Fuzzy Risk Assessment of Earthquake Hazard on City

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Abstract: Based on the concept of fuzzy risk suggested in paper[1], we discuss how to estimate the fuzzy risk of earthquake hazard on city. This is the application of the model in paper[1]. This paper shows how to get earthquake fuzzy risk $\mu_m(m, x)$ and calculate hazard fuzzy risk $\pi_C(l, x)$.

Keywords and phrases: Earthquake, hazard factor, fuzzy risk, hazard-formative environment, hazard-effected body, possibility distribution.

1. Introduction

China is a country where earthquake often occurs. Earthquake is one of main hazard factors with cause hazard on city. So, appropriate analysis on it is of momentous significance.

Risk assessment of earthquake hazard is usually called earthquake dangerous analysis. Probability and statistics methods is the main traditional means be used to deal with uncertain factor in earthquake risk.

Analysing hazard-effected body is usually called earthquake hazard prediction, which can be done by analysing data of historic earthquake hazard and researching characteristics of hazard-effected body. Generally, to analyse uncertainness of prediction, the means employed is based on probability theory.

The reason why the method of probability risk is used generally in earthquake hazard analysis is that probability theory has been being perfected and it is easy to be used in many occasion. But an important point is ignored, that is, although there may be statistical laws which control earthquake activity and earthquake hazard, it would be very difficult to recognize the laws. From the inaccuracy of earthquake forecast and plenty of much scattered data of historic earthquake, we can see there is a large distance between the facts and our knowledge. Under these circumstances, the result from the methods of probability risk is not a good one to help scientific decision. In this paper, we promote the fuzzy set theory into risk assessment of earthquake hazard on city, which can supply much information.

2. Fuzzy Risk Model

In paper[1], the theory and general way of calculation of fuzzy risk is suggested. In order to quoting conveniently in this paper, we simply repeat it.

The definition of fuzzy risk: Let Y be the discourse universe of hazard factor z, and x be the probability of surpassing y, where $y \in Y$, during T years . $\pi_z(y,x)$ is the possibility of surpassing y with a probability x about z.

$$R_z = \{\pi_z(y,x) | y \in Y, x \in [0,1]\} \tag{2.1}$$

is called fuzzy risk about z.

The common probability risk is a general relationship within $Y \times [0,1]$, but the fuzzy risk is a fuzzy relationship.

Let B be a hazard-effected body, L be the discourse universe of hazard, and $POSS_B(l,y)$ be the possibility when B is struck by force y, where $y \in Y$, $l \in L$.

$$d_B(y) = \{POSS_B(l,y) | l \in L, y \in Y\} \tag{2.2}$$

is called fuzzy distribution of hazard prediction about B.

Denote

$$\begin{cases} r_1(x,y) = \pi_z(y,x) \\ r_2(y,l) = POSS_B(l,y) \end{cases} \tag{2.3}$$

Then, hazard fuzzy risk is defined as

$$\begin{cases} R_B = \{r_B(l,x) | l \in L, x \in [0,1]\} \\ r_B(l,x) = \sup_{y \in Y}\{\min\{r_1(x,y), r_2(y,l)\}\} \end{cases} \tag{2.4}$$

If there are n independent hazard-effected bodies in city C, as

$$C = \{B_1, B_2, \cdots, B_n\} \tag{2.5}$$

then the hazard risk of city C is

$$\begin{cases} R_C = \{\pi_C(l,x) | l \in L, x \in [0,1]\} \\ \pi_C(l,x) = \sup_{l_1+l_2+\cdots+l_m=l}\{\min_{B_i \in C}\{r_{B_i}(l_i,x)\}\} \end{cases} \tag{2.6}$$

3. Fuzzy System Model of Hazard-Formative Environment

The earthquake which cause hazard on city is mainly constructive earthquake. This kind of earthquake is caused by fault suddenly breaking. So the research would start from analysing moving fault. If we take probability method, the first step is to study the relationship of magnitude m and frequency $N(m)$ by the statistics formula:

$$\log_{10} N(m) = p - qm \tag{3.1}$$

The second step is to let $\alpha = 2.3p, \beta = 2.3q$ and get the probability density function as the following:

$$f_M(m) = \begin{cases} \frac{\beta \exp[-\beta(m-m_0)]}{1-\exp[-\beta(m_u-m_0)]}; & m_0 \leq m \leq m_u; \\ 0, & \text{else.} \end{cases} \tag{3.2}$$

where m_0 is the minimum magnitude, and m_u is the maximum in the district.

