Fuzzy Almost Semicontinuous Functions and Fuzzy Strongly Regular Topological Spaces

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Abstract: In this paper, fuzzy almost semicontinuous functions, a new class of functions, is introduced. some characteristic properties of this class of functions is investigated. The composition of fuzzy almost semicontinuous functions with other functions is studied. The concept of fuzzy strongly regular topological space is also introduced and its characteristics and relationships with other kind of fuzzy topological space are discussed. The relationships of some kind of fuzzy compactness under fuzzy almost semicontinuous functions are studied.

Keywords: Fuzzy semi-open, fuzzy δ-open, Fuzzy regular open, fuzzy semi -θ-closed, fuzzy almost continuous, fuzzy almost semicontinuous.

1. Introduce

Zadeh in [18] introduced the fundamental concept of a fuzzy set. Weaker forms of continuity in fuzzy topological spaces have been studied by many scholars in [1-10, 12]. In this paper we introduce a new class of functions, called fuzzy almost semicontinuous, as a generalization of fuzzy almost continuous and fuzzy semicontinuous. The characteristics of fuzzy almost semicontinuous are investigated. The composition of fuzzy almost semicontinuous functions with other functions is studied. The concept of fuzzy strongly regular topological spaces is also introduced, and its relationships with other fuzzy topological spaces are studied. The relationships of some kind of fuzzy compactness under fuzzy almost semicontinuous functions are discussed.

For general terminologies and the basic concepts, not explained here, we refer to [1, 2, 11] Some definitions and results which will be needed are recalled here.

Throughout this paper, X and Y mean fuzzy topological spaces.

Definition 1.1[1] Let A be a fuzzy set in a fuzzy topological space X. A is called:

(i) fuzzy semiopen [1] if there is a open set B such that $B \leqslant A \leqslant clB$.

(ii) fuzzy semiclosed [1] if there is a closed set B, such that $intB \leqslant A \leqslant B$.

(iii) fuzzy regular open [1] if $int(cl(A)) = A$.

(iv) fuzzy regular closed [1] if cl(int(A)) = A.

Theorem 1 · 2[15] For a fuzzy set A in a fuzzy space X, the following are equivalent:

(i) A is a fuzzy semiclosed set.

(ii) A′ is a fuzzy semiopen set.

(iii) int(cl(A)) $\leqslant$ A.

(iv) cl(int(A′)) $\geqslant$ A′.

Definition 1. 3[2] Let A be a fuzzy set in X, and define the following sets:

S-clA = $\bigcap$ {B | A $\leqslant$ B, B fuzzy semiclosed}

S-intA = $\bigcup$ {B | B $\leqslant$ A, B fuzzy semiopen}

A is a fuzzy semiopen set iff A = S-intA

A is a fuzzy semiclosed set iff A = S-clA

Definition 1. 4[15] A fuzzy set A on fuzzy topological space X is said to be δ-open if for each fuzzy point $x_a \in A$, there exists a regular open fuzzy set B in X, such that $x_a \in B \leqslant A$.

It follows from the definition that a δ-open fuzzy set is a union of fuzzy regular open sets and a fuzzy regular open set is a δ-open fuzzy set.

Definition 1. 5[15]. A fuzzy point x_a in a fuzzy topological space X is said to be a δ-adherent point of a fuzzy set A in X if every regular open quasi-neighbourhood of x_a is quasi-concident with A.

The union of all δ-adherent fuzzy points of a fuzzy points of a fuzzy set A in a fuzzy topological space X is called the δ-closure of A and is denoted by δ-cl(A). If A = δ-cl(A), then A is called fuzzy δ-closed.

Lemma 1. 6[15] The δ-closure of a fuzzy set in an fuzzy topological space is δ-closed.

Definition 1. 7[4] Let A be a fuzzy set in a fuzzy topological space X, the fuzzy semi, θ-closure of A, denoted by cls-θ(A) is defined as\{$x_a \in X$ | for every fuzzy semi-open semi quasi-neighbourhood B of x_a, s-clB q A} and X is fuzzy semi θ-closed iff A = cls-θ(A)

Definition 1. 8 Let f : X→Y be a function between two fuzzy topological spaces, then f is called.

(i) a fuzzy semicontinuous function [1] iff $f^{-1}(A)$ is a fuzzy semi-open set of X for each fuzzy open set A in Y.

