assume that the functional relationship between the dose and a adverse entity response can be expressed as:

$$y = g(w) \tag{6.1}$$

where w is the site intensity, y is the measure of damage. The relationship can be improved by using a fuzzy relationship of W, and Y

$$R_2 = R_{W,Y} = \{r^{(2)}(w, y)\} \tag{6.2}$$

which can be obtained by the experts. Where W, and Y is the universe of discourse w, and y respectively.

The entity fuzzy response $\underset{\sim}{Y}$ can be got by using:

$$\mu_Y(y) = \sup_{w \in W} \{r^{(2)}(w, y) \wedge \mu_W(w)\} \tag{6.3}$$

where $\mu_W(w)$ is the membership function of site fuzzy intensity as in (5.3).

If $\underset{\sim}{W}$ is a estimator of fuzzy risk, namely,

$$\mu_W(w) = \mu_W(w, x) = \sup_{m \in M, d \in D} \{r^{(1)}(m, d, w) \wedge \pi_z(m, x) \wedge \mu_D(d)\}$$

we can get the fuzzy risk of the entity response as the following:

$$\pi_Y(y, x) = \sup_{w \in W} \left\{ r^{(2)}(w, y) \wedge \sup_{m \in M, d \in D} \{r^{(1)}(m, d, w) \wedge \pi_z(m, x) \wedge \mu_D(d)\} \right\} \tag{6.4}$$

7. Case calculation

The studied city of this case calculation is the author's imagination according to characteristics of Chinese cities. Suppose the agent is earthquake. Let us calculate its fuzzy risk of the entity response.

Let there be 50 entities in city C. And suppose all entities are buildings. That is

$$C = \{c_1, c_2, \cdots, c_{50}\} \tag{7.1}$$

7.1. release assessment

The risk source can be regarded as a seismic active belt around or nearby the city. In the belt, 12 epicenters of historic earthquakes with $M \geq 5.0$ in T years were recorded. The set of these historic earthquakes is:

$$\{M_1, M_2, \cdots, M_{12}\} = \{5.5, 6.8, 5.1, 5.7, 5.0, 6.5, 6.5, 6.0, 6.0, 5.2, 7.4, 5.2\} \tag{7.2}$$

which is called a sample set.

Let $M_0 = 4.9$ be the minimum magnitude which used in engineering, and $M_\mu = 7.4$ be the maximum magnitude in the belt. The universe of discourse earthquake magnitude in the belt is $[M_0, M_\mu] = [4.9, 7.4]$. According to the capacity of the set of these historic earthquakes, take step $\Delta = 0.5$, and let

$$U = \{u_1, u_2, \cdots, u_6\} = \{4.9, 5.4, 5.9, 6.4, 6.9, 7.4\} \tag{7.3}$$

Then, the universe $[M_0, M_\mu]$ of discourse earthquake magnitude has been changed into the discrete universe U. Using information distribution method[9], U can absorb information from the set of these historic earthquakes and show its information structure.

$\forall u_j \in U$ is called a controlling point which absorbs information of the neighboring samples in some fashion. In other words, a sample may be distributed to some controlling points. The simplest model is to distribute a sample to two points which are near the value of the sample.

We use the simplest formula as (7.4) to distribution the samples in (7.2) to the controlling points in (7.3), namely,

$$q_{ij} = 1 - \frac{|M_i - u_j|}{\Delta}, \quad |M_i - u_j| \leq \Delta \tag{7.4}$$

where q_{ij} is called information gain of point u_j from sample M_i.

After 12 earthquake data have been treated with this simple process and information gains at each controlling point have been summed up, a distribute of information gains will turn out. That is

$$Q = \{Q_1, Q_2, \cdots, Q_6\} = \{2.2, 3.0, 2.4, 2.2, 1.2, 1\}$$

where $Q_j = \sum_{i=1}^{12} q_{ij}$.

In fact, Q_j means that there are Q_j earthquakes whose magnitude is about u_j. Namely, Q_j is the number of earthquake with magnitude u_j. Using Q, the number of earthquake with magnitude greater than or equal to u_j can be obtained as:

$$N_j = \sum_{i=j}^{6} Q_i \tag{7.5}$$

They can constitute a number distribution of exceeding magnitude as:

$$N = \{N_1, N_2, \cdots, N_6\} = \{12, 9.8, 6.8, 4.4, 2.2, 1\} \tag{7.6}$$

Obviously, the probability value of exceeding u_j is

$$p_j = \frac{N_j}{12}$$

where 12 is the number of the samples in (7.2). We can obtain a exceeding probability distribution as

$$P = \{p_1(M \geq u_1), p_2(M \geq u_2), \cdots, p_1(M \geq u_6)\} = \{1, 0.82, 0.57, 0.37, 0.18, 0.08\} \tag{7.7}$$

Because we only have 12 samples, exceeding probability distribution in (7.7) is unreliable. The reason is that the knowledge sample set in (7.2) is incomplete which carries fuzzy information. Using two dimensions information distribution method[4], P can be optimally changed to fuzzy risk of earthquake.

