# Comparison of Some Fuzzy Composition Operators

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**Abstract:** This paper gives some Fuzzy composition operators and gets order of seven composition operators on Mamdani's composition algorithm. Theoretical analysis shows that the paper extends Mamdani's composition algorithm and obtains more rational composition operators.

## 1. Introduction

Mamdani's composition algorithm is a kind of Fuzzy reasoning method which widely is studied and used in Fuzzy control theory and industrial application. At present, most of controllers are desiged by Mamdani's composition algorithm. However, theory and practical application shows that Mamdani's method isn't the best Fuzzy reasoning method which fits our intuition. The paper improves Mamdani's composition algorithm for Fuzzy inference in preciseness and uniformity. By changeing Mamdani's max— min composition operator into a series of other composition operators, we obtain a set of composition operators and set up the order of Fuzzy reasoning. So, we provide some Fuzzy composition operators with comparison for designing Fuzzy controllers and lead to rational selection of Fuzzy reasoning method in Fuzzy control.

## 2. Unifomi criterium of Fuzzy reasoning

(1) Mamdani's composition algorithm

Zadeh presented compositional rule of inference in 1975. It transforms a Fuzzy conditional proposition for instance "If $X$ is $A$ then $Y$ is $B$" into a Fuzzy relation accoding to Fuzzy logic implication operators. later, it reduces on basic of composition of Fuzzy relation.

Up to now, there have been a lot of Fuzzy composition reasoning algorithm. Mamdani's max—min composition algorithm is popular in Fuzzy control and application. In the following, we will introduce Mamdani's composition algorithm.

Suppose $X_i \in F(U)$, $Y_i \in F(V)$, $i \in I$(index set), $i=1,2,\cdots,n$ to be Fuzzy set in domain $U=\{U_1,U_2,\cdots,U_n\}$ and $V=\{V_1,V_2,\cdots,V_n\}$. $F(U)$ and $F(V)$ are Fuzzy power sets of domain $U$ and $V$ respectively. $A_i$, $B_i$, $i=1,2,...,n$ express Fuzzy variable of convex regular in $F(U)$ and $F(V)$ respectively. Now, we consider the following Fuzzy reasoing form:

Ant 1 : If $X_1=A_1$ then $Y_1=B_1$

Ant 2 : If $X_2=A_2$ then $Y_2=B_2$

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Ant n : If $X_n=A_n$ then $Y_n=B_n$ (1)

Ant : If $X$ is $A'$

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Cons : $Y$ is $B'$

So, $B'=A \circ R=A' \circ \bigcup_{i=1}^{n}(A_i \times B_i)$. Where " $\circ$ " is max—min composition operators. X is cartesion product. $R=(r_{ij})_{m\times n}$, $r_{ij}=\bigvee_{i=1}^{n}(\mu_{Ak}(i) \wedge \mu_{Bk}(j))$. This is a Mamdani's composition algorithm.

(2) Uniformity of inference

At present, many scholars have presented fifteen Fuzzy reasoning methods and compared their properties [2,3,4]. However a lot of papers didn't propose evaluating criterium to these Fuzzy reasoning methods and leaded to selecting Fuzzy reasoning methods's arbitrary in industrial application. Hence, we give a kind of evaluating method about Fuzzy reasoning.

Definition 1: Unifomi criterium of Fuzzy reasoning.

Suppose (1)'s reasoning form: "If $A'=A_i$ then $B'=B_i(i=1, 2, \cdots, n)$". The Fuzzy reasoning method is called satisfing uniformi criterium. If "$A'=A_i$ then $B'\neq B_i$", the case is called dissatisfing unifomi creterium. Unifomi creterium also is called reappearing of rule. A kind of Fuzzy reasoning method is good or bed, best verifing method is reappearing of original rule. Because fundment of Fuzzy reasoning is on basic of original rule. It reflects characters of practical problem which we deal with. So, this evaluating method is effect.

Definition 2: The Fuzzy reasoning method is called complete reappearing : If $A' = A_i$ then $B' = B_i (i=1,2,\cdots,n)$.

Definition 3: The Fuzzy reasoning method is called uncomplete reappearing : If $A' = A_i$ then $B' \subseteq B_i$ or $B' \supseteq B_i, i=1,2,\cdots,n$.

Proposition 1. Mamdani's composition algorithm is uncomplete reappearing.

Proof. Only to get $A_i \circ R \supseteq B_i$, $i=1,2,\cdots,n$.

Because
$$A_i \circ R = \bigvee_{k=1}^{m} A_i(u_k) \wedge (\bigvee_{p=1}^{n} A_p(u_k) \wedge B_p(v))$$
$$\geqslant \bigvee_{k=1}^{m} A_i(u_k) \wedge A_i(u_k) \wedge B_i(v)$$
$$= \bigvee_{k=1}^{m} A_i(u_k) \wedge B_i(v)$$
$$= B_i(v)$$ (Because $A_i$ is a Fuzzy set of convex regular)

$$A_i \circ R \supseteq B_i(v) \quad \square$$

From proposition 1, we may find that Mamdani's composition algorithm's result conclude original rule's result. Futhermore, by increasing rule's numbers the difference between Mamdani's reasoning result and original rule's result are more and more. So, Mamdani's method isn't best one.

