FUZZY ALMOST STRONG SEMICONTINUOUS MAPPINGS

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ABSTRACT

In this paper,we introduce the fuzzy almost strongly semicontinuous, fuzzy almost strongly semiopen and fuzzy almost strongly semiclosed mappings, and also establish some of their characteristic properties, and discuss relations between fuzzy almost strongly semicontinuous mapping and some other mappings.

Keywords: Fuzzy topological spaces; fuzzy strongly semiopen sets; fuzzy regular open sets; fuzzy almost strongly semicontinuous mappings.

1. PRELIMINARIES

In this work, A^{o}, A^{-} and A' will denote respectively the interior, closure and complement of the fuzzy set A in a fuzzy topological space (fts,for short). Simply by X, Y we shall denote fuzzy topological spaces (X,δ), (Y,τ).

Definition 1.1[5]. Let A be a fuzzy set of an fts X. Then A is called (1) a fuzzy strongly semiopen set of X iff there is a fuzzy open set B in X such that $B\leqslant A\leqslant B^{-o}$; (2) a fuzzy strongly semiclosed set of X iff there is a fuzzy closed set B in X such that $B^{o-}\leqslant A\leqslant B$.

Definition 1.2[5]. Let A be a fuzzy set of an fts X and define the following sets:

$A^{\Delta}=\bigcup\{\ B \mid B\leqslant A,\ B$ fuzzy strongly semiopen $\}$

$A^{\sim}=\bigcap\{\ B \mid A\leqslant B,\ B$ fuzzy strongly semiclosed $\}$.

We call A^{Δ} the fuzzy strong semi-interior of A and $A^{\sim}$ the fuzzy strong semi-closure of A.

2. MAPPINGS ON FUZZY SPACES

Definition 2.1. Let $f: X \to Y$ be a mapping from an fts X to another fts Y, f is said to be

(1) fuzzy almost strongly semicontinuous (f.a.s.s.c., for short) if $f^{-1}(B)$ is a fuzzy strongly semiopen set of X for each fuzzy regular open set B of Y.

(2) fuzzy almost strongly semiopen if $f(A)$ is a fuzzy strongly semiopen set of Y for each fuzzy regular open set A of X.

(3) fuzzy almost strongly semiclosed if $f(A)$ is a fuzzy strongly semiclosed set of Y for each fuzzy regular closed set A of X.

Definition 2.2. Let $f: X \to Y$ be a mapping. f is said to be f.a.s.s.c. at a fuzzy point p in X if fuzzy regular open set B of Y and $f(p) \leqslant B$, there exists a fuzzy strongly semiopen set A of X such that $p \leqslant A$ and $f(A) \leqslant B$.

Theorem 2.3. Let $f:(X,\delta) \to (Y,\tau)$ be a mapping. Then the following are equivalent:

(1) f is f.a.s.s.c.

(2) $f^{-1}(B)$ is a fuzzy strongly semiclosed set of X for each fuzzy regular closed set B of Y.

(3) $(f^{-1}(B^{o-}))^{\sim} \leqslant f^{-1}(B)$ for each $B' \in \tau$.

(4) $f^{-1}(B) \leqslant (f^{-1}(B^{-o}))^{\Delta}$ for each $B \in \tau$.

(5) there is a base η for τ such that $f^{-1}(B) \leqslant (f^{-1}(B^{-o}))^{\Delta}$ for each $B \in \eta$.

(6) there is a base η for τ such that $(f^{-1}(B^{o-}))^{\sim} \leqslant f^{-1}(B)$ for each $B' \in \eta$.

(7) $(f^{-1}(B))^{\sim} \leqslant f^{-1}(B^{-})$ for each fuzzy semiopen set B of Y.

(8) $f^{-1}(B^{o}) \leqslant (f^{-1}(B))^{\Delta}$ for each fuzzy semiclosed set B of Y.

(9) f is f.a.s.s.c. for each fuzzy point p in X.

Theorem 2.4. Let $f:(X,\delta) \to (Y,\tau)$ be a mapping. Then the following are equivalent:

(1) f is fuzzy almost strongly semiopen.

(2) $f(A) \leqslant (f(A^{-\circ}))^{\Delta}$ for each $A \in \delta$.

(3) there is a base η for δ such that $f(A) \leqslant (f(A^{-\circ}))^{\Delta}$ for each $A \in \eta$.

(4) $f(A^{\circ}) \leqslant (f(A))^{\Delta}$ for each fuzzy semiclosed set A of X.

(5) for each fuzzy set B of Y and each fuzzy regular closed set A of X, when $f^{-1}(B) \leqslant A$, there exists a fuzzy strongly semiclosed set C of Y such that $B \leqslant C$ and $f^{-1}(C) \leqslant A$.

Theorem 2.5. Let $f: X \rightarrow Y$ be a mapping. Then the following are equivalent:

(1) f is fuzzy almost strongly semiclosed.

(2) $(f(A^{\circ -}))^{\sim} \leqslant f(A)$ for each fuzzy closed set A of X.

(3) $(f(A))^{\sim} \leqslant f(A^{-})$ for each fuzzy semiopen set A of X.

(4) for each fuzzy set B of Y and each fuzzy regular open set A of X, when $f^{-1}(B) \leqslant A$, there exists a fuzzy strongly semiopen set C of Y such that $B \leqslant C$ and $f^{-1}(C) \leqslant A$.

