Using fuzzy logic for knowledge representation at control synthesis*

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Abstract

A simple approach to knowledge representation based on Petri nets is presented in this paper. The possibility how the bivalued or/and fuzzy knowledge can be represented in a uniform way by means of a model in analytical terms is pointed out. The algorithms of the truth propagation as well as the knowledege inference are given in analytical terms. An illustrative example is introduced.

1 Introduction

To synthetize the control for technical systems, especially for some kinds of discrete event dynamic systems (DEDS) like flexible manufacturing systems (FMS), transport systems, communication systems, etc., a suitable form of knowledge representation is needed in addition to the system model. It is necessary in order to express or/and specify additional information concerning the control task, some external circumstances and influences, human experience, etc. Manytimes such knowledge can be fuzzy. Usually it can be expressed by means of the production *IF-THEN* rules. A system of the production rules creates the suitable knowledge base (KB).

This paper points out a simple approach to the knowledge representation by means of logical Petri nets (LPNs) and fuzzy Petri nets (FPNs). The FPNs were originally introduced in [3]. However, the approach based on such classical FPNs makes possible (see e.g. [1], [2]) to express only the fuzzy truth propagation. To express also the knowledge inference a modified approach is used here.

2 The knowledge description

Knowledge can be understand to be consisting of some pieces - e.g. some statements S_i, $i = 1, n_1$. Causality among such pieces of knowledge can be expressed by the *IF-THEN* production rules - e.g. R_j, $j = 1, m_1$. When the analogy between the FPNs positions (i.e. places) and the statements as well as the analogy between the FPNs transitions (taken

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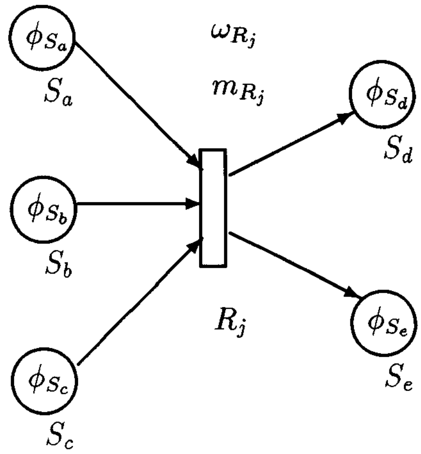

Figure 1: The rule R_j with the input and output statements

together with their input and output positions) and the rules are made, the knowledge can be represented by the FPNs. In order to illustrate this analogy, let us introduce Fig. 1.

Consequently, the structure of the KB can be formally defined to be the following quadruplet

$$\langle S, R, \Psi, \Gamma \rangle \quad ; \quad S \cap R = \emptyset \quad ; \quad \Psi \cap \Gamma = \emptyset \tag{1}$$

where

S $= \{S_1, ..., S_{n_1}\}$ is a finite set of the statements.
S_i , $i = 1, n_1$, are the pieces of knowledge (the elementary statements).

R $= \{R_1, ..., R_{m_1}\}$ is a finite set of the rules.
R_j , $j = 1, m_1$, are the rules either in the form of implications:
R_j : $(S_a \, and \, S_b \, and \, ... \, and \, S_c) \Rightarrow (S_d \, and \, S_e)$
or in the form of *IF-THEN* structures:
R_j : **IF** $(S_a \, and \, S_b \, and \, ... \, and \, S_c)$ **THEN** $(S_d \, and \, S_e)$.
where S_a, S_b, ..., S_c are the input statements of the rule R_j, and the S_d, S_e are the output statements of the rule R_j.

