Some Note on N-compact Sets in L-fuzzy Topological Spaces

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1. Introduction

The N-compactness in fuzzy topological spaces introduced by Wang [2] is the most reasonable fuzzy compactness in various kinds of fuzzy compactness. Zhao [1] has generalized it to the general L-fuzzy topological spaces (L-fts's, for short), and given some characterizations of N-compact L-fuzzy subsets. Based on this, a series of works have been launched [4,5]. But it is necessary to point out that N-compact L-fuzzy subsets have no the characterizations by means of covers and the family of (closed) L-fuzzy subsets which has finite intersection property. The purpose of this note is to give this three kinds of characterizations of N-compact L-fuzzy subsets.

2. Preliminaries

Our terminology and symbols follows [1]. Specifically, L always denote a fuzzy lattice, its smallest element and greatest element are 0 and 1 respectively. X always denote a non-empty crisp set. The collection of all the L-fuzzy subsets on X, denoted by L^X, can be naturally seen as a fuzzy lattice $(L^X, \leqslant, \vee, \wedge, ')$, its smallest element and greates element are 0_X and 1_X respectively, where $0_X(x) \equiv 0$ and $1_X(x) \equiv 1$ for any $x \in X$. The set of all the nonzero union-irreducible elements of L is denoted by $M(L)$.The elements in $M(L,X)=\{x_a:x\in X,a\in M(L)\}$ are called points.Put $p(L)=\{p\in L:1\neq p$ is prime elements of L$\}$ It is easy to check that $a\in M(L)$ iff $a'\in p(L)$.For $A\in L^X$ and $x_a\in M(L,X)$,x_a is called the point in A ,if $x_a\in A$, i.e., $a\leqslant A(x)$.For each $\Omega\subset L^X$,we define $\Omega'=\{A':A\in\Omega\}$,$\vee\Omega=\vee\{A:A\in\Omega\}$,$\wedge\Omega=\wedge\{A:A\in\Omega\}$.For $A\in L^X$ and $a\in L$,$A_{[a]}=\{x\in X:A(x)\geqslant a\}$. Let (L^X,δ) be an L-fts,$A\in L^X$, $\Omega\subset L^X$.We denote by A^- the closure of A in (L^X,δ),and define $\Omega^-=\{A^-:A\in\Omega\}$.

Definition 2.1 [3]. A subset B of L is called a maximal set of $a\in L$, if $\wedge B=a$ and for each subset C of L with $\wedge C\leqslant a$ and each $x\in B$,there is $y\in C$ such that $y\leqslant x$.The union of all the maximal sets of a is denoted by $\alpha(a)$,and put $\alpha^*(a)=\alpha(a)\cap p(L)$.

Lemma 2.2 [3]. (1) For each $a\in L$, there always exists a maximal set $\alpha(a)$ of a.

(2) For any $r\in p(L)$, $\wedge\alpha^*(r)=r$.

(3) For any $r\in p(L)$, $\alpha^*(r)=(\beta^*(r'))'$, where $\beta^*(r')=\beta(r')\cap M(L)$, $\beta(r')$ is the minimal set of $r'\in M(L)$.

3. Some characterizations of N-compact L-fuzzy sets

Definition 3.1. Let (L^X,δ) be an L-fts,$A\in L^X$, $r\in p(L)$. $\Omega\subset L^X$ is called an r-cover of A,if for each $x\in A_{[r']}$ there exists $U\in\Omega$ such that $x\in\iota_r(U)$. Ω is called an r^+-cover of A,if there exists $t\in\alpha^*(r)$ such that Ω is an t-cover of A.

Definition 3.2. Let (L^X,δ) be an L-fts,$A\in L^X$, $r\in p(L)$. $\Omega\subset L^X$ is called the family which has finite r^+-intersection property (or briefly, f.r^+-i.p.) in A, if for each $\psi \in 2^{(\Omega)}$ and every $t \in \alpha^*(r)$, there is $x \in A_{[t']}$ such that $(\wedge \psi)(x) \geqslant t'$.

Theorem 3.3. Let (L^X, δ) be an L-fts, $A \in L^X$. Then the following are equivalent:

(1) A is N-compact;

(2) For each $r \in p(L)$ and every r-cover Ω of A there exists $\psi \in 2^{(\Omega)}$ such that ψ is an r^+-cover of A;

(3) For each $r \in p(L)$ and every family $\Omega \subset \delta'$ which has f.r^+-i.p. in A, there is $x \in A_{[r']}$ such that $(\wedge \Omega)(x) \geqslant r'$.

(4) For each $r \in p(L)$ and every family $\Omega \subset L^X$ which has f.r^+-i.p. in A, there exists $x \in A_{[r']}$ such that $(\wedge \Omega^-)(x) \geqslant r'$.