Then, according to the type of different focuses and the distance relation between site and focus, using analytic algorithm, surpassing probability of a certain earthquake parameter (acceleration etc.) can be calculated [2,3,4].

$$P(Y > y) = \int_{m_1}^{m_2} \frac{x}{l} f_M(m)dm + \int_{m_2}^{m_u} f_M(m)dm \tag{3.3}$$

where m_1 is the larger of m_0 and m_1', where m_1' corresponds to the closest distance, whereas m_2 corresponds to the fault length.

This method is popular extensively in earthquake engineering. But it doesn't fit to the city which hasn't rich financial capacity. The method require earthquake engineers to analyse sites one by one as San Francisco. It is impossible and unnecessary for the cities of developing countries. Even having done it, there would be some wrong in the conclusion which worked out by expending a huge sum of money, because it is not easy to analyse reliability of parameters which are connected which a series of physical hypothesis and the selection of random model.

In fact, hazard-formative environment is a fuzzy system. We would use fuzzy method to analyse earthquake risk by which only description of main tendency is required, but not full accurate.

To fuzzy risk method, it is not necessary to find analytic function among parameters, and only fuzzy relation is required.

Suppose the district where the city lies in has n earthquake records

$$M' = \{M_1, M_2, \cdots, M_n\} \tag{3.4}$$

Let m_0 be the minimum magnitude which used in engineering, (m_0 can be 4), m_u be the maximum of magnitude in this district. Take $[m_0, m_u]$

to be the universe of magnitude and ΔM to be step according to n, the capacity of M'. Some discrete points with equal distance can be got in $[m_0, m_u]$.

$$X = \{x_1, x_2, \cdots, x_k\} \tag{3.5}$$

is defined as discrete universe of magnitude, where $x_j, j = 1, 2, \cdots, k$ is called controlling points.

Using equation (3.6), we can distribute[5] information of M in X,

$$q_{ij} = \begin{cases} 1 - \frac{|M_i - x_j|}{\Delta M}, & |M_i - x_j| \leq \Delta M; \\ 0, & \text{others.} \end{cases} \tag{3.6}$$

and obtain primary information distribution vector Q:

$$Q = (Q_1, Q_2, \cdots, Q_J) \tag{3.7}$$

where $Q_j = \sum_{i=1}^{j} q_{ij}$. Let $N_j = \sum_{i=1}^{j} Q_i$, we call

$$N = (N_1, N_2, \cdots, N_J) \tag{3.8}$$

primary frequency distribution of magnitude.

In the district where the city lies in , if the area is not very large, n is usually small which leads to N only rough approximation in frequency of magnitude. If the area is too large, the frequency distribution can not be used because district character would vanish. For risk assessment of urban earthquake hazard, the area would be smaller, and we would improve frequency distribution N by using information diffusion method[6].

Discrete universe is required for fuzzy approximating inference when we use fuzzy matrix. Let U and V be the discrete universe of magnitude and of frequency which are written as the following.

$$\begin{cases} U = \{u_1, u_2, \cdots, u_E\}, & u_e \leq u_{e+1} \\ V = \{v_1, v_2, \cdots, v_S\}, & v_s \leq v_{s+1} \end{cases} \tag{3.9}$$

Diffusing (x_j, N_j) in $U \times V$ by (3.10),

$$q_j(u, v) = \frac{1}{2\pi h_1 h_2} \exp[-\frac{(u - x_j)^2}{2h_1^2}] \exp[-\frac{(v - N_j)^2}{2h_2^2}] \tag{3.10}$$

we say that (x_j, N_j) gives the information with quantity $q_j(u, v)$ to point (u, v). Where, h_1, h_2 are called diffusing coefficients. When $J \geq 10$,

$$h_1 = 1.4208 \frac{m_u - m_0}{J - 1}, \qquad h_2 = 1.4208 \frac{N_1 - N_J}{J - 1} \tag{3.11}$$

Let

$$q(u,v)=\sum_{j=1}^{J} q_j, \qquad q_u=\sum_{s=1}^{S} q(u,v_s) \tag{3.12}$$

and

$$r(u,v)=q(u,v)/q_u \tag{3.13}$$

then

$$R_{m\to N}(u,v)=\{r(u,v)|u\in U, v\in V\} \tag{3.14}$$

is called fuzzy relationship matrix between magnitude and frequency.