(ii) a fuzzy almost continuous function [1] iff $f^{-1}(A)$ is a fuzzy open set of X for each regular open set A in Y.

(iii) a fuzzy weakly continuous function [1] iff $f^{-1}(A) \leqslant \text{int } f^{-1}(\text{cl}A)$ for each open set A in Y.

(iv) a fuzzy weakly semicontinuous function [9] iff $f^{-1}(A) \leqslant \text{S-int} f^{-1}(\text{S-cl}A)$ for each open set A in Y.

(v) a fuzzy R-map [12] iff $f^{-1}(A)$ is a fuzzy regular open set of X for each fuzzy regular open set A in Y.

(vi) a fuzzy semi irresolute function [4] iff $f^{-1}(A)$ is a fuzzy semi-open of X for each fuzzy semi θ-open set A in Y.

(vii) a fuzzy irresolure function [1] iff $f^{-1}(A)$ is a fuzzy semi-open set of X for each fuzzy semi-open set A in Y.

(viii) a fuzzy completely irresolute function [10] if $f^{-1}(A)$ is a fuzzy regular open set of X for each fuzzy semi θ-open set A in Y.

(iX) a fuzzy completely weakly irresolute function [10] iff $f^{-1}(A)$ is a fuzzy regular open set of X for each semi θ-open set A in Y.

Definition 1.9[1] A fuzzy topological space(X,δ) is called a fuzzy semiregular space iff the collection of all fuzzy regular open sets X forms a base for fuzzy topology δ.

A fuzzy topological space X is called a fuzzy regular space iff each fuzzy open set A of X is a union of fuzzy open sets λ_i' s of X such that cl $\lambda_i \leqslant \lambda$.

Definition 1. 10[15] A fuzzy topological space X is normal if for every closed fuzzy set C in X and a fuzzy open set A in X containing C, there exists a fuzzy open set B in X such that $C \leqslant B \leqslant clB \leqslant A$.

2. Fuzzy almost semicontinuous functions.

Definition 2. 1 A fuzzy function $f: X \to Y$ from a fuzzy topological space X to another fuzzy topological space Y is said to be a fuzzy almost semicontinuous function if $f^{-1}(A)$ is a fuzzy semi-open set of X for such fuzzy regular open set A in Y.

Theorem 2. 2 If $f: X \to Y$ is a fuzzy almost continuous function then f is a fuzzy almost semicontinuous.

Proof: Obvious.

The converse of the above need not be true is shown by the following Example 2. 3.

Exanlple2. 3 Let $X=\{a,b\}$, $Y=\{a,b,c\}$ and $f: X \to Y$ be defined as $f(a)=a$, $f(b)=b$. Let us define fuzzy sets A in X and B in Y as follows.

$A(a)=0.3$ $A(b)=0.4$

$B(a)=0.3$ $B(b)=0.4$ $B(c)=0.5$.

Then $\{0, A, 1_X\}$ is a fuzzy topology on X and $\{0, B, 1_Y\}$ is a fuzzy topology on Y. It can be verified that f is fuzzy almost semicontinuous, but the inverse image of a fuzzy regular open B is not a fuzzy open set in X.

Theorem 2. 4 If $f: X \to Y$ is a fuzzy almost semicontinuous function from a fuzzy topological space X to a fuzzy semiregular space, then f is almost continuous.

Proof: The result follows from the definition 1. 9 and 2. 1.

Theorem2. 5 Fuzzy weakly semicontinuous functions and fuzzy almost semicontinuous functions is independent notions.

This is shown by the following Example 2. 6 and Example 2. 7.

Example 2. 6 Refer to Example 2. 4 f is shown a fuzzy almost semicontinuous function. Also by computations it follows that $f^{-1}(B) \leqslant$ S-int f^{-1}(s-clB) = S-int $f^{-1}(B)$.

Thus f is not a fuzzy weakly semicontinuous function.

Example 2. 7 A fuzzy weakly semicontinuous function need not be a fuzzy almost semicontinuous function. Let $X=\{a,b,c\}$, $\delta_1=\{0,A,B,1\}$, $\delta_2=\{0,D,1\}$,

where

$A(a)=0.4, A(b)=0.2, A(c)=0.1$

$B(a)=0.5, B(b)=0.5, B(c)=0.5$

$D(a)=0.5, D(b)=0.5, D(c)=0.6$

Consider the identify function $f:(X,\delta_1)\rightarrow(Y,\delta_2)$. Simple computations give.

