Crispification: Defuzzification of Intuitionistic Fuzzy Sets

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Abstract A new operation under intuitionistic fuzzy sets is defined in the paper. *Crispification* is analogical to the defuzzification operation of fuzzy sets. Its introduction allows development of various engineering applications of intuitionistic fuzzy sets: in control, optimization, expert systems etc. Four different definitions of this operation are introduced and analyzed.

Key words: intuitionistic fuzzy set, defuzzification, center of area, mean of maximum, BADD method

1. Introduction

Intuitionistic fuzzy set (IFS) [1] is an extension and generalization of the ordinary fuzzy set concept. It considers not only the degree of membership to a given set, but also the degree of non-membership such that the sum of both values is less than one [2]. In the last decade various applications of IFS concept to different fields have appeared: in classification [3,4], decision making [5], optimization [6,7], expert systems [8,9], logic programming [10,11], systems theory [12], graph theory [13], generalized net theory [14], neural networks [15] etc..

From the other hand, the apparatus of IFS is not full and is under investigation. The purpose of this paper is to introduce and analyze some definitions of *crispification* which is an analog to the basic operation of fuzzy sets - defuzzification. This definitions in combination with the conjunction and disjunction operations in IFS [2] allow to develop new engineering applications of IFS such as intuitionistic fuzzy (IF) controllers, IF expert systems etc.

2. Crispification : definitions

For a fixed universe E, the IFS (A) can be interpreted as a mapping E ---> [0;1]x[0;1] and it can be defined by a pair $<\mu_A ; \nu_A>$ where for $x \in E$ $\mu_A(x)$ denotes the degree of membership of x to the set A and $\nu_A(x)$ - the degree of non-membership of x to A; and $\mu_A(x)$ and $\nu_A(x)$satisfy the condition: $\mu_A(x) + \nu_A(x) \leq 1$. The set B is fuzzy set, in case when $\mu_B(x)+\nu_B(x) =1$. Now, by analogy with defuzzification operation of fuzzy sets, we introduce crispification operation as a map [0;1]x[0;1] ---> R, where R is the set of real numbers. Here it will be treated IFSs over universe E=R. The result of this operation is a crisp value which is representative for the given IFS as whole. This operation is necessary in controllers to derive the final control action (fuzzy or IF set-point can not be given to the servo in a control system) which will be realized. It is necessary also in decision making and expert systems for elicitation of information.

In fuzzy set theory, two basic defuzzification operators exist: center of area (COA) and mean of maximum (MOM) [16]. COA defuzzification is defined as follows:

$$x^*_{COA} = \frac{\sum_{i=1}^{N} \mu(x_i) x_i}{\sum_{i=1}^{N} \mu(x_i)} \qquad N = card(x_i)$$

where x* denotes defuzzified (crisp) value

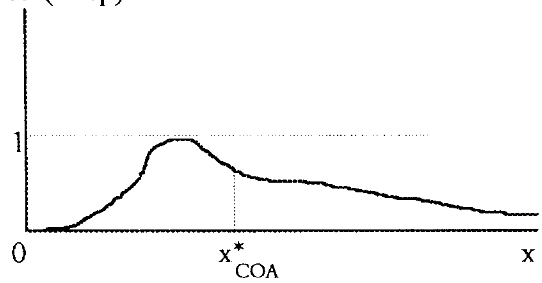

Fig.1

MOM defuzzification method is given by:

$$x^*_{MOM} = \frac{\sum_{j=1}^{M} x_j^m}{m} \qquad x^m = \{x \mid \mu(x^m) = \max_{i=1}^{N} \mu(x_i)\}$$

where M denotes number of maximums

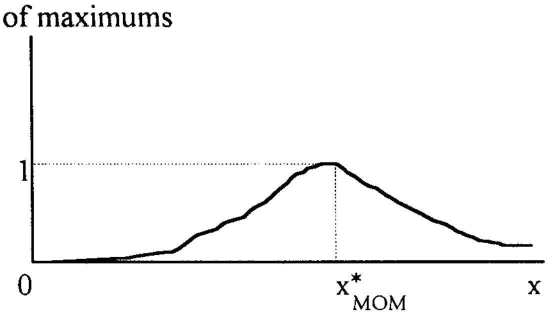

Fig.2

Recently, these two operators was successfully generalized by so called BADD operator [9]:

$$x^*_{BADD} = \frac{\sum_{i=1}^{N} \mu^\alpha(x_i) x_i}{\sum_{i=1}^{N} \mu^\alpha(x_i)} \qquad \alpha \in [0;\infty)$$

where α denotes power coefficient

It should be mentioned that for α=1 BADD implies COA and for $\alpha \longrightarrow \infty$ it approaches MOM method.