Proof. (1) ==> (2) Suppose that A is N-compact and Ω is an r-cover of A ($r \in p(L)$). Then $\Theta = \Omega'$ is an r'-RF of A (see Definition 4.2 of [1]). In fact, for each point $x_{r'} \in A$, we see that $x \in A_{[r']}$. Then there is $U \in \Omega$ such that $x \in \iota_r(U)$, thus $U' \in \eta(x_{r'})$ (see Definition 2.3 of [1]). This shows that Θ is an r'-RF of A. From the N-compactness of A, there is $\psi = \{U_1, \ldots, U_n\} \in 2^{(\Omega)}$ such that $\Phi = \psi' \in 2^{(\Theta)}$ is an $(r')^-$-RF of A, i.e., there is $t \in \beta^*(r')$ such that Φ is an t-RF of A. Now we will prove that ψ is an r^+-cover of A. Put $s = t'$, then $s \in (\beta^*(r'))' = \alpha^*(r)$. For each $x \in A_{[s']} = A_{[t]}$, x_t is a point in A, thus there is $U_i \in \psi$ such that $U'_i \in \eta(x_t)$, so $x \in \iota_s(U_i)$. This shows ψ is an s-cover of A, and hence ψ is an r^+-cover of A.

(2) ==> (3) Suppose that (3) is untenable, then there exist $r \in p(L)$ and some $\Omega \subset \delta'$ which has f.r^+-i.p. in A such that $(\wedge \Omega)(x) \not\geqslant r'$ holds for each $x \in A_{[r']}$, i.e., $x \in \iota_r(\vee \Omega')$, and so there is $P \in \Omega$ such that $x \in \iota_r(P')$. This shows that $\Omega' \subset \delta$ is an r-cover of A. By (2),

there is $\psi=\{P_1,\ldots,P_n\}\in 2^{(\Omega)}$ such that ψ' is an r^+-cover of A, i.e., there is $t\in\alpha^*(r)$ such that ψ' is an t-cover of A, Hence for any $x\in A_{[t']}$, there is $P_i\in\psi$ such that $x\in\iota_t(P'_i)$, so $x\in\iota_t(\vee\psi')$, $(\wedge\psi)(x)\not\geqslant t'$. This contradicts that Ω has f.r^+-i.p. in A.

(3) ==> (4) Suppose that $\Omega\subset L^X$ has f.r^+-i.p. in A, then it is clear that $\Omega^-\subset\delta'$ has f.r^+-i.p. in A. From (3) we see that there is $x\in A_{[r']}$ such that $(\wedge\Omega^-)(x)\geqslant r'$.

(4) ==> (1) Suppose that A is not N-compact, then from definition 4.4 in [1], there exist $\alpha\in M(L)$ and some α-RF $\Phi\subset\delta'$ of A such that any $\psi\in 2^{(\Phi)}$ is not an α^--RF of A, i.e., ψ is not an γ-RF of A for any $\gamma\in\beta^*(\alpha)$. Hence there is some point $x_\gamma\in A$ such that $P\overline{\in}\eta(x_\gamma)$ for any $P\in\psi$, i.e., $\gamma\leqslant P(x)$, and so $\gamma\leqslant(\wedge\psi)(x)$. Note that $x\in A_{[\gamma]}$ and $\gamma'\in(\beta^*(\alpha))'=\alpha^*(\alpha')$. From this we see that Φ has f.$(\alpha')^+$-i.p. in A. By (4) there is $x\in A_{[\alpha]}$ such that $(\wedge\Phi^-)(x)=(\wedge\Phi)(x)\geqslant\alpha$, and thus $P(x)\geqslant\alpha$ holds for each $P\in\Phi$. This shows that $P\overline{\in}\eta(x_\alpha)$ holds for each $P\in\Phi$, this contradicts that Φ is an α-RF of A, and hence A is an N-compact set, and the proof is completed.

References

[1] Zhao Dongsheng, The N-compactness in L-fuzzy topological spaces, J. Math. Anal. Appl. 128(1987), 64--79.

[2] Wang Guojun, A new fuzzy compactness defined by fuzzy nets, J. Math. Anal. Appl. 94(1983), 1--23.

[3] Wang Guojun, The Theory of L-Fuzzy Topological Spaces, Shanxi Normal University Press, Xi'an, China, 1988 (in Chinese).

[4] Meng Guangwu, Lowen's compactness in L-fuzzy topological spaces, Fuzzy Sets and Systems, 53(1993), 329--333.

[5] Zhang Xingfang,Covering thpe characterizations for some kinds of L-fuzzy compact subsets in L-fuzzy topologicalspaces,J. Liaocheng Teacher's College,3(1994),18--24 (in Chinese).