High frequency correspond to high probability, on the contrary, low probability is corresponded. Assuming T is the length of time during it we get M'. And suppose that in next T years this district will follow the same law of earthquake activeness. So, earthquake which magnitude not less than m_0 must occur, it is said, the probability is 1 to meet an earthquake which magnitude not less than m_0. Because of this, we can turn V into probability universe:

$$P=\{p_1,p_2,\cdots,p_S\} \tag{3.15}$$

where $p_s=v_s/v_S, s=1,2,\cdots,S$. Denote $p=v/v_S$. Let $m=u$, $x=p$, and $\mu_m(u,p)=r(u,v)$, then:

$$r_m=\{\mu_m(m,x)|m\in U, x\in P\} \tag{3.16}$$

is the earthquake fuzzy risk of where the city lies during T years.

If we write m as magnitude, and x as probability,Because U is magnitude universe, and P is probability universe

Usually, there is a distance between the earthquake focus and the city we analysed, which causes a certain attenuation of earthquake intensity.

Make use of the fuzzy relationship between epicentral intensity I_0 and magnitude M, we can change earthquake risk into intensity risk. In China, Fuzzy relationship $R(M,I_0)$ was got[7] ,where $I_0=\{VI,VII,VII,\cdots,XII\}$ and $M=\{m_1,m_2,\cdots,m_{14}\}=\{4.6,4.9,\cdots,8.5\}$. Denote $R(M,I_0)$ by $\{r(m,i)|m\in M, i\in I_0\}$. With the help of (3.16) and $R(M,I_0)$, we can get intensity risk as

$$\begin{cases} R_{I_0}=\{\mu_{I_0}(i,x)|i\in I_0, x\in[0,1]\} \\ \mu_{I_0}(i,x)=\sup_{m\in M}\{\min\{r(m,i),\mu_m(m,x)\}\} \end{cases} \tag{3.17}$$

This relationship only shows the situation of focus. With the character of earthquake intensity, we can work out what it be after attenuation.

4. Fuzzy Mathematical Model of Earthquake Intensity Attenuation

Because we are not able to exactly know where earthquake would occur before it, the distance between the city and the focus is a fuzzy number. Let d_1, d_2 is the nearest and farthest distance, the simplest fuzzy number is a bell function:

$$\underset{\sim}{D} = \int_{d_1}^{d_2} \mu(d)/d \tag{4.1}$$

where

$$\mu(d) = \exp[-\frac{(\frac{d_2+d_1}{2} - d)^2}{\frac{(d_2-d_1)^2}{6}}] = exp[-1.5(\frac{d_2+d_1-2d}{d_2-d_1})^2] \tag{4.2}$$

We mainly analyse the intensity attenuation on city because it is convenient to use intensity to predict damage of hazard-effected bodies existing in large number.

The relationship of attenuation can be return from the estimation of intensity area. In a certain district, if earthquake in m will produce intensity I with area a, we know that the intensity at the site where distance to focus in $d = \sqrt{a/\pi}$ would be I.

Let M, A be magnitude and area universe, respectively. To site intensity i whose universe I is the same as I_0 we can get the primary information distribution matrix[8] $Q'_i(M, A)$, its elements is $q'_i(m, a)$. Let $d = \sqrt{a/\pi}$ and note $q_i(m, d) = q'_i(m, a)$, then we can construct the primary information distribution matrix $Q_i(M, D)$ between M and D which is the universe of distance.

$$Q_i(M, D) = \{q_i(m, d) | m \in M, d \in D\} \tag{4.3}$$

Using the normal method[8], a fuzzy relationship matrix from M to D can be got as

$$R_i(M, D) = \{r_i(m, d) | m \in M, d \in D\} \tag{4.4}$$

Because there is the fuzzy relationship $R(M, I_0)$ between M and I_0 in existing, we can get a fuzzy relationship from I_0 to D for I as the following

$$\begin{cases} R_i(I_0, D) = \{r_i(i_0, d) | i_0 \in I_0, d \in D\} \\ r_i(i_0, d) = \sup_{m \in M}\{\min\{r(m, i_0), r_i(m, d)\}\} \end{cases} \tag{4.5}$$