Ψ $\subseteq S \times R$ is a set of the causal interconnections between the statements entering the rules and the rules themselves. It can be expressed by means of the incidence matrix $\mathbf{\Psi}_K = \{\psi_{ij}^K\}$ $i = 1, n_1$; $j = 1, m_1$. In the analogy with the LPNs $\psi_{ij}^K \in \{0, 1\}$ and in case of the LPNs with fixed structure they are constant (in case of the LPNs with variable structure they are the bivalued functions). In the analogy with the FPNs $\psi_{ij}^K \in < 0, 1 >$. It means that the element ψ_{ij}^K represents the absence (when 0), presence (when 1) or a fuzzy measure of existence (when the value is between these boundary values) of the causal relation between the input statement S_i and the rule R_j. In other words, each ψ_{ij}^K, $K = 1, N_1$ are members of a fuzzy set with the corresponding membership functions $\mu_{\psi_{ij}}(\psi_{ij}^K)$.

Γ $\subseteq R \times S$ is a set of the causal interconnections between the rules and the statements emerging from them. It can be expressed by means of the incidence matrix

$\mathbf{\Gamma}_K = \{\gamma_{ij}^K\}$, $\gamma_{ij}^K \in \{0,1\}$ in analogy with LPNs or $\gamma_{ij}^K \in <0,1>$ in analogy with FPNs, $i = 1, m_1$; $j = 1, n_1$, i.e. very analogically (to the matrix $\mathbf{\Psi}_K$) expressing the occurrence of the causal relation between the rule R_i and its output statement S_j. Each γ_{ij}^K, $K = 1, N_1$ are members of a fuzzy set with the corresponding membership functions $\mu_{\gamma_{ij}}(\gamma_{ij}^K)$.

$\emptyset$ is an empty set.

The KB "dynamics" development (i.e. the statements truth propagation) can be formally expressed as follows

$$\langle \Phi, \Omega, \delta_1, \mathbf{\Phi}_0, \mathcal{M} \rangle \quad ; \quad \Phi \cap \Omega = \emptyset \tag{2}$$

where

$\Phi = \{\mathbf{\Phi}_0, ..., \mathbf{\Phi}_{N_1}\}$ is a set of the state vectors of the KB.

$\mathbf{\Phi}_K = (\phi_{S_1}^K, ..., \phi_{S_{n_1}}^K)^T$; $K = 0, N_1$ is the state vector of the KB (the state of the statements truth propagation) in the step K.

K is the discrete step of the KB dynamics development

T symbolizes the vector or matrix transposition

N_1 is an integer representing the number of different situations during the KB dynamics development (i.e. during the statements truth propagation)

$\phi_{S_i}^K$, $i = 1, n_1$ is the state of the truth of the elementary statement S_i in the step K. It means that the statement is false (when 0), true (when 1) or that the statement is true with a fuzzy measure (when this parameter acquires its value from the real interval between these two boundary values). In other words, for each S_i the $\phi_{S_i}^K$, $K = 1, N_1$ are members of a fuzzy set with the corresponding membership functions $\mu_{\phi_{S_i}}(\phi_{S_i}^K)$.

$\Omega = \{\mathbf{\Omega}_0, ..., \mathbf{\Omega}_{N_1}\}$ is a set of the "control" vectors of the KB.

$\mathbf{\Omega}_K = (\omega_{R_1}^K, ..., \omega_{R_{m_1}}^K)^T$; $K = 0, N_1$ is the "control" vector of the KB (i.e. the state of the rules evaluability) in the step K.

$\omega_{R_j}^K$, $j = 1, m_1$ is the state of the rule R_j evaluability in the step K. It means that the rule is not able to be evaluated (when 0), the rule is able to be evaluated (when 1) or that the rule is able to be evaluated with a fuzzy measure (when this parameter acquires its value from the interval between these two boundary values). It depends on the fuzzy values of the intput statements truth.

$\delta_1 : \Phi \times \Omega \longmapsto \Phi$ is a transition function of the KB.

$\mathbf{\Phi}_0$ is the initial state vector of the KB.

$\mathcal{M} = \{\mathbf{m}_1, ..., \mathbf{m}_{N_1}\}$ is the set of the vectors representing the rules truth values in the step K.