Let discrete universe of discourse magnitude be

$$\{m_1, m_2, \cdots, m_{14}\} = \{4.6, 4.9, 5.2, 5.5, 5.8, 6.1, 6.4, 6.7, 7.0, 7.3, 7.6, 7.9, 8.2, 8.5\} \tag{7.8}$$

and discrete universe of discourse probability be

$$\{x_1, x_2, \cdots, x_6\} = \{0, 0.2, 0.4, 0.6, 0.8, 1\} \tag{7.9}$$

We use the simplest formula as (7.10) to distribute sample (u_j, p_j) to discrete point (m_i, x_k), namely,

$$\tilde{f}_j(m_i, x_k) = \begin{cases} (1 - \frac{|m_i - u_j|}{0.3})(1 - \frac{|x_k - p_j|}{0.2}), & |m_i - u_j| \leq 0.3 \text{ and } |x_k - p_j| \leq 0.2 \\ 0, & \text{others} \end{cases} \tag{7.10}$$

where 0.3 and 0.2 is step of magnitude and probability respectively. Let

$$\tilde{f}(m_i, x_k) = \sum_{j=1}^{6} \tilde{f}_j(m_i, x_k)$$

and

$$g_i = max\{f(m_i, x_k)|k = 1, 2, \cdots, 6\}$$

If $g_i = 0$, let $g_i = 1$. Then,

$$\pi_z(m_i, x_k) = \frac{\tilde{f}(m_i, x_k)}{g_i}$$

is fuzzy risk of earthquake which is

$$\Pi_z = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \\ m_6 \\ m_7 \\ m_8 \\ m_9 \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \left(\begin{array}{cccccc} 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.15 & 1.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.94 \\ 0.00 & 0.00 & 0.07 & 0.38 & 1.00 & 0.11 \\ 0.00 & 0.00 & 0.18 & 1.00 & 0.42 & 0.05 \\ 0.00 & 0.13 & 0.93 & 1.00 & 0.00 & 0.00 \\ 0.02 & 0.34 & 1.00 & 0.16 & 0.00 & 0.00 \\ 0.10 & 1.00 & 0.63 & 0.00 & 0.00 & 0.00 \\ 0.32 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.84 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 1.00 & 0.67 & 0.00 & 0.00 & 0.00 & 0.00 \\ 1.00 & 0.67 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{array} \right) \end{array} \qquad (7.11)$$

7.2 exposure assessment

In earthquake engineering, we usually use the attenuation relationship of seismic intensity to exposure assessment. The first of all is to transform earthquake magnitude into epicentral intensity. In China, there is a fuzzy relationship[5], we denote

$$R_{M,I_0} = \{r'(m, i)|m \in M, i \in I_0\} \qquad (7.12)$$

Obviously, the fuzzy risk of epicentral intensity in seismic active belt can be obtained by using the below formula:

$$\pi_{I_0}(i, x) = \sup_{m \in M} \{\pi_z(m, x) \wedge r'(m, i)\} \qquad (7.13)$$

where $\pi_z(m, x) \in \Pi_z$ in (7.11), $x \in \{x_1, x_2, \cdots, x_6\}$ in (7.9), and $i \in I_0$.

From (7.11) and (7.12), we obtain the fuzzy risk of epicentral intensity as the following:

$$\Pi_{I_0} = \begin{array}{c} \\ VI \\ VII \\ VIII \\ IX \\ X \\ XI \\ XII \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \left(\begin{array}{cccccc} 0.00 & 0.00 & 0.07 & 0.38 & 0.90 & 1.00 \\ 0.02 & 0.14 & 0.37 & 0.67 & 1.00 & 0.33 \\ 0.31 & 0.36 & 1.00 & 0.91 & 0.42 & 0.11 \\ 0.51 & 1.00 & 0.63 & 0.16 & 0.00 & 0.00 \\ 0.84 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 1.00 & 0.67 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{array} \right) \end{array} \quad (7.14)$$

Let the universe of discourse site intensity be $I = \{V, VI, VII, VIII, IX\}$, and the universe of discourse distance be

$$D = \{v_1, v_2, \cdots, v_6\} = \{9, 15, 20, 40, 80, 140\}$$

Suppose the nearest and farthest distance from the city to the belt is $d_1 = 0$ km. and $d_2 = 30$ km. respectively. Recall (5.4). Then, the fuzzy distance is

$$\underset{\sim}{D} = 0.79/9 + 1/15 + 0.85/20 + 0.02/40 + 0/80 + 0/140 \quad (7.15)$$

According to the materials of intensity attenuation relating to the seismic active belt, we can obtain a intensity attenuation relationship[6] as the following.

$$R_{I_0,D,I} = \{r^{(1)}(i_1, d, i_2) | i_1 \in I_0, d \in D, i_2 \in I\} \quad (7.16)$$