When epicentral intensity is i_0, after attenuation of $\underset{\sim}{D}$, intensity of the site would be

$$\begin{cases} R(I_0, I) = \underset{\sim}{D} \circ R_i(I_0, D) \\ r(i_0, i) = \sup_{d \in D}\{\min\{\mu(d), r_i(i_0, d)\}\} \end{cases} \tag{4.6}$$

Where $\mu(d)$ can be got from equation (4.2). So, to $\underset{\sim}{D}$, fuzzy risk of intensity can be calculated. That is

$$\begin{cases} R_I = \{\mu(i, x) | i \in I, x \in [0, 1]\} \\ \mu(i, x) = \sup_{i_0 \in I_0}\{\min\{r(i_0, i), \mu_{I_0}(i_0, x)\}\} \end{cases} \tag{4.7}$$

Where $r(i_0, i)$ comes from (4.6), and $\mu_{I_0}(i_0, x)$ from (3.17).

5. Method of Fuzzy Earthquake Disaster Prediction

The method of fuzzy earthquake hazard prediction must be selected according to the type of object of hazard effect and earthquake input. To object of hazard effect B, the result of prediction is showed in (2.2). To earthquake input y, many cases would appear, they can be showed by using possibility distribution $POSS_B(l, y)$ which means there is uncertainness for destructive.

The degree of earthquake hazard is usually recorded with language. It can be turned into fuzzy subset on the earthquake hazard index by using (5.1) which has an universe $U = \{u_1, u_2, \cdots, u_{11}\} = \{0, 0.1, 0.2, \cdots, 1\}$.

$$\begin{cases} A_1 = \text{Good condition} = 1/0 + 0.7/0.1 + 0.2/0.2 \\ A_2 = \text{Light destruction} = 0.2/0 + 0.7/0.1 + 1/0.2 + 0.7/0.3 + 0.2/0.4 \\ A_3 = \text{General destruction} = 0.2/0.2 + 0.7/0.3 + 1/0.4 + 0.7/0.5 + 0.2/0.6 \\ A_4 = \text{Heavy destruction} = 0.2/0.4 + 0.7/0.5 + 1/0.6 + 0.7/0.7 + 0.2/0.8 \\ A_5 = \text{Collapse} \doteq 0.2/0.6 + 0.7/0.7 + 1/0.8 + 0.7/0.9 + 0.2/1 \end{cases} \tag{5.1}$$

Generally, for a special kind of hazard-effected body, there must be a fuzzy relationship R between earthquake parameter and hazard. For instance, if hazard-effected body is single layer brick pillar factory-building, and x is dynamic reaction then $A = x \circ R$. Comparing A with the fuzzy sets of (5.1) in closing way, the possibility of every type of earthquake hazard can be got. In general, we can use valence closing method to do it.

Let $\underset{\sim}{A}$ be a fuzzy set which is calculated by dynamic reaction on the earthquake hazard universe U, write it in $\underset{\sim}{A}(u)$. Suppose $\underset{\sim}{B}$ is a fuzzy set in (5.1)and write it in $\underset{\sim}{B}(u)$.

$$\underset{\sim}{A} \cdot \underset{\sim}{B} = \vee_{u \in U}(\underset{\sim}{A}(u) \wedge \underset{\sim}{B}(u)), \quad \underset{\sim}{A} \odot \underset{\sim}{B} = \wedge_{u \in U}(\underset{\sim}{A}(u) \vee \underset{\sim}{B}(u)) \tag{5.8}$$

are called inter-times and outer-times of A and B separately. The valence closing of A and B is defined as

$$(\underset{\sim}{A}, \underset{\sim}{B}) = \frac{1}{2}(\underset{\sim}{A} \cdot \underset{\sim}{B} + (1 - \underset{\sim}{A} \odot \underset{\sim}{B})) \tag{5.9}$$

If $\underset{\sim}{B}$ is the degree of earthquake hazard, which occurred to a single layer brick pillar building , and x is dynamic reaction , then $(\underset{\sim}{A}, \underset{\sim}{B})$ is the possibility of earthquake prediction where $\underset{\sim}{B} \in \{A_1, A_2, \cdots, A_5\}$. Usually there are more than one $\underset{\sim}{B}$ make $(\underset{\sim}{A}, \underset{\sim}{B})$ be lager than zero, so several possibilities of destruction exited.

6. City System Analysis

According to the above formulas, to each hazard-effected body, we can calculate their risk of earthquake hazard. Now, we begin to discuss the hazard risk of the whole city as (2.6).