$\mathbf{m}_K = (m_{R_1}^K, ..., m_{R_{m_1}}^K)^T$; $K = 0, N_1$ is the vector of the rules truth values in the step K.

$m_{R_j}^K \in <0, 1>$, $j = 1, m_1$ expresses the fuzzy truth value of the rule R_j in the step K. In other words, for each R_j the $m_{R_j}^K$, $K = 1, N_1$ are members of a fuzzy set with the corresponding membership functions $\mu_{m_{R_j}}(m_{R_j}^K)$.

It is better for us to write $\mathbf{m}_K$ in the form of the (m_1 x m_1)-dimensional diagonal matrix $\mathbf{M}_K = diag\{m_{R_1}^K, ..., m_{R_{m_1}}^K\}$.

To imagine the introduced facts see Fig. 2. It can be said that the LPNs are (using the fuzzy sets terminology) a crisp form of the FPNs.

As to the membership functions of the statements truth, the rules truth and the fuzzy measures of existence of the causal interconnections are not analysed in details because they strongly depend on the actual application and they must be set by an expert from the actual domain.

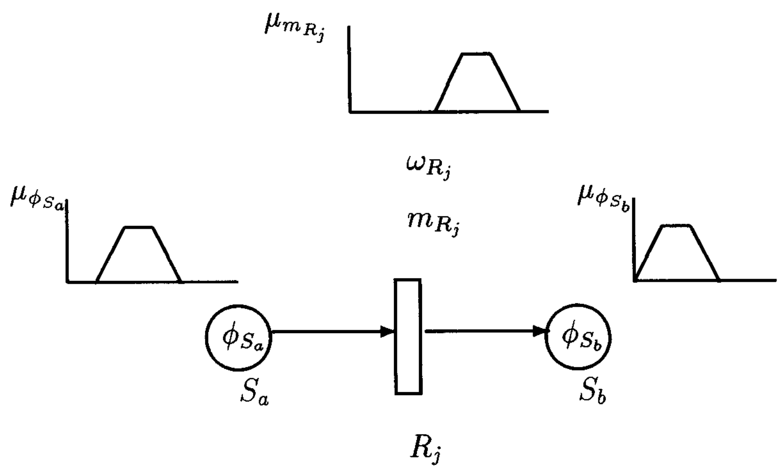

Figure 2: The simple fuzzy rule R_j with corresponding membership functions

The KB dynamics development (more precisely the transition function δ_1) can be expressed in analytical terms as follows

$$\mathbf{\Phi}_{K+1} = \mathbf{\Phi}_K \; \underline{or} \; \mathbf{\Delta}_K \; \underline{and} \; \mathbf{\Omega}_K \quad , \quad K = 0, N_1 \quad , \quad \mathbf{\Phi}_{K_{|K=0}} = \mathbf{\Phi}_0 \tag{3}$$

$$\mathbf{\Delta}_K = \mathbf{\Gamma}_K^T \; \underline{or} \; \mathbf{\Psi}_K \tag{4}$$

$$\mathbf{\Psi}_K \; \underline{and} \; \mathbf{\Omega}_K \; \leq \; \mathbf{\Phi}_K \tag{5}$$

where

and is the operator of logical multiplying in general. For both the bivalued logic and the fuzzy one it can be defined (for scalar operands) to be the minimum of its operands. For example the result of its application on the scalar operands a, b is a scalar c which can be obtained as follows: $a \; \underline{and} \; b \; = \; c \; = \; min \, \{a, \, b\}$.

or is the operator of logical additioning in general. For both the bivalued logic and the fuzzy one it can be defined (for scalar operands) to be the maximum of its operands. For example the result of its application on the scalar operands a, b is a scalar c which can be obtained as follows: $a \; \underline{or} \; b \; = \; c \; = \; max \, \{a, \, b\}$.