$D=f^{-1}(D)\leqslant S\text{-int } f^{-1}(S\text{-cl}(D))\leqslant S\text{-int } f^{-1}(1))=1$.

Hence f is a fuzzy weakly semicontinuous function. Also by easy computations it follows that the inverse image of the regular open set D is not semi-open in X. Thus f is not a fuzzy almost semicontinuous function.

From [9]we know that fuzzy almost continuous function and fuzzy weakly semicontinuous are independent notions. Since a fuzzy continuous function is a fuzzy almost semicontinuous function, a fuzzy almost semicontinuous function need not be fuzzy weakly continuous.

The following Example 2.8 shows that a fuzzy weakly continuous need not be a fuzzy almost semicontinuous function.

Example 2.8 Let $X=\{a,b,c\}$, $\delta_1\{0,B,1\}$ and $\delta_2=\{0,A,1\}$, where

$A(a)=0.3, A(b)=0.1, A(c)=0.4$

$B(a)=0.6, B(b)=0.7, B(c)=0.5$

Consider the identity function $f:(X,\delta_1)\rightarrow(X,\delta_2)$

Simple computations give

$A=f^{-1}(A)\leqslant \text{int} f^{-1}(\text{cl}A)=\text{int}A'=B$

Hence f is a fuzzy weakly continuous function. Also by computations, it follows that the inverse image of the regular open set A is not semi-open, thus f is not a fuzzy almost semicontinuous function. So we obtain theorem 2.9.

Theorem 2.9 Fuzzy weakly continuous functions and fuzzy almost semicontinuous function is independent notions.

Theorem 2.10 If $f:X\rightarrow Y$ is a semicontinuous function, then f is a fuzzy almost semicontinuous.

Proof: Noting a open set is semi-open, a regular open set is open.

The converse of this theorem need not be true is shown by Example 2.11.

Example 2.11 Let $X=\{a,b,c\}$, $\delta_1=\{0,D,1\}$ and $\delta_2=\{0,A,B,1\}$, where

$A(a)=0.4, A(b)=0.2, A(c)=0.1$

$B(a)=0.5, B(b)=0.5, B(c)=0.5$

$D(a)=0.3, D(b)=0.2, D(c)=0.2$

Consider the identity function $f:(X,\delta_1)\rightarrow(Y,\delta_2)$ by computations, it follows that B is regular open and A is not. The inverse of each regular open set is semi-open. However, the inverse of the open set A is not semi-open. Therefore, f is almost semicontinuous, but it is not semicontinuous.

Definition 2.12 A function $f:X\rightarrow Y$ from a fuzzy topological space X to a fuzzy topological space Y is said to be fuzzy almost semicontinuous at a fuzzy point x_α in X if for each regular open set A of Y containing $f(x_\alpha)$, there exists a fuzzy semi-open set B containing x_α such that $f(B)\leqslant A$.

Theorem 2.13 Let $f:X\rightarrow Y$ be a fuzzy function, Then the following are equivalent.

(i) f is a fuzzy almost semicontinuous function.

(ii) f is a fuzzy almost semicontinuous at each fuzzy point in X.

(iii) $f^{-1}(A)$ is a fuzzy semi-closed set for each regular closed set A in Y.

(iv)$f^{-1}(A)$ is a fuzzy semi-closed set for each δ-closed fuzzy set A in Y.

(v) $f^{-1}(A)$ is a fuzzy semi-open set for each δ-open fuzzy set A in Y.

(vi) $intcl(f^{-1}(A)) \leqslant f^{-1}(\delta\text{-}clA)$, for each fuzzy set A in Y.

(vii)$f(intclA) \leqslant \delta\text{-}clf(A)$, for each set A in X.

(viii) $f^{-1}(A) \leqslant clintf^{-1}(intclA)$, for each open set A in X.

(iX) $f^{-1}(A) \geqslant int\ cl\ f^{-1}(clintA)$, for each closed set A in Y.

Proof: (i)⇔(iv) It is easy from the definitions.

(i)⇔(iii) Noting that $f^{-1}(A') = (f^{-1}(A))'$ for any fuzzy set A of Y. This is obvious.