6.1. Classifying method

Assuming hazard-effected bodies in (2.5) can be classified into O categories. Responding to every B, we suppose that these O categories can be queued by degree of loss. The category which has the most serious degree of loss is sorted at the first. For instance, single layer brick building won't have a great loss when a middle earthquake occurs, instead gas pipes often cause large loss to cities. Therefore, gas pipe should be in front of single layer brick building.

To earthquake, the damage degree is usually recorded with language. It can be turned into fuzzy sets on the earthquake hazard index by using (5.1).

When we use classifying method to analyse the loss of the whole city on single hazard factor, we always suppose that the loss of hazard-effected body is in direct proportion to its volume in a category. Make use of historic data, we can have loss coefficient $c(k, A_j)$ to each category where A_j is damage degree and k is a serial number of categories.

Assuming the volume of B is V. If the damage degree is A_j, the loss of B would be $l = c(k, A_j)V$. If earthquake damage prediction of B is $POSS_B(A_j, i)$ where $i \in I$ is earthquake intensity, the loss of B would be a distribution as

$$r_B(i,l) = \sup_{A_j \in \{A_1, A_2, \cdots\}} \{\min\{POSS_B(A_j, i), r(A_j, l)\}\} \tag{6.1}$$

where

$$r(A_j, l) = \begin{cases} 1, & \text{when } l = c(k, A_j)V; \\ 0, & \text{others} \end{cases}$$

Obviously, fuzzy loss risk is

$$r_B(l, x) = \sup_{i \in I}\{\min\{\mu(i, x), r_B(i, l)\}\} \tag{6.2}$$

where i is a hazard factor variable, and x is probability. When hazard factor is earthquake, i is earthquake intensity, and $\mu(i, x)$ can be get from Eq.(4.7).

Now, the loss risk of city C would be

$$\begin{cases} R_C = \{\pi_C(l, x) | l \in L, x \in [0, 1]\} \\ \pi_C(l, x) = \sup_{l_1, l_2 + \cdots + l_m = l}\{\min_{B_k \in C}\{r_{B_k}(l_k, x)\}\} \end{cases} \tag{6.3}$$

where L is the discourse universe of economic loss.

6.2. Direct loss method

Classifying method is a rough way. If there are enough data, we ought to use direct loss method.

Assuming the construction value of B is $e^{(1)}$, equipment value is $e^{(2)}$, and enlarge coefficient is λ. $e^{(1)}$ can be got mainly according to building cost of B. In general, $e^{(1)}$=cost to build B−depreciation charge. $e^{(2)}$ can be got according to instrument, equipment, etc. which are in B. λ is relate to the social function of B.

Therefore, the whole value of B is

$$e = (1+\lambda)(e^{(1)} + e^{(2)}) \tag{6.4}$$

In fact, hazard index is defined by percentage. Let damage A_j ($j = 1, 2, \cdots$) are fuzzy sets on the discourse universe $U = \{u_1, u_2, \cdots, u_S\}$ of hazard index. If we know that B will face hazard as A_j, the loss would be

$$\pi_j(u_s, l) = \frac{\mu_{A_j}(u_s)}{lu_s} \tag{6.5}$$

If the damage prediction of B is $POSS_B(A_j, i)$, then

$$r_B(i, l) = \sup_{A_j \in \{A_1, A_2, \cdots\}} \{\min\{POSS_B(A_j, i), r(A_j, l)\}\} \tag{6.6}$$

is the loss distribution, where

$$r(A_j, l) = \sup_{u_s \in U} \{\pi_j(u_s, l)\} \tag{6.7}$$

Make use of (6.2) and (6.3), the whole city's loss risk on single hazard factor can be got.

7. Concluding Remarks

If we use classic method to analyse risk of earthquake hazard, we have to face hypothesis in every step. But it is difficult to judge if it is suitable in engineering field. Classic method is a idealized theory method.

The main point of fuzzy risk method is to use fuzzy relationship replacing analytic function pattern among parameters. A series of mathematical hypothesis can be omitted and the method is universal significance. Fuzzy relationship can be got from lots of historic earthquake data instead of mathematical hypothesis. In China, rich data make methods suggested in this paper is more suitable. Improvement of computer functions also provide guarantee for this method. Due to limited space, analysis of example will be suggested in papers afterwards.

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