To derive the knowledge inference suppose that the inference mechanism consists of two parts:

1. the mechanism of the statements truth propagation (something like a carrier-wave) - when the rules truth values are crisp (equal to 1).

2. the influence of the fuzzy values of the rules truth (something like a modulation wave).

2.1 The truth propagation

The automatic mechanism of the statements truth propagation can be analytically described as follows

$$\begin{aligned}
\overline{\mathbf{\Phi}}_K &= \underline{neg}\, \mathbf{\Phi}_K = \mathbf{1}_{n_1} - \mathbf{\Phi}_K && (6)\\
\mathbf{v}_K &= \mathbf{\Psi}_K^T \underline{and}\, \overline{\mathbf{\Phi}}_K && (7)\\
\mathbf{w}_K &= \underline{neg}\, \mathbf{v}_K = \mathbf{1}_{m_1} - \mathbf{v}_K = \mathbf{1}_{m_1} - (\mathbf{\Psi}_K^T \underline{and}\, (\mathbf{1}_{n_1} - \mathbf{\Phi}_K)) && (8)\\
&= \underline{neg}(\mathbf{\Psi}_K^T\, \underline{and}\, (\underline{neg}\, \mathbf{\Phi}_K)) && (9)\\
\mathbf{\Omega}_K &= \mathbf{w}_K && (10)
\end{aligned}$$

where the meaning of quantities is the following

$\mathbf{v}_K$ is a m_1-dimensional auxiliary vector pointing out (by its nonzero elements) the rules that cannot be evaluated, because there is at least one false (of course in the LPNs analogy) statement among its input statements

$\mathbf{w}_K$ is a m_1-dimensional "control" vector pointing out the rules that have all their input statements true and, consequently, they can be evaluated in the step K of the KB dynamics development. This vector is a base of the inference, because it contains information about the rules that can contribute to obtaining the new knowledge - i.e. to transfer the KB from the state $\mathbf{\Phi}_K$ of the truth propagation into another state $\mathbf{\Phi}_{K+1}$. These rules correspond to the nonzero elements of the vector $\mathbf{w}_K$.

$\underline{neg}$ is the operator of logical negation in general. For both the bivalued logic and the fuzzy one it can be defined (for scalar operands) to be the complement of its operand. For example the result of its application on the scalar operands a is a scalar b which can be obtained as follows: $\underline{neg}\, a = b = 1 - a$.

After imbedding (10) into equation (3) we have

$$\mathbf{\Phi}_{K+1} = \mathbf{\Phi}_K\, \underline{or}\, \mathbf{\Delta}_K\, \underline{and}\, (\underline{neg}(\mathbf{\Psi}_K^T\, \underline{and}\, (\underline{neg}\, \mathbf{\Phi}_K))) \qquad (11)$$

2.2 The knowledge inference

The automatic mechanism of the knowledge inference can be described as follows

$$\begin{aligned}
\overline{\mathbf{\Phi}}_K &= \underline{neg}\, \mathbf{\Phi}_K = \mathbf{1}_{n_1} - \mathbf{\Phi}_K && (12)\\
\mathbf{v}_K &= \mathbf{\Psi}_K^T \underline{and}\, \overline{\mathbf{\Phi}}_K && (13)\\
\mathbf{w}_K &= \underline{neg}\, \mathbf{v}_K = \mathbf{1}_{m_1} - \mathbf{v}_K = \mathbf{1}_{m_1} - (\mathbf{\Psi}_K^T \underline{and}\, (\mathbf{1}_{n_1} - \mathbf{\Phi}_K)) && (14)\\
&= \underline{neg}(\mathbf{\Psi}_K^T\, \underline{and}\, (\underline{neg}\, \mathbf{\Phi}_K)) && (15)\\
\mathbf{\Omega}_K &= \mathbf{M}_K \,.\, \mathbf{w}_K = \mathbf{M}_K \,.\, (\underline{neg}(\mathbf{\Psi}_K^T\, \underline{and}\, (\underline{neg}\, \mathbf{\Phi}_K))) && (16)
\end{aligned}$$