Using formula (5.7), so we see that the fuzzy risk of the site intensity might be

$$\Pi_{I} = \begin{array}{c} \\ V \\ VI \\ VII \\ VIII \\ IX \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \left(\begin{array}{cccccc} 0.51 & 0.56 & 0.85 & 0.85 & 1.00 & 0.90 \\ 0.51 & 0.82 & 0.91 & 0.91 & 1.00 & 0.56 \\ 0.66 & 0.92 & 1.00 & 0.91 & 0.42 & 0.14 \\ 0.85 & 1.00 & 0.63 & 0.36 & 0.36 & 0.11 \\ 0.79 & 0.79 & 0.51 & 0.16 & 0.00 & 0.00 \end{array} \right) \end{array} \quad (7.17)$$

7.3 consequence assessment

We can define fuzzy damage as:

$$\begin{cases} A_1 = \text{Good condition} = 1/l_1 + 0.2/l_2 \\ A_2 = \text{Light destruction} = 0.2/l_1 + 1/l_2 + 0.2/l_3 \\ A_3 = \text{General destruction} = 0.2/l_2 + 1/l_3 + 0.2/l_4 \\ A_4 = \text{Heavy destruction} = 0.2/l_3 + 1/l_4 + 0.2/l_5 \\ A_5 = \text{Collapse} = 0.2/l_4 + 1/l_5 + 0.2/l_6 \end{cases} \quad (7.18)$$

In China, the fuzzy relationship[7] between the site intensity and fuzzy damage of a single layer brick pillar factory-building is:

$$R_{I',A} = \begin{array}{c} \\ VI \\ VII \\ VIII \\ IX \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \left(\begin{array}{ccccc} 1.00 & 0.43 & 0.14 & 0.00 & 0.00 \\ 0.21 & 1.00 & 0.36 & 0.00 & 0.00 \\ 0.21 & 0.36 & 1.00 & 0.14 & 0.13 \\ 0.00 & 0.14 & 0.43 & 1.00 & 0.57 \end{array} \right) \end{array} \tag{7.19}$$

where $I' = \{VI, VII, VIII, IX\}$, and $A = \{A_1, A_2, A_3, A_4, A_5\}$.

Using formula (6.3) and according to (7.17), we can obtain the fuzzy risk of the entity response as the following:

$$\Pi_A = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \left(\begin{array}{cccccc} 0.51 & 0.82 & 0.91 & 0.91 & 1.00 & 0.56 \\ 0.66 & 0.92 & 1.00 & 0.91 & 0.43 & 0.43 \\ 0.85 & 1.00 & 0.63 & 0.36 & 0.36 & 0.14 \\ 0.79 & 0.79 & 0.51 & 0.16 & 0.14 & 0.11 \\ 0.57 & 0.57 & 0.51 & 0.16 & 0.13 & 0.11 \end{array} \right) \end{array} \tag{7.20}$$

7.4 loss assessment

Suppose that the loss of a building is in direct proportion to its area and damage index. Moreover, let us presume that every square meter is worth 490 dollars in city C. If the area of all buildings in city C totalled 50,000 square meters, the buildings in city C is worth 24.5 million dollars. Corresponding with the universe L of discourse damage index in (7.18), we can obtain the universe of discourse losses of the city as:

$$Y_C = \{y_1, y_2, y_3, \cdots y_6\} = \{0, 4.9, 9.8, 14.7, 19.6, 24.5\} \tag{7.22}$$

where a unit of loss is million dollars.

By using (7.18) and (7.21), it is easy to obtain the fuzzy relationship $R_{Y,A}$ between the loss and fuzz damage.

Obviously, corresponding to the fuzzy risk of the entity response, the fuzzy risk of losses of the city is

$$\Pi_Y = R_{Y,A} \circ \Pi_A \tag{7.23}$$

where operator "∘" is max – min type. That is

$$\Pi_Y = \begin{array}{c} \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \left(\begin{array}{cccccc} 0.51 & 0.82 & 0.91 & 0.91 & 1.00 & 0.56 \\ 0.66 & 0.92 & 1.00 & 0.91 & 0.43 & 0.43 \\ 0.85 & 1.00 & 0.63 & 0.36 & 0.36 & 0.20 \\ 0.79 & 0.79 & 0.51 & 0.20 & 0.20 & 0.14 \\ 0.57 & 0.57 & 0.51 & 0.16 & 0.14 & 0.11 \\ 0.20 & 0.20 & 0.20 & 0.16 & 0.13 & 0.11 \end{array} \right) \end{array} \tag{7.24}$$

According to Π_Y, we know that the probability of exceeding losses is not one value but a fuzzy set. For example, when $y = y_3 = 9.8$(million dollars), the fuzzy probability of loss is:

$$\underset{\sim}{P}(\xi \geq 9.8) = 0.85/0 + 1.00/0.2 + 0.63/0.4 + 0.36/0.6 + 0.36/0.8 + 0.20/0.9$$

The benefit of this result is that one can easily understand impreciseness of risk assessment of natural hazard in case of lacking of earthquake hazard data. It might be useful to set a flexible and more economical strategy, plan and action on disaster reduction.

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