THE PROTRUDING PROPERTY IN FUZZY COMPREHENSIVE JUDGMENT

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ABSTRACT

When applying the comprehensive judgment model in making policies, we find that its protruding property is quite essential. Without this protruding property, policy-making be comes impossible and even contradictions emerge. this paper deals the protruding property and its influence on the comprehensive judgment, gives the full conditions for producing the protruding property and improves the model of the comprehensive judgment from the theory and the application.

KEYWORD: Fuzzy, Comprehensive judgment, Model, protruding property

1 INTRODUCTION

Comprehensive judgment, as a model for many-sided policy-making, has been widely used [1--9], but the study on the basic theory of the comprehensive judgment is far from enough. We find in our study that the protruding property in comprehensive judgment is quite essential. without the protruding property, policy-making becoms impossible and even contradictions emerge e.g. When judging a student's study results with the comprehensive judgment model we may assume that the elements sets U={ maths u_1, physics u_2, chemistry u_3, English u_4 }, the comment sets V={A level a_1, B level a_2, C level a_3}, in which $a_1, a_2, a_3 \in [0, 100]$. their subordinate functions are

$$A1(x)=\begin{cases} 1 & x\in [90,100], \\ (x-70)/20, & x\in [70,90], \\ 0 & x\in [0,70]. \end{cases} \qquad (1)$$

$$A2(x)=\begin{cases} 1-|x-70|/20, & x\in [50,90], \\ 0 & x\in [0,50]. \end{cases} \qquad (2)$$

$$A3(x)=\begin{cases} 1, & x\in [0,50], \\ (100-x)/50, & x\in (50,100], \end{cases} \qquad (3)$$

Now a student's study results are: maths 95; physics 85; chemistry 80; english 65. Assuming the assumed weight of u_1 to be 0.3; u_2 to be 0.3; u_3 to be 0.2; u_4 to be 0.2. We get the following simgle element judging matrix by filling (1) (2) (3) with the results:

$$R=\begin{bmatrix} 1 & 0 & 0.1 \\ 0.75 & 0.25 & 0.3 \\ 0.5 & 0.5 & 0.4 \\ 0 & 0.75 & 0.7 \end{bmatrix} \qquad (4)$$

If A=(0.3 0.3 0.2 0.2), then the judgment results are:

(1) When 'o' is assumed to be '$\vee$--$\wedge$', B=AoR=(0.3 0.25 0.3) (5)
(2) When 'o' is assumed to be '$\oplus$--$\wedge$', B=AoR=(0.8 0.65 0.8) (6)
(3) When 'o' is assumed to be '$\oplus$--.', B=AoR=(.63 .33 .34) (7)

From(5) (6) and (7) we know that it's impossible to judge the student's results reasonably since the vectors of the three judgments are contradictory within themselves.

Why is it impossible to make policy by usinng the model above? We try to give the ansuwer with the help of the conception of protruding property.

2 PROTRUDING PROPERTY IN FUZZY COMPREHENSIVE JUDGMENT

Definition: assume $\{p_1, p_2, \ldots p_n,\} \in Rx[0,1]$, in which $P_i=P_i(X_i,Y_i)$ $(i=1,2,3 \ldots, n)$. $x_1<x_2<x_3<\ldots<X_n$, call the sets $\{p_1, p_2, \ldots, P_n\}$ general protrusion, if $Y_{i0}=\bigvee_{i=1}^{n} Y_i$ and $y_1<y_2<y_3<\ldots<Y_{i0}$, $Y_{i0}>Y_{i0+1}>\ldots>Y_{n-1}>Y_n$

Theorem 1 assume $A\in F(R)$, then A is the protruding fuzzy sets $\Leftrightarrow$ $\forall n\in N$, $\forall \{p_1, p_2, \ldots P_n\} \in \{(x, A(x))/x\in R\}$, $\{p_1, p_2, \ldots, P_n\}$ general protrusion.