3. MUTUAL RELATIONSHIPS

Proposition 3.1. Let $f: X \rightarrow Y$ be f.a.s.s.c. Then f is also fuzzy weakly continuous.

The converse of Proposition 3.1 need not be true. It is shown by Example 3.4.

Remark 3.2. From Proposition 3.1 and Definitions of fuzzy semicontinuous [1], fuzzy almost continuous[1], fuzzy weakly continuous[1], fuzzy strongly semicontinuous[5], fuzzy almost semicontinuous[2] and fuzzy almost precontinuous[3] (f.s.c., f.a.c., f.w.c., f.s.s.c., f.a.s.c. and f.a.p.c., for short respectively) it is clear that the following implications are true:

$$\begin{tikzcd}
\text{f.s.s.c.} \arrow[r, Rightarrow] \arrow[dr, Rightarrow] & \text{f.s.c.} \arrow[r, Rightarrow] & \text{f.a.s.c.} \\
\text{f.a.c.} \arrow[r, Rightarrow] & \text{f.a.s.s.c.} \arrow[r, Rightarrow] \arrow[ur, Rightarrow] \arrow[dr, Rightarrow] & \text{f.a.p.c.} \\
 & & \text{f.w.c.}
\end{tikzcd}$$

None of these implications is reversible (see [2],[5] and the following examples). The following Examples 3.3 and 3.4 also show that the f.a.s.s.c. and f.s.c. are independent notions.

Example 3.3. Let $X=\{a,b,c\}$, $\delta=\{0,A,B,1\}$ and $\tau=\{0,C,D,1\}$ where

$A(a)=0.1,\quad A(b)=0,\quad A(c)=0.2;$

$B(a)=0.5,\quad B(b)=0.5,\quad B(c)=0.5;$

$C(a)=0.2,\quad C(b)=0.3,\quad C(c)=0.4;$

$D(a)=0.6,\quad D(b)=0.5,\quad D(c)=0.7.$

Consider the identity mapping $f:(X,\delta)\rightarrow(Y,\tau)$. Then f is f.a.s.s.c., but neither f.a.c. nor f.s.c. Certainly f is not f.s.s.c.

Example 3.4. Let $X=\{a,b,c\}$, $\delta=\{0,A,B,1\}$ and $\tau=\{0,C,1\}$ where

$A(a)=0.2,\quad A(b)=0,\quad A(c)=0.1;$

$B(a)=0.6,\quad B(b)=0.5,\quad B(c)=0.7;$

$C(a)=0.4,\quad C(b)=0.3,\quad C(c)=0.2.$

Consider the identity mapping $f:(X,\delta)\rightarrow(Y,\tau)$. Then f is f.w.c. Obviously f is also f.s.c., so f is f.a.s.c. Yet f is not f.a.s.s.c.

Example 3.5. Let $X=\{a,b,c\}$, $\delta=\{0,A,1\}$ and $\tau=\{0,B,C,1\}$ where

$A(a)=0.6,\quad A(b)=0.8,\quad A(c)=0.5;$

$B(a)=0.3,\quad B(b)=0.2,\quad B(c)=0.1;$

$C(a)=0.5,\quad C(b)=0.5,\quad C(c)=0.5.$

Consider the identity mapping $f:(X,\delta)\rightarrow(Y,\tau)$. Clearly f is f.a.p.c., but not f.a.s.s.c.

Theorem 3.6. Let $f:X\rightarrow Y$ be a mapping from a fuzzy space X to a fuzzy semiregular space Y[1]. Then f is f.a.s.s.c. iff f is f.s.s.c.

Theorem 3.7. Let X_1, X_2, Y_1 and Y_2 be fuzzy spaces such that X_1 is product related to X_2 and Y_1 is to Y_2. Then the product $f_1\times f_2$: $X_1\times X_2\rightarrow Y_1\times Y_2$ of fuzzy almost strongly semicontinuous mappings f_1: $X_1\rightarrow Y_1$ and $f_2:X_2\rightarrow Y_2$ is f.a.s.s.c.

Proof. Let $B=\bigcup(A_i\times B_j)$, $i\in I$, $j\in J$, I and J are index sets, where the A_i's and B_j's are fuzzy open sets of Y_1 and Y_2 respectively. B is a fuzzy open set of $Y_1 \times Y_2$. Using Lemmas 2.3, 3.1 and Theorem 3.10 of [1], Theorem 2.5 of [5] and Theorem 2.3 we have

$$\begin{aligned}(f_1 \times f_2)^{-1}(B) &= \bigcup (f_1^{-1}(A_i) \times f_2^{-1}(B_j)) \\ &\leqslant \bigcup [(f_1^{-1}(A_i^{-o}))^{\Delta} \times (f_2^{-1}(B_j^{-o}))^{\Delta}] \\ &\leqslant [\bigcup (f_1^{-1}(A_i^{-o}) \times f_2^{-1}(B_j^{-o}))]^{\Delta} \\ &= [(f_1 \times f_2)^{-1}(\bigcup (A_i \times B_j)^{-o})]^{\Delta} \\ &\leqslant [(f_1 \times f_2)^{-1}((\bigcup (A_i \times B_j))^{-o})]^{\Delta} \\ &= [(f_1 \times f_2)^{-1}(B^{-o})]^{\Delta}.\end{aligned}$$

Thus $f_1 \times f_2$ is f.a.s.s.c.

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