A STUDY ON THEORY OF FUZZY SYSTEM GAME

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ABSTRACT The classical game theory, which is based on the double value logic theory, lose sight of much fuzzy information or grey information. In most problems of complicated game, the states of each side of the game systems are not certain. Much fuzzy information or grey informationed with in the real game grocess. Hence, distortion of decision making used the classical game theory is considerable under fuzzy enviroment or fuzzy meaning.

In theis paper, we introduce the fuzzy sets and possibility theor for dealing with the game problems under fuzzy circumstances or fuzzy states the some rough models of fuzzy game theory for solving problems involving fuzzy number and imprecies variables are presented and the relevant illustrations are given.

KEY WORDS: Fuzzy matrix game, Fuzzy system game theory, Imprecise variable, Process Control

1. INTRODUCTION

The game theory is widely applied to military affairs, physical training, commercial production and so on. But the classical game theor, which is based on the double value logic theor, lose sight of much fuzzy information and grey information. In most problems of complicated game, the states of each side of the game systems are not certain. Much fuzzy information or grey informution is contained with in the real game grecess. For example, in the commercial production competition, the profit distribution of ench side of competition is fuzzineas and in the military competition, not only gains and losses of the competition is fuzziness but also poasibility applied tacticeses is not certain. Hence, distortion of decision making used the classical game theroy is considerable under fuzzy enviroment or fuzzy meaning.

How do we consider the fuzzy information and the grey information of game problems into the processes and make theoretic decision accord with the actual game situation. This sudy task would be a important study tack of game theroy research.

Inthis paper, we introduce the fuzzy sets and possibility theory for dealling with the game problems under fuzzy circumstances or fuzzy states. The some rough models of fuzzy game theory for solving problms involving fuzzy number ond imprecise variables are presented and the relevant illustration are given.

2. GAME PROBLEMS UNDER FUZZY CIRCUMSTAHCE AND FUZZY MEANING

In the most game problems, uncertain elements contained with the game grocesses are common coccurrence, this uncertain elements are frequently shown fuzziness. Let us see a concrete example following:

Example 1. Acertain factory will determine a plan of the product output of the latter half of the year. According to past experience and market forecast, the produc market predicted will be three possibility: A good sale possibility as P(B1) and a common sale possibility as P(B2) and a bad sale possibility as P(B3). We use three tacticeses possibility to be : a big batch process as P(A1) and a middle batch process as P(A2) and a small batch process as P(3). Possible profis under the situation (Ai,Bj)composed each pure tactics can be estimated also, to see table 1.

Table 1. product output decision table

		product market		
		B1(good)	B2(common)	B3(bad)
		P(B1)	P(B2)	P(B3)
A1(big)	P(A1)	VH	H	VH
A2(middle)	P(A2)	VL	L	H
A3(small)	P(A3)	M	VL	H

HERE: VH—VERY HICH; H—HICH; M—MEDIUM; L—LOW; VL—VERY LOW

How do you make decision analysis and rational, selection of product tacticses, let business profit be maximum? supose that the product tacticses are one aide (I) of the game problem and the natural states is other one (II). There are tow outstanding characteristics in the above—mentioned example: (I) the states of the both side of the game system are fuzziness and one is presented by imprecise variables or fuzzy subsets. The description words: "big, middle, middle, small" of the product tacticses (I) are imprecise language and the denicription words "good, common, bad" of the na-tural states (II) are also. There are not outstanding limit between tacticese, the sircumstance of the game system is fuzziness call fuzzy circumstance. (II) The profits of each situation (Ai, Bj) are vague, and it must be described by fuzzy numbers or imprecise variables (i. e fuzzy subsets) not distinct numbers. The profits are vague to call fuzzy profit (or say fuzzy meaning).

Definition 1. The game problems under fuzzy circumstance and fuzzy meaning are fuzzy game problems. The game theory dealling with the fuzzy game problems call fuzzy game theory.

Following we will discuss the fuzzy matrix game problems.

3. FUZZY MATRIX GAME

Definition 2. call fuzzy matrix game G, if $G=\{\underset{\sim}{S1}, \underset{\sim}{S2}, \underset{\sim}{A}\}$ where $\underset{\sim}{S1}=\{Ai\}$ $i=1,2,3,\cdots\cdots m$; $\underset{\sim}{S2}=\{Bj\}$ $j=1,2,3,\cdots\cdots n$; $\underset{\sim}{A}=\{C_{ij}\}$ mxn only as n, m is limited positive integer and $\underset{\sim}{C_{ij}}$ is fuzzy numbers or imprecise varibles (i,e. fuzzy subsets). $\underset{\sim}{A}$ is called the fuzzy profit matrix. Let us intruduce μ: $\underset{\sim}{C_{ij}} \rightarrow [0,1]$ and thus $\mu=\{\mu_{ij}\}$ mxn it is called the fuzzy profit membership fuction (or grade) matrix. So the fuzzy matrix game $C=\{S1, \underset{\sim}{S2}, A\}$ might become $C=\{\underset{\sim}{S1}, \underset{\sim}{S2}, \mu\}$.