By analogy, three crispification operators are defined in the paper: IF_COA, IF_MOM, MOE (mean of extremum), IF_BADD (IF_xxx means Intuitionistic Fuzzy version of xxx). The COA crispification operation over IFS is defined as follows

$$x^0_{IF\_COA} = \frac{\sum_{i=1}^{N} (\mu(x_i) - \nu(x_i)) x_i}{\sum_{i=1}^{N} (\mu(x_i) - \nu(x_i))} \qquad N = card(x) \qquad \text{for} \quad \mu(x_i) > \nu(x_i)$$

This operator is defined basing on the difference between the degree of membership and non-membership which have to be positive.

The IF_MOM operator is defined by analogy with the MOM defuzzification operator over $(\mu(x_i)-\nu(x_i))$. Obviously, it guarantees the maximization of $\mu(x_i)$ and minimization of $\nu(x_i)$.

$$x^0_{IF\_MOM} = \frac{\sum_{j=1}^{L} x_j^l}{2L} \qquad x^l = \{x \mid \mu(x^l) - \nu(x^l) = \max_{i=1}^{N} (\mu(x_i) - \nu(x_i))\}$$

A new operator (by differ from defuzzification) MOE is introduced which averages all points with maximal $\mu(x_i)$ (the most acceptable points) and these with minimal $\nu(x_i)$ (the less non-acceptable):

$$x^0_{MOE} = \frac{\sum_{j=1}^{M} x_j^m}{2M} + \frac{\sum_{j=1}^{K} x_j^k}{2K} \qquad x^m = \{x \mid \mu(x^m) = \max_{i=1}^{N} \mu(x_i)\}$$

$$x^k = \{x \mid \nu(x^k) = \min_{i=1}^{N} \nu(x_i)\}$$

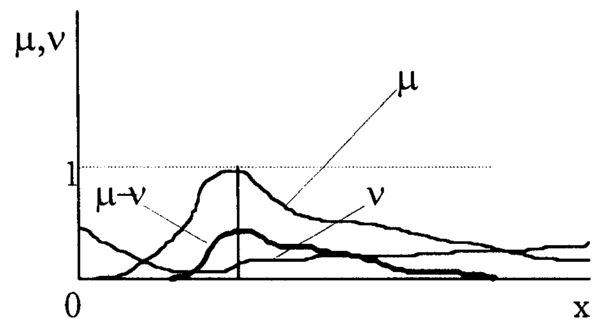

Fig.3

Finally, we introduce an analog of BADD operator over $\mu(x_i)-\nu(x_i)$:

$$x^0_{IF\_BADD} = \frac{\sum_{i=1}^{N} (\mu(x_i) - \nu(x_i))^{\alpha} x_i}{\sum_{i=1}^{N} (\mu(x_i) - \nu(x_i))^{\alpha}} \qquad \text{for } \mu(x_i) > \nu(x_i)$$

3. Crispification : properties

The basic features of these operators are similar with these of their analogs:

The IF_COA operator gives all possible solutions in which the degree of acceptance is higher than the degree of non-acceptance ($\mu(x_i)>\nu(x_i)$). However, it averages *good* and *poor* solutions (although it gives them different weights).

The IF_MOM and MOE operations gives information about the best solution(s) (MOE gives it in the sense of higher degree of acceptance and lower degree of non-acceptance). However they ignore information about the rest possible solutions.

The IF_BADD operator has the analogical property as BADD defuzzification operator:

- it implies IF_COA operator for $\alpha=1$ (it implies COA, if $\nu=0$ also);
- it approaches IF_MOM operator for $\alpha \longrightarrow \infty$ and approaches MOM when $\nu=0$ also;
- it implies middle average (MA) when $\alpha=0$:

IF_BADD($\alpha=1;\nu>0$) = IF_COA

IF_BADD($\alpha=1;\nu=0$) = COA

IF_BADD($\alpha \longrightarrow \infty;\nu>0$) = IF_MOM

IF_BADD($\alpha \longrightarrow \infty;\nu=0$) = MOM

IF_BADD($\alpha = 0$) = MA

α/ν	**0**	**positive**
0	IF_COA	COA
1	IF_MOM	MOM
∞	MA	

Table 1

4. Conclusion

A new operation under intuitionistic fuzzy sets is defined in the paper. *Crispification* is analogical to the defuzzification operation of fuzzy sets. These definitions will be very helpful for development of various engineering applications of intuitionistic fuzzy sets: in control, optimization, expert systems etc. Four definitions are introduced and analyzed. They have analogical properties with defuzzification operators.

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