After imbedding (16) into equation (3) we have

$$\mathbf{\Phi}_{K+1} = \mathbf{\Phi}_K\, \underline{or}\, \mathbf{\Delta}_K\, \underline{and}\, (\mathbf{M}_K \,.\, (\underline{neg}(\mathbf{\Psi}_K^T\, \underline{and}\, (\underline{neg}\, \mathbf{\Phi}_K)))) \qquad (17)$$

3 An illustrative example

Consider a set of the following statements:

$S_1 = "A" \quad S_2 = "B" \quad S_3 = "C" \quad S_4 = "D"$

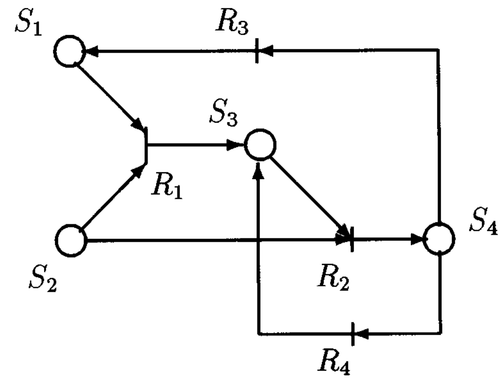

Figure 3: The PN-based representation of the KB

connected by the following systems of rules:

$R_1: \; IF(S_1 \underline{and} S_2) THEN(S_3)$ $\quad R_2: IF(S_2 \underline{and} S_3) THEN(S_4)$

$R_3: \; IF S_4 THEN S_1$ $\quad R_4: IF S_4 THEN S_3$

The situation is illustrated on Fig. 3. Let us consider the case of the KB where the structure is given as crisp. However, both the statements truth and the rules truth values are fuzzy. Consider the initial state of the statements truth in the form: "A" is *true with fuzzy measure* 0.3 (i.e. $\phi^0_{S_1} = 0.3$); "B" is *true with the fuzzy measure* 0.5 (i.e. $\phi^0_{S_2} = 0.5$).

Hence,

$\mathbf{\Phi}_0 = (0.3, 0.5, 0.0, 0.0)^T$; $\underline{neg}\, \mathbf{\Phi}_0 = (0.7, 0.5, 1.0, 1.0)^T$

$\mathbf{v}_0 = (0.7, 1.0, 1.0, 1.0)^T$; $\mathbf{w}_0 = (0.3, 0.0, 0.0, 0.0)^T$

When we consider the rules truth values $\mathbf{m}_K = (0.8, 0.7, 0.9, 1.0)^T$; $K = 0, N_1$

$\mathbf{\Omega}_0 = (0.24, 0.0, 0.0, 0.0)^T$; $\mathbf{\Phi}_1 = (0.3, 0.5, 0.24, 0.0)^T$

$\underline{neg}\, \mathbf{\Phi}_1 = (0.7, 0.5, 0.76, 1.0)^T$; $\mathbf{v}_1 = (0.7, 0.76, 1.0, 1.0)^T)$

$\mathbf{w}_1 = (0.3, 0.24, 0.0, 0.0)^T$; $\mathbf{\Omega}_1 = (0.24, 0.168, 0.0, 0.0)^T$

$\mathbf{\Phi}_2 = (0.3, 0.5, 0.24, 0.168)^T$

....

It means that (as a consequence of the state of the statements truth: "A" is *true with the fuzzy measure* 0.3 and "B" is *true with the fuzzy measure* 0.5) the statements "C" will be *true with the fuzzy measure* 0.24 and "D" will be *true with the fuzzy measure* 0.168 (i.e. $\phi^0_{S_3} = 0.24$ and $\phi^0_{S_4} = 0.168$) .

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