Book Review

Fuzzy Measure Theory

by Zhenyuan Wang and George J. Klir

Plenum Press, New York, 1992, ix + 354 pages

This book intends to be an extensive presentation of the notion of inclusion-monotonic set functions, coined as "fuzzy measures" by Sugeno in 1974, and especially the so-called "fuzzy integral", a concept invented by Sugeno himself, as a qualitative counterpart of Lebesgue integral. In Sugeno's integral the minimum operation plays the role of the product while the supremum is a substitute to the (infinite) summation. The book appears under the form of a monograph supplemented by appendices, two of which reproduces 6 already published papers. Five of them are from other authors.

The monograph part of the book is a careful and systematic presentation of the results obtained by Zenyuan Wang on the fuzzy measure and integral since the early eighties. It is a piece of pure mathematics. The brief first chapter is an interesting discussion that casts the book in historical perspective, and position fuzzy measures inside some aspects of the emerging trend in non-additive probabilities. Chapter 2 reviews results on infinite sets; an original typology of families of subsets of an infinite set is supplied. It is original because it contains the new concept of a plump class (closed under arbitrary unions and intersections), that the first author is supposed to have introduced. Plump classes contrasts with more usual constructs such as sigma-algebras, closed under countable intersections, and complementations. Chapter 3 is devoted to fuzzy measures and variants thereof. The point is that in the infinite case, fuzzy measures have been introduced by Sugeno as monotonic set functions that are continuous from below *and* from above, just as probability measures. The most typical example of fuzzy measure is the λ-fuzzy measure, which enjoys a distorted form of additivity, parametrized by the real number λ. Wang and Klir notice that this concept can be systematized by means of a bijective mapping that distorts an additive measure (called quasi-measures by the authors). It is recalled that well-known classes of monotonic set functions such as belief functions and possibility measures do not quite fit in the fuzzy measure framework. Indeed, possibility measures are not necessarily continuous from above. The authors also point out that belief functions are only continuous from above (but this make sense in the non-finite setting, which is rather unusual for belief functions).

Chapter 4 deals with so-called extensions of various classes of set functions, such as quasi-measures and possibility measures. The idea is that, given two families of subsets, one of which includes the other, how to extend the definition of a set function living on the smaller family over to the bigger one that contains it. Interestingly such extensions do not always exist for fuzzy measures. The chapter gives mathematical conditions under which this extension exists and is unique.

Chapter 5 is devoted to subclasses of monotonic set functions that are characterized by properties weaker than sub-additivity. Especially, the notion of auto-continuity introduced by the first author is presented in details. Interestingly, sub-additive set-functions are auto-continuous, while they are generally not continuous (as fuzzy measures are supposed to be).

Chapter 6 studies measurable functions in the setting of fuzzy measures. Several notions of convergence of functions with respect to a fuzzy measure are studied and compared. A section is devoted to "convergence in possibility" (as opposed to convergence in probability).

Sugeno's fuzzy integral is presented in Chapter 7, with special emphasis on convergence theorems (monotonic convergence, but also everywhere convergence). Preliminary investigations on fuzzy measures defined by fuzzy integrals are reported.

Chapter 8 hints on a much more ambitious topic, namely the imbedding of both Lebesgue integral and Sugeno's integral, in a more general setting, with a generalized sum (subsuming the maximum operation) and a generalized product (subsuming the minimum operation). The name "pan-integral" (maybe it means "universal", from the Greek?) is coined for that generalization.

There is a Chapter 9 entitled "applications". It essentially illustrates Dempster-Shafer theory and possibility theory by means of examples. It also reviews the role of fuzzy integrals in subjective evaluation processes, and sketches the basis of fuzzy number theory. Namely the sum of fuzzy numbers is obtained as a convolution with respect to a fuzzy integral. Three appendices complement this material in a very useful way, with some background on classical measure theory, a glossary of concepts and a glossary of notations. An appendix on fuzzy sets is also supplied. However it is not useful at all in the reading of the first part of the book, but is relevant for an easier reading of some of the papers reproduced at the end of the book. These papers (which are not explicitly mentioned in the table of contents) are as follows: two papers by Qiao Zhong (one of them cosigned by the first author) provide complementary material on fuzzy integrals of fuzzy sets. The four other papers are taken from the engineering literature: the 1990 paper by John Yen generalizing belief functions to fuzzy sets, the 1990 paper by Tom Strat on decision analysis using belief functions, a 1987 paper by Henri Prade and Claudette Testemale on the use of possibility and necessity measures in information retrieval, and finally, a 1990 paper by Tahani and Keller on the use of fuzzy integrals for the purpose of data fusion in computer vision.

This book is a valuable contribution to the literature on non-additive probabilities and its main merits are to make the works by Wang Zhenyuan easily available. These works were indeed published for a major part in not-easily accessible periodicals. However, the monograph is not completely satisfactory because as a whole, it is rather heterogeneous. While the eight first chapters of the book stand as a coherent theoretical contribution, the presence of the reprinted papers is not really justified. Namely the theoretical concepts introduced in the first 200 pages are not really needed to understand the applications. The reason is that while the mathematical part of the book focuses on the handling of infinite sets in fuzzy measure theory, the applications only use a finite setting, where the subtle continuity notions introduced in the first part become void. The first part of the book will clearly be of interest to a mathematically sophisticated readership which may not find much interest in the applied papers. On the contrary, an engineering-oriented readership might be deterred from digging in the book because the gap between the theory and what looks really useful in it for the application papers is rather significant.

Another debatable aspect of the book is that the relationship between fuzzy measure theory and other works on non-additive probabilities are not really discussed. A typical example is with Chapter 8. There are older works by Weber that are rather similar to the "pan-integral", and more recent works by Murofushi and Sugeno. These works are cited but not really analyzed. Also the concept of quasi-measure is closely related to the concept of "distorted probability" developed in the literature of non-classical utility theory, by Quiggin, Yaari and colleagues, who are not cited in the book. Lastly, Choquet's integral is mentioned in the glossary but could have been discussed in more details as an alternative to the fuzzy integral, for the sake of better locating the latter in the multifaceted-literature on non-additive theory. Despite these reservations this book is worth reading by people interested in measure theory, decision theory, and uncertainty modelling. Interestingly, another mathematically-oriented, fully devoted to Choquet's integral has been recently published (Denneberg, 1994). It would be quite instructive to achieve a comparative readings of the two books.

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Reference

Denneberg D. Non-Additive Measure and Integral. Kluwer Academic Publishers, Dordrecht, 1994.