Prove $\Rightarrow$, assume $A(X_{i0})=Y_{i0}=\bigvee_{i=1}^{n} A(X_i)$, then

$$Y_{i0}>Y_{i0-1}=A(X_{i0-1})>A(X_{i0})\wedge A(X_{i0-2})=Y_{i0}\wedge Y_{i0-2}$$
$$Y_{i0-2}=A(X_{i0-2})>A(X_{i0})\wedge A(X_{i0-3})=Y_{i0}\wedge Y_{i0-3}=Y_{i0-3}$$
$$\ldots \ldots$$
$$Y_2=A(X_2)>A(X_{i0})\wedge A(X_1)=Y_{i0}\wedge Y_1=Y_1$$

In the same way we can prove, $Y_{i0}>Y_{i0+1}>\ldots>Y_{n-1}>Y_n$

$\Leftarrow$ If A is nonfuzzy protrusion, then, $\exists x_1, x_2, x_3\in R$, $x_1<x_2<x_3$, $A(x_2)<A(x_1)\wedge A(x_3)$, therefore,. $\{p_1=p_1(x_1, A(x_1)), p_2=p_2(x_2, A(x_2)), p_3=(x_3, A(x_3))\}$ non-general protrusion, be contradictory.

Theorem 2 Assume $A.\in f(x)$, $R\in F(X,R)$. If $\vee x\in X$, $R(x,y)$ is the protruding fuzzy sets on R. then, B=AoR is the protruding fuzzy sets on R, in which 'o' is assumed to be '$\vee$--T'.

proof $\forall a\in [0,1]$, $$\bigvee_{x\in X}[A(x)TR(X, aY_1+(1-a)Y_2)]>\bigvee_{x\in X}[A(x)T(R(x,Y_1)\wedge R(x,Y_2))$$
$$=\bigvee_{x\in X}\{[A(x)TR(X,Y_1)]\wedge[A(x)TR(x,Y_2)]\}$$
$$=[\bigvee_{x\in X}[A(x)TR(x,Y_1)]\wedge \bigvee_{x\in X}[A(x)TR(x,Y_2)]$$

i.e. $B(2Y_1+(1-a)Y_2>B(Y_1)\wedge B(Y_2)$

Theorem 3 Assume function f_1 be increase on $[a,b]$, f_2 be decrease on $[a,b]$ $f_1(a)<f_2(a)$ and $f_1(b)>f_2(b)$, then $\exists t\in[a,b]$, When $x\in[a,t]$, $f_1(x)<f_2(x)$; when $x\in[t,b]$, $f_1(x)>f_2(x)$, in which t is called the dividing point between f_1 and f_2. since this proposition can be easily understood, the proof is omitted.

Theorem 4 Assume $A_i \in F(R)$ is regular flat-top-mount-like function [5], the core of A_i is $[a_i, b_i]$ $(i=1,2,\ldots,n)$ and

$$a_1 < b_1 < \ldots < a_i < b_i < a_{i+1} < b_{i+1} < \ldots < a_n < b_n.$$

then, $\forall x \in R$, $\{(1, A_1(x)), (2, A_2(x)), (3, A_3(x)), \ldots, (n, A_n(x))\}$ general protrusion.

proof : From theorem 3 we know that A_i and A_{i+1} have a dividing point on R and the point is marked t_i $(i=1,2,\ldots,n-1)$, $\forall x \in R$, assume $A_{io}(x) = \bigvee_{i=1}^{n} A_i(x)$.

Therefore, $t_1 < t_2 < \ldots < t_{io-1} < x < t_{io} < \ldots < t_{n-1}$, then,

$$A_1(x) < A_2(x) < \ldots < A_{io-1}(x) < A_{io}(x)$$

and

$$A_{io}(x) > A_{io+1}(x) > \ldots > A_{n-1}(x) > A_n(x),\ \text{i.e.}$$

$\{(1, A_1(x)), (2, A_2(x)), \ldots, (n, A_n(x))\}$ general protrusion.