Definition 3. Supose that there is a fuzzy game $G^{\wedge}=\{\underset{\sim}{S1^{\wedge}}, \underset{\sim}{S2^{\wedge}}, \underset{\sim}{A^{\wedge}}\}$ and $G=\{\underset{\sim}{S1}, \underset{\sim}{S2}, \underset{\sim}{A}\}$, $G^{\wedge}$ is a G alternate state. Among wich $\underset{\sim}{S1^{\wedge}}=\{Ai^{\wedge}\}$ is the $\underset{\sim}{S1}=\{Ai\}$ expand tnctics set and $\underset{\sim}{S2^{\wedge}}=\{Bj^{\wedge}\}$ is the $\underset{\sim}{S2}=\{Bj\}$ expand tactics set, $\underset{\sim}{A^{\wedge}}=\{E_{ij}\}$, element E_{ij} is the profits of the situation $(Ai^{\wedge}, Bj^{\wedge})$. If the situation (Ai'', Bj'') make

$$E1\ (Ai'', Bj'')=\max_{i}\ \min_{j} E._{ij}$$

$$E2\ (Ai'', Bj'')=\min_{i}\ \max_{i} E._{ij}$$

to tenable. So we consider the situation (Ai, Bj'') be a optimum (or satisfactory) situation of the fuzzy matrix game G.

The optimun (or satisfactory) situation (Ai'', Bj'') of various fuzzy matrix games might be determined by differentiate dealling with. The following we will fut forward the way that find the optimum situation (Ai'', Bj'') thet the fuzzy matrix game have a saddle point.

4. FUZZY GAMES OF SADDLE FIORT EXISTED

Let us consider a fuzzy matrix game $C=\{\underset{\sim}{S1}, \underset{\sim}{S2}, \underset{\sim}{A}\}$ where $\underset{\sim}{A}=\{\underset{\sim}{C_{ij}}\}_{m\times n}$ supose that elements $\underset{\sim}{C_{ij}}$ have a special structure, we might imitate the method that is to find optimum situation with the classical game theory to find the optimum situation (Ai″, Bj″) of the fuzzy matrix game.

Definition 4. Supose that the fuzzy matrix game $G=\{\underset{\sim}{S1}, \underset{\sim}{S2}, \underset{\sim}{A}\}$ or $G=\{\underset{\sim}{S1}, \underset{\sim}{S2}, \mu\}$ where $\underset{\sim}{S1}=\{Ai\}$ $i=1,2,3,\cdots\cdots,m$; $\underset{\sim}{S2}=\{Bj\}$ $j=1,2,3,\cdots\cdots,n$ $\underset{\sim}{A}=\{\underset{\sim}{C_{ij}}\}_{m\times n}$ $\mu=\{\mu_{ij}\}_{m\times n}$ if situayion (Ai″, Bj″) make equation

$$\max_i \min_j \underset{\sim}{C_{ij}}=\min_j \max_i \underset{\sim}{C_{ij}} \text{ and } \max_i \underset{\sim}{C_{ij''}}=\min_j \underset{\sim}{C_{i''j}}$$

or

$$\max_i \min_j \mu_{ij}=\min_j \max_i \mu_{ij} \text{ and } \max_i \mu_{ij''}=\min_j \mu_{i''j}$$

be tenable, its relevant value mark VG call the value of the fuzzy ganme G. The situation (Ai″, Bj″) is a saddle piont of the game G and name the optimun (satisfactory) situetion. If $\underset{\sim}{C_{ij}}$ is fuzzy number, we might use the way that find maximum or minimum fuzzy number to find a saddle piont of tha fuzzy maxtrix game.

Example 2. Find a saddle piont of the G and the game value VG and the optimum situation. Supose that $G=\{\underset{\sim}{S1}, \underset{\sim}{S2}, \underset{\sim}{A}\}$ where $\underset{\sim}{S1}=\{A1, A2, A3, A4\}$ $\underset{\sim}{S2}=\{B1, B2, B3\}$

$$A=\begin{bmatrix} -7 & 1 & -8 \\ 3 & 2 & 4 \\ 16 & -1 & -3 \\ -3 & 0 & 5 \end{bmatrix}$$

Solution 1. Decide whether or not a saddle piont exist. First we find the minmum fuzzy number of each rows of the matrix A.

$\min_j \underset{\sim}{C_{ij}}=\min_j(\underset{\sim}{-7}, \underset{\sim}{1}, \underset{\sim}{-8})=\underset{\sim}{-8}$ $\quad$ $\min_j \underset{\sim}{C2j}=\min_j(\underset{\sim}{3}, \underset{\sim}{2}, \underset{\sim}{4})=\underset{\sim}{2}$

$\min_j \underset{\sim}{C3j}=\min_j(\underset{\sim}{16}, \underset{\sim}{-1}, \underset{\sim}{-3})=\underset{\sim}{-3}$ $\quad$ $\min_j \underset{\sim}{C4j}=\min_j(\underset{\sim}{-3}, \underset{\sim}{0}, \underset{\sim}{5})=\underset{\sim}{-3}$

next we find the maximum fuzzy number in each minimum fuzzy numbers.