(i) ⇨ (v) Let A be a δ-open fuzzy set in Y, there exists fuzzy regular open set B_i($i \in I$ is an index) such that $A = \bigcup_{i \in I}(B_i)$

Now $f^{-1}(A) = f^{-1}(\bigcup_{i \in I} B_i) = \bigcup_{i \in I} f^{-1}(B_i)$

for each $B_i(i \in I)$, $f^{-1}(B_i)$ is a fuzzy semi-open set, so $f^{-1}(A)$ is a semi-open set.

(iii)⇔(iv) This is obvious, being a complentent of each other.

(iv) ⇨ (vi) Since the δ-closure of the fuzzy set A in Y is δ-closed, $f^{-1}(\delta\text{-}clA)$ is a fuzzy semi-open set. Hence $f^{-1}(\delta\text{-}clA) \geqslant intcl(f^{-1}(\delta\text{-}clA)) \geqslant intcl(f^{-1}(A))$.

vi ⇨ vii Let $f(x_\alpha) \not\leqslant \delta\text{-}clf(A)$, be a fuzzy point in Y, then $x_\alpha \not\leqslant f^{-1}(\delta\text{-}clf(A))$.

Since $f^{-1}(\delta\text{-}clf(A)) \geqslant intclf^{-1}(f(A)) \geqslant intclA$ from (vi), it follows that $x_\alpha \not\leqslant f(intclA)$

Hence $f(intclA) \leqslant \delta\text{-}clf(A)$

(i) ⇨ (viii) since intclA is a fuzzy regular open set when A is any open set when A is any open set in Y,

$f^{-1}(intclA)$ is a fuzzy semi-open set.

Hence $f^{-1}(intclA) \leqslant clintf^{-1}(intclA)$.

Now A is fuzzy open, so $intclA \geqslant intA = A$.

Hence $f^{-1}(A) \leqslant clintf^{-1}(intclA)$.

(viii) ⇨ (i) Let A be any regular open set in Y, then $A = intclA$, by (viii), $clintf^{-1}(intclA) = clintf^{-1}(A) \geqslant f^{-1}(A)$, which shows that $f^{-1}(A)$ is a fuzzy semi-open set.

(iii)⇔(iX) It is anagous to the proof of (i)⇔(viii).

theorem 2. 14 Every fuzzy R-map is a fuzzy almost semi-continuous function.

Proof: Obvious.

The converse of the above is not true by Example 2. 3.

3. Composition of fuzzy almost semicontinuous functions.

Theorem 3. 1. If $f: X \to Y$ is fuzzy almost semicontinuous and $g: Y \to Z$ is fuzzy almost semicontinuous.

Proof: The theorem follows from the definitions.

Theorem 3. 2 If $f: X \to Y$ is fuzzy irresolute and $g: Y \to Z$ is fuzzy almost semicontinuous, then $gof: X \to Z$ is fuzzy almost semicontinuous.

Proof: The theorem follows from the definitions.

Theorem 3. 3 If $f: X \to Y$ is completely irresolute and $g: Y \to Z$ is fuzzy almost semicontinuous, then gof: $X \to Z$ is R-map.

Proof: Let B be a fuzzy regular open set of Z, then $g^{-1}(B)$ is a fuzzy semi-open in Y.

Now $f^{-1}(g^{-1}(B)) = (gof)^{-1}(B)$ is a fuzzy regular open set in X, since f is a fuzzy completely irresolute function. Hence the theorem.

Theorem3. 4, If $f: X \to Y$ is fuzzy almost semicontinuous and $g: Y \to Z$ is weakly completely irresolute, then gof: $X \to Z$ is semi-irresolute.

Proof: Obvious.

4. Fuzzy strongly regular space

Definition 4. 1 A fuzzy topological space X is fuzzy strongly regular if for each fuzzy point x_a and each fuzzy semi-open set A in X containing x_a, there exists a fuzzy open set B, such that $x_a \in B \leqslant clB \leqslant A$.

Here, we give useful a chracterization of a fuzzy strongly regular space.

Theorem 4. 1 X is a fuzzy strongly regular space iff for each fuzzy semi-open B, there exist fuzzy open sets B_i's ($i \in I$ an index set) such that.