# 3. Discussing other composition operators

Consider two kind of operators

| product operators | sum operators |
| --- | --- |
| $a \wedge b = \min\{a,b\}$ | $a \vee b = \max(a,b)$ |
| $a \cdot b = ab$ | $a \dotplus b = a+b-ab$ |
| $a \odot b = 0 \vee (a+b-1)$ | $a \oplus b = 1 \wedge (a+b)$ |
| $a \mathbin{\dot\wedge} b = \begin{cases} a, & b=1 \\ b, & a=1 \\ 0, & a,b<1 \end{cases}$ | $a \mathbin{\dot\vee} b = \begin{cases} a, & b=0 \\ b, & a=0 \\ 0, & a,b>1 \end{cases}$ |

Obviously, all of above operators are Fuzzy operators.

**Proposition 2.** The following ineqalities are correct for product operators and sum operators:

$$\mathsf{A} \leqslant \odot \leqslant \cdot \leqslant \wedge \leqslant \vee \leqslant + \leqslant \oplus \leqslant \forall$$

Result is obvious.

Now, we begin to discuss other operators on basic of Mamdani's method. Change Mamdani's max—min operators into the following six kind of operators: $\vee - \cdot$, $\vee - \odot$, $\vee - \mathsf{A}$, $+ - \wedge$, $\oplus - \wedge$, $\forall - \wedge$. So, we may form six kind of composition reasoning methods and set up their relations between above six methods and Mamdani's method. We obtain the following propositions.

**Proposition 3.** $\vee (A_i \wedge R) \supseteq \vee (A_i \cdot R) \supseteq \vee (A_i \odot R) \supseteq \vee (A_i \mathsf{A}\ R) = B_i$

**Proof.** Because

$$\begin{aligned}
\vee (A_i \wedge R) &= \bigvee_{k=1}^{m} A_i(U_k) \wedge [\bigvee_{p=1}^{n} A_p(U_k) \wedge B_p(V)] \\
&\geqslant \bigvee_{k=1}^{m} A_i(U_k) \cdot [\bigvee_{p=1}^{n} A_p(U_k) \wedge B_p(V)] \\
&= \vee (A_i \cdot R) \\
&\geqslant \bigvee_{k=1}^{m} A_i(U_k) \odot [\bigvee_{p=1}^{n} A_p(U_k) \wedge B_p(V)] \\
&= \vee (A_i \odot R) \\
&\geqslant \bigvee_{k=1}^{m} A_i(U_k)\ \mathsf{A}\ [\bigvee_{p=1}^{n} A_p(U_k) \wedge B_p(V)] \\
&= \vee (A_i \mathsf{A} R) \\
&\geqslant \bigvee_{k=1}^{m} A_i(U_k)\ \mathsf{A}\ [\vee A_i(U_k) \wedge B_i(V)] \\
&= \bigvee_{k=1}^{m} A_i(U_k)\ \mathsf{A}\ B_i(V) \\
&= B_i(V)
\end{aligned}$$

Result is correct. □

**Proposition 4.** $\forall\ (A_i \wedge R) \supseteq \oplus\ (A_i \wedge R) \supseteq +\ (A_i \wedge R) \supseteq \vee (A_i \wedge R)$

**Proof.** The proving method is the same as proposition 3.

We may get the following proposition from above two propositions.

**Proposition 5.** $\forall\ (A_i \wedge R) \supseteq \oplus\ (A_i \wedge R) \supseteq +\ (A_i \wedge R) \supseteq \vee (A_i \wedge R) \supseteq \vee (A_i \cdot R) \supseteq \vee (A_i \odot R) \supseteq \vee (A_i \mathsf{A} R) = B_i$

From proposition 5, we may find that best composition reasoning method is

$\vee - \wedge$ operator, the bad method is $\vee - \wedge$ operators and Mamdani's method is only in middle state. Futhermore, $\vee - \wedge$ composition algorithm content with complete reappearing of rule.

## 4. Conclution

The paper gives six kind of Fuzzy composition reasoning methods and sets up their reasoning order between the six methods and Mamdani's method on unifomi criterium. Futhermore, the paper obtains the best composition reasoning method. The good method extends Mamdani's composition algorithm. It's conclusion is significance to Fuzzy theory research and practical application.

## References

[1] E. H. mamdani and B. R. Gains, Fuzzy reasoning and $A_p$plication, London: Academic, 1981.

[2] M. Mizumoto and H. J. Zimmermam, comparision of Fuzzy Reasoning Methods, Fuzzy sets and Systems, 8(1982), 253—283.

[3] M. Mizumoto, Fuzzy controls by Product—Sum— Gravity Method, Advancement of Fuzzy theory and Systems in China and Japan. PP, cl. 1—cl. 4. International Acadimic Publisher, 1990.

[4] A. Piskunov, Fuzzy Implication in Fuzzy Systems control, Fuzzy and Systems, 45(1992), 25—35.

[5] J. B. Kiszka, etal., The Inference of some Fuzzy implication operators on Accuracy of a Fuzzy Model—Part Ⅰ, Part Ⅱ, Fuzzy sets and Systems, 15 (1985), 111—129, 223—240.

[6] M. Mizumoto, Fuzzy Mathematics and $A_p$plication, Liu Feng Pu Translate, Publishing House of science, 1986.

[7] Zou Kai Qi and Xu Yang, Fuzzy Systems and Expert Systems. Publishing House of West—South Traffic University, 1989.