$\max_i(\underset{\sim}{-8}, \underset{\sim}{2}, \underset{\sim}{-3}, \underset{\sim}{-3})=\underset{\sim}{2}$ $\quad$ so $\quad$ $\max_i \min_j \underset{\sim}{C_{ij}}=\underset{\sim}{C22}=\underset{\sim}{2}$

use tha same method, we find

$\min_j \max_i \underset{\sim}{C_{ij}}=\underset{\sim}{C22}=2$ and $\max_i \underset{\sim}{Ci2}=\min_j \underset{\sim}{C2j}=\underset{\sim}{2}$ So situation (A2, B2) is a saddle piont and $VG=\underset{\sim}{2}$ and the optimum situation is (A2, B2). Supose that we definite profit mumbership grades

$$\mu_{ij}=\frac{\underset{\sim}{C_{ij}}+|\underset{\sim}{C_{ij''}}|}{\max(\underset{\sim}{C_{ij}}+|\underset{\sim}{C_{ij}''}|)} \text{ here } |\underset{\sim}{C_{ij}''}|=\begin{cases} |\min \underset{\sim}{C_{ij}}| & \text{if } \min C_{ij}<0 \\ \underset{\sim}{0} & \text{if } \min C_{ij}>=0 \end{cases}$$

thus we can find a saddle piont according to the profit mumbership grades. According to the above definition, the fuzzy profit matrix $\underset{\sim}{A}$ can become

$$\mu=\begin{bmatrix} 0.041 & 0.375 & 0 \\ 0.458 & 0.416 & 0.5 \\ 1 & 0.291 & 0.209 \\ 0.209 & 0.333 & 0.541 \end{bmatrix}$$

Solution 2. we use profit membership function matrix to determine a saddle piont of the fuzzy game problem. First we obtain $\max_i \min_j(\mu_{ij})=\max_i(0, 0.416, 0.209, 0.209)=0.416=\mu22$ next obtain $\min_j\max_i(\mu_{ij})=\min_j(1, 0.416, 0.541)=0.416=\mu22$ and $\max_i \mu12=\min_j \mu2j=0.416=\mu22$ So we obtain the saddle piont to be (A2, B2) and the result is same with solution 1. the profit membership grade of optimum situation is 0.416 that express a specific value of VG with the maximum gains. We consider this expression still more express the gains grade of optimum situation.

If $\underset{\sim}{C_{ij}}$ is imprecise variable (fuzzy subsets), we might imitate the classical method to obtain the saddle piont of the game, according to the variables preference order provided.

Example 3. obtain the VG and the optimum situation (Ai", Bj"), supose that the fuzzy matrix game $G=(\underset{\sim}{S}1, \underset{\sim}{S}2, \underset{\sim}{A})$ where $\underset{\sim}{S}1=(A1, A2, A3)$ $\underset{\sim}{S}2=(B1, B2, B3)$

$$\underset{\sim}{A}=\begin{bmatrix} VH & H & VH \\ VH & L & M \\ L & VL & H \end{bmatrix}$$

Solution: we introduce priority order that is $VI \propto L \propto M \propto H \propto VH$. So we have

$\min_j \underset{\sim}{C}_{ij}=\min_j(VH, H, VH)=H$ $\min_j \underset{\sim}{C}2j=\min_j(VH, L, M)=L$

$\min_j \underset{\sim}{C}3j=\min_j(L, VL, H)=VL$ thus $\max_i \min_j \underset{\sim}{C}_{ij}=\max_i(M, L, VL)=H$

uss the same method, we find $\min_j \max_i \underset{\sim}{C}_{ij}=H$ and $\min_j \underset{\sim}{C}_{ij}=\max_i \underset{\sim}{C}i2=H$

Sove obtain the saddle piont to be (A1, B2) that is theoptimum situation of the game G and VG=H. Supose that we introduce the specific of among imprecise variables, the fuzzy gain matrix that is prencented by imprecis variables might transform the gains membership function matrix $\underset{\sim}{\mu}=(\mu_{ij})_{m\times n}$ Use the method $\max_i \min_j \mu_{ij}=\min_j \max_i \mu_{ij}$ and $\max_i \mu_{ij}=\min_j \mu_{ij}$ determene or not the end dle piont existed.

5. CONCUDING RENARRS

In the above——mentioned sections, we have put the basic concepts of fuzzy game and the method finding the optimum situation of the fuzzy matrix game. It is one of the fundamental contents of fuzzy game theory.

The method finding the optimum situation is bassed on the real application, its result is directly perceived through the senses. The theory about fuzzy games and the method finding the optimum situation of other fuzzy prblem will be put separately.

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