$A = \bigcup_{i \in I} B_i$ and $clB_i \leqslant A$

Proof: Let A be any fuzzy set in X and x_a be any fuzzy point contained in A, then there exist open set B_i in X such that $x_a \in B_i \leqslant clB_i \leqslant A$ from the definition of a fuzzy strongly regular space.

Taking union over all the fuzzy points contained in A, we have $A = \bigcup_{x_a \in A} x_a \leqslant \bigcup_{i \in I} B_i \leqslant \bigcup_{i \in I} clB_i \leqslant A$

Therefore $\bigcup_{i \in I} B_i = A$ and $clB_i \leqslant A$.

Converse, let x_a be any fuzzy point contained in a semi-open set A of X, then there exist fuzzy open sets $B_i (i \in I)$ such that

$A = \bigcup_{i \in I} B_i$ and $clB_i \leqslant A$

since $x_a \in A = \bigcup_{i \in I} B_i$, there exists $B_i (i \in I)$ such that $x_a \in B_i$, therefore

$x_a \in B_i \leqslant clB_i \leqslant A$.

Theorem 4. 2. Each fuzzy strongly regular space X is a fuzzy regular space.

Proof: It is easy. Since a fuzzy open set is a fuzzy semi-open set.

The inverse of the above is not true by following Example 4. 3.

Example 4. 3 Let $X = R, \varepsilon \in R^+, A_\varepsilon(x) = \begin{cases} 1 & x \in (-\varepsilon, \varepsilon) \\ 0 & \text{otherwise} \end{cases}$

then $B = \{A_\varepsilon \mid \varepsilon \in R^+\}$ forms a base for a fuzzy topology on X. It is verified that (X, δ) is fuzzy regular.

Taking $B(x) = \begin{cases} 1 & x \in (0,1] \\ 0 & \text{otherwise} \end{cases}$

We obtain that the fuzzy set B is semi-open and fuzzy point $1_{\frac{1}{2}} \in B$ there exists no open set D in X such that $1_{\frac{1}{2}} \in D \leqslant clD \leqslant B$.

Thus the fuzzy topological space(X,δ) is not fuzzy strongly regular.

We know that fuzzy normal spaces need not be fuzzy regular spaces. Since fuzzy strongly spaces contain fuzzy regular spaces, fuzzy normal spaces need not be fuzzy strongly regular spaces. Moreover, fuzzy strongly regular space need not be fuzzy normal spaces. The following theorem 4. 4 shows that fuzzy strongly regular and fuzzy compact spaces is fuzzy normal spaces.

Theorem 4. 4 If a fuzzy topological space X is fuzzy strongly regular and fuzzy compact spaces, then the fuzzy topological space X is fuzzy normal.

Proof: Let A be any fuzzy close set of X contained in a fuzzy open set B in X. $\forall x_\alpha \in A$, then $x_\alpha \in B$, From the definition of fuzzy strongly regular space, there exists a fuzzy open set D_i such that $x_\alpha \in D_i \leqslant clD_i \leqslant A$.

Taking union over all fuzzy points contained in A, we have $A = \bigcup\limits_{x_\alpha \in B} x_\alpha \leqslant \bigcup\limits_{i \in I} D_i \leqslant \bigcup\limits_{i \in I} clD_i = B$.

As X is fuzzy compact and A is closed, there exists a finite subcollection of open $\{B_i | i = 1, 2, \cdots, n\}$ whose union contains the fuzzy closed set A.

Let C denote this finite union, then C is a fuzzy open set in X. Hence

$A \leqslant C \leqslant clC \leqslant B$

Therefore X is fuzzy normal.

From the proof of this theorem, we obtain that a fuzzy regular and fuzzy compact space is fuzzy normal.

Theorem 4. 5 If $f: X \to Y$ is a fuzzy almost semicontinuous and fuzzy closed function from a strongly regular space X onto an fuzzy-topological space Y such that $f^{-1}(y_\alpha)$ is fuzzy compact for each fuzzy point y_α in Y, then Y is fuzzy strongly regular.

