Processing of Sources of Fuzzy Quantities

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The submitted brief contribution presents an idea which should be elaborated and published (let us hope) in a more detailed paper being under preparation by professors J. Jacas and J. Recasens from Barcelona and by the author. That idea is as follows: Quantitative fuzzy data are emitted by different sources each of them displays some kind of homogenity representable by a generating map. Then the algebraic processing of such fuzzy data can be represented by some analogous manipulation with the generating maps.

1 Introduction

Due to [3] and other works a fuzzy quantity a is a fuzzy subset of the real line R with membership function $\mu_a : R \to [0,1]$ fulfilling

(1) $$\exists\, x_a \in R : \mu_a(x_a) = 1$$

(2) $$\exists\, x_1,\, x_2 \in R,\ x_1 < x_2,\ \forall\, x \notin [x_1, x_2],\ \mu_a(x) = 0.$$

In this paper we interpret these assumptions so that the fuzzy quantity a is the fuzzy extension (or fuzzification) of the crisp modal value x_a. The set of all fuzzy quantities fulfilling (1) and (2) will be denoted by $I\!R$.

The linear algebraic operations over fuzzy quantities from $I\!R$ can be derived from the extension principle [1] and defined in the classical way, namely, for $a,\ b \in I\!R,\ r \in R, r \neq 0$, also $r \cdot a$ and $a \oplus b$ belong to $I\!R$, and

(3) $$\mu_{r \cdot a}(x) = \mu_a(x/r),$$

(4) $$\mu_{a \oplus b}(x) = \sup_{y \in R}(\min(\mu_a(y),\ \mu_b(x-y))),$$

for any $x \in R$. That is the traditional approach elaborated and discussed in many papers.

In practical applications of the fuzzy numbers theory, namely if the relevant fuzzy quantities are neither triangular nor trapezoidal, the practical calculation of (3) and especially (4) is not generally simple. The calculation of membership function of the convex

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combinations like

(5) $$b = r_1 \cdot a_1 \oplus r_2 \cdot a_2 \oplus \cdots \oplus r_n \cdot a_n, \quad r_i \in R,\ a_i \in I\!R,\ i = 1, \dots, n,$$

repeated always whenever new fuzzy data $a_1, \dots, a_n$ are obtained can lead to serious difficulties. Even the application of other computation formulas based on more general t-norms, instead of (3) and (4), usually does not simplify the situation.

It would be desirable to derive another approach to such situation, and to find a more universal mathematical structure allowing to construct the membership functions

$$\mu_{r_1 \cdot a_1 \oplus \cdots \oplus r_n \cdot a_n}$$

of (5) in a simpler way.

2 Generating functions

Jacas and Recasens in some of their papers, let us mention here namely [2], have elaborated a concept of generating map. Besides other useful properties investigated by Jacas and Recasens, generating maps can serve as a tool for solving the problem mentioned above.

Generating map, or generating function, is a non-decreasing real-valued function defined for any $x \in R$ and such that

(6) $$f(0) = 0,$$

(7) $$\lim_{x \to \infty} f(x) = \infty, \quad \lim_{x \to -\infty} f(x) = -\infty.$$

If $a \in I\!R$ is a fuzzy quantity with modal value x_a then its membership function μ_a is generated from the function f by means of some operation, e. g.

(8) $$\mu_a(x) = \max\left(0,\ 1 - |f(x_a) - f(x)|\right).$$

The set of all fuzzy quantities generated by a function f is denoted by $I\!R_f$.

This procedure can be interpreted as follows. Each generating function f represents some source of fuzzy quantities which is in some sense selfconsistent. If f and g are two generating functions then each of them represents its own fuzziness of numbers. For $x_a = 3$, e. g., "fuzzy 3" from $I\!R_f$ can be quite different from "fuzzy 3" in $I\!R_g$.

If the solution of some applied problem demands to calculate a combination of fuzzy quantities like (5) then its particular elements $a_1, \dots, a_n$ can be generated by different sources, i. e. by different generating functions $f_1, f_2, \dots, f_n$. It inspires the following idea.

If we succeed to extend the linear operations over fuzzy quantities to analogous operations over the generating maps then it will be possible to derive from $f_1, \dots, f_n$ (in the case of (5)) a new function $h : R \to R$ fulfilling (6) and (7), and generating the results of combination (5). In such case it would be possible to do a simple calculation over crisp modal values

$$r_1 \cdot x_{a_1} + \cdots + r_n \cdot x_{a_n} = x_b$$

and then to generate the membership function μ_b of b using the generating map h by means of (8) or another formula of that type. Such procedure is evidently easier than the repetitive application of (3) and (4).

The operations over the generating functions reflecting the analogous processing of fuzzy quantities are now investigated by author and by J. Jacas and J. Recasens, and the results will be published in due time. In this moment it seems to be rational to define the necessary operations in the following way. The multiplication of f by a crisp real coefficient $r \in R$, $r \neq 0$, as

$$(9) \qquad \langle r \cdot f \rangle (x) = f(x/r) \quad \text{if } r > 0, \qquad = -f(x/r) \quad \text{if } r < 0.$$

Similarly, the "sum" of two sources of fuzzy quantities can be defined by the following operation over their generating functions f and g

$$(10) \qquad \langle f \boxplus g \rangle (x) = \sup_{y \in R}(\min(f(y), g(x-y))).$$

Evidently, in both cases the brackets $\langle\ \rangle$ denote one generating function derived from the objects in their interior by means of the operations in question.

3 Conclusive comment

The method described above displays some quite useful properties and the resulting fuzzy quantities possess properties consistent with the intuitive expectation of their natural features. More detailed results concerning their properties are to be published in a paper within a short time.

References

[1] D. Dubois, H. Prade: Fuzzy numbers: an overview. In: J. Bezdek (Ed.), Analysis of Fuzzy Information. CRC, Boca Raton, FA, (1987), Vol. I, 3–39.

[2] J. Jacas, J. Recasens: Fuzzy numbers and equality relations. Transactions of Second IEEE International Conference on Fuzzy Systems 1993, IEEE Neural Network Council 1993.

[3] M. Mareš: Computation over Fuzzy Quantities. CRC Press, Boca Raton. Florida, 1994.