Theorem 4 provides a full condition for general protruoion, if the condition of theorem 4 is ment by A_i $(i=1,2,\ldots,n)$, then from theorem 1 and theorem 2 we know if $R \in F(X, R)$ is constructed with A_i $(i=1,2,\ldots,n)$ and 'o' is assumed to be '$\vee$--T', then, B=AoR must be the protruding fuzzy sets on R. Therefore, we can easily make reasonable policies acwrding to the comprehensive judgment model.

3 AN EXAMPLE

Now let's settle the problem of making policy mentioned in the introduction again. We know that the condition in theorem 4 is not met by the production . therefore slightly change A_1, A_2, A_3 and assume

$$A1(x) = \begin{cases} 1 & x \in [90, 100], \\ x/90 & x \in [0, 90], \end{cases} \tag{8}$$

$$A2(x) = \begin{cases} x/70 & x \in [0, 70], \\ (4(85-x)+15)/75 & x \in (75, 85], \\ 2(100-x)/150 & x \in (85, 100], \end{cases} \tag{9}$$

$$A3(x) = \begin{cases} 1 & x \in [0, 50], \\ (9(85-x)+35)/350 & x \in (50, 80], \\ (100-x)/150 & x \in (85, 100], \end{cases} \tag{10}$$

the rest in the production remains the same, then we have

$$R = \begin{pmatrix} 1 & 1/15 & 1/30 \\ 17/18 & 0.2 & 0.1 \\ 8/9 & 7/15 & 8/35 \\ 13/18 & 13/14 & 43/70 \end{pmatrix} \tag{11}$$

When 'o' is assumed to be '..,$\vee$--$\wedge$'

$$B=A.R=(0.3\ \ 0.2\ \ 0.2)$$

The study results of the student is judged to be A level according to the broadest subordinate ptinciple. When 'o' is assumed to be '..$\oplus$--$\wedge$',

$$B=AoR=(\ 0.3\ \ 13/70\ \ 43/350\ )$$

The student's results are still judged to be A level according to the broadest principle.

4 CONCLUSION

This reseach shows us that if the function constructing the single element judging matrix is met with special requirement when the comprehensive judgment model is used , the result of comprehensive judgment gets the protruding property and reasonable policies can be made. Other wise poliey policy-making is comes impossible and even contradictions emerge.

ACKNOWLEDGMENT

The authors would like to thank professor Lin Raorui of Tsinghua university for his helpful comments. the authors also would like to thank associatc professor Ma Shaoping of tsinghua university for his valuable discussions.

REFERENCES

[1] Wang peizhuang, On fuzzy sets and its application, Shanghai publishing House, 1983.

[2] Wu Wangming, The methods of applying fuzzy sets, beijing teachers university publishing House, 1985.

[3] Wang Guangyuan, On the nature and application of comprehensive judgment according to a few mathematical models, Fuzzy Mathematics, 4 (1984).

[4] Feng Kejun, An example showing the principle of identifying the conter of models and its application in map drawing, Wuhan Survey and Dawing Scienle and Technology University journal, 1 (1989)

[5] Zou Kaiqi, Xu Yang, Fuzzy system and expert system, Southnest Traffic University publishing House, 1989.

[6] Wang Aimin, The quantitative evaluation on scicntists and technicians By means of the fuzzy sets and commputer treatment, PROCEEDIN OF NAFIPS' 88, 1988 , America.

[7] L.A. Zadch, Fuzzy sets, Information and control, 8 (1965)

[8] Stcphen E, Sheridan and poul skjoth, Automatic kiln control at Oregon prti-and Cement company's Darkce plant, Utilizing fuzzy logic, for presentantion at the 25th IEEE Cement industry Technical confer ccc, san Antionic, Texas, May 1983.

[9] Wang Aimin , The current quantized approach for assessment and the computer managements system, International conference on information and systems' 92, Dalian Maritime University Publishing hose , dalian, china, 1992.