Proof: Let y_α be any fuzzy point contained in a regular open fuzzy set B in Y. Since f is fuzzy almost semicontinuous, $f^{-1}(B)$ is a fuzzy semi-open set in X. Moreover, $f^{-1}(B)$ contains the fuzzy set $f^{-1}(y_\alpha)$. As X is a fuzzy strongly regular space, we obtain fuzzy open set A_i in X for each fuzzy point $x_\alpha \in f^{-1}(y_\alpha)$ such that

$x_\alpha \in A_i \leqslant clA_i \leqslant f^{-1}(B)$

Taking union over all the fuzzy points contained in $f^{-1}(y_\alpha)$, we have $f^{-1}(y_\alpha) = \bigcup\limits_{x_\alpha \in f^{-1}(y_\alpha)} x_\alpha \leqslant \bigcup\limits_{i \in I} A_i = \bigcup\limits_{i \in I} clA_i \leqslant f^{-1}(B)$

Due to fuzzy compactness of $f^{-1}(y_\alpha)$, there exists a finite subcollection of open fuzzy sets $\{A_i | i = 1, 2, \cdots, n\}$, whose union contain the fuzzy set $f^{-1}(y_\alpha)$. Let D denote this finite union, the D is a fuzzy open set in X and $clD \leqslant f^{-1}(B)$.

Hence $f^{-1}(y_\alpha) \leqslant D \leqslant clD \leqslant f^{-1}(B)$

As f is fuzzy closed, [17, theorem 3. 2], there exists a fuzzy open set C in Y, satifying $y_\alpha \in C$ and $f^{-1}(C) \leqslant D$

which gives

$C \leqslant f(D) \leqslant f(clD) \leqslant f(f^{-1}(B))$

Moreover, since f is onto,

$f(f^{-1}(B)) = B$ and as f is fuzzy closed, $clC \leqslant clf(clD) = f(clD)$ Therefore $y_\alpha \in C \leqslant clC \leqslant B$

5. Fuzzy almost semicontinuous and some kind of compactness of fuzzy topological spaces.

Here we study relationships of some kind of compactness of fuzzy topological spaces under the fuzzy almost semicontinuous, Let we called the following defintions:

A fuzzy topological space X is fuzzy semi-compact iff each fuzzy semi-open cover of X has a finite subcover and X is fuzzy nearly compact iff each fuzzy regular open cover of X has a finite subcover.

Theorem 5.1 If $f: X \to Y$ is a fuzzy almost semicontinuous function of a fuzzy semi-compact space X onto a fuzzy topological space Y, then Y is fuzzy nearly compact.

Proof: Let $\{G_i | i \in I\}$ be any fuzzy regular open cover of Y, then $\{f^{-1}(G_i) | i \in I\}$ is a fuzzy semi open cover of X. Since X is a fuzzy semi-compact, there exists a finite subfamily $\{f^{-1}(G_j) | j = 1, 2, \cdots, k\}$ of $\{f^{-1}(G_i) | i \in I\}$ which cover X. It follows that $\{G_j | = 1, 2, \cdots, k\}$ is a finite subfamily of $\{G_i | i \in I\}$ which covers Y. Hence Y is fuzzy nearly compact.

Lemma 5.2 A fuzzy semi-regular and nearly fuzzy compact space X is a fuzzy compact.

Proof: Let $\{U_i | i \in I\}$ be any fuzzy opencover of Y. Since X is semi-regular, $\forall\ i \in I$, there exists a family of regular open set $\{A_{ij} | j \in J_i\}$

such that $U_i = \bigcup_{j \in J_i} A_{ij}$ Let $\varphi = \bigcup_{i \in I} \{A_{ij} | j \in J_i\}$, then φ is a regular open cover of X. As X is nearly compact, there exists a family $\{A_{ij} | i \in I' \subset I, j \in J_i' \subset J_i, I'$ and J_i' are all finite set$\}$ which covers X.

Taking $U_i \supset A_{ij} (i \in I')$,

So $\{U_i | i \in I'\}$ is a finite subcover of X. Hence X is fuzzy compact.

Theorem 5.3. If $f: X \to Y$ is a fuzzy almost semicontinuous function of a fuzzy semi-compact space onto a fuzzy semi-irregular space Y, then Y is fuzzy compact.

Proof: It is clear from lemma 5.2 and theorem 5.1

Theorem 5.4 If $f: X \to Y$ is a almost semicontinuous function of a fuzzy strongly regular and fuzzy compact space onto a fuzzy topological space Y, then Y is nearly compact.

Proof: It is similar to the Lemma 5.2.

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