WORLD MODELLING AND PLANNING IN UNCERTAIN 3D ENVIRONMENT

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Abstract. The problem of 3D path planning for a robot has been studied in recent literature. Much of the work in this area is limited to the case of accurate environment model. In this paper, we develop a new method to treat uncertain 3D environment by using fuzzy set and theory. It is assumed that a fuzzy obstacle is represented by a set of fuzzy half-spaces. Based on this representation, a decision function which aims towards finding an acceptable 3D trajectory is constructed. The local minus gradient information of the decision function is used to guide the robot to search for its goal. Simulation studies indicate the feasibility of the proposed work.

Key words: 3D path planning, fuzzy environment, fuzzy half-space, fuzzy number, decision-making.

1. Introduction

Path planning for robots is an interesting subject in artificial intelligence and robotics. In some applications, such as robots operating in space station or doing undersea exploration, it becomes necessary to model a three dimensional environment. Most existing methods for representation (e.g. visibility graph[1-2] or crystal map [3-4]) assumes that precise information of the environment is available. But in practice this is not always true. Usually a robot can not obtain the precise description of obstacles in its workspace. This is mainly because 1) robots usually have insufficient or incomplete knowledge about its workspace; 2) error in sensing modalities will introduce position and orientation estimation problems in the modelling of robot's workspace; 3) borders of some obstacles can not be clearly defined. In these situations, a flexible model that can accomodate blurry information about the environment and the corresponding robust planning scheme are required.

In this paper, a fuzzy approach to treating an imprecisely known environment is presented. The proposed approach models every object in the workspace as a fuzzy obstacle. A 3-dimensional fuzzy obstacle can be considered as the intersection of a set of fuzzy half-spaces. The coefficients of the fuzzy half- spaces are described by fuzzy numbers and stored as basic world description. Every point in the 3D space could be assigned a membership value indicating the degree of that point belonging to the obstructing region. Based on this description, a decision function is constructed which represents an aggregate of the goals of the path planning problems. The minus gradient information can be used to find an acceptable trajectory for the robot from starting point to destination point. The results of a simulation test are given to demonstrate the capability of the proposed approach.

2. Representation of Uncertain 3D Environment

In uncertain 3D space, we can use fuzzy equation $\underset{\sim}{f}(x,y,z)$ to describe a fuzzy camber, whose definition is given below:

Definition 1: A fuzzy camber described by fuzzy equation $\underset{\sim}{f}(x,y,z)=0$ is a fuzzy set $S\subset R^3$, which satisfies:

$$\forall\ (x,y,z)\in R^3,\quad \mu_S(x,y,z)=\mu_{\underset{\sim}{A}}(0) \tag{1}$$

where $\underset{\sim}{A}=\underset{\sim}{f}(x,y,z)$

Example As a simple illustration assume that

$\underset{\sim}{f}(x,y,z)=x+y+z-(1,2,3)$

Then

$\mu_S(1,2,2)=\mu_{\underset{\sim}{A}_1}(0)=0,\qquad \underset{\sim}{A}_1=(2,3,4)$

$\mu_S(1,1,0)=\mu_{\underset{\sim}{A}_2}(0)=1,\qquad \underset{\sim}{A}_2=(-1,0,1)$

$\mu_S(1,1,0.5)=\mu_{\underset{\sim}{A}_3}(0)=0.5,\qquad \underset{\sim}{A}_3=(-0.5,0.5,1.5)$

Under uncertainty, obstacles in 3D space could not be modelled precisely. The uncertainty arises from ill-defined boundary cambers and location of a fuzzy obstacle. Using the concept of fuzzy sets, any fuzzy obstacle can be represented by a set of fuzzy cambers. Since a fuzzy camber divides the whole space into two fuzzy half-spaces, the interior area of a fuzzy obstacle can be considered as the intersection of several fuzzy half-spaces produced by its fuzzy cambers. In this paper, a fuzzy half-space is described by a linear inequality whose coefficients are fuzzy numbers, and its membership function is defined through the concept of agreement index of a fuzzy number with regard to an upper or lower bound[6]. The formal definition is as follows:

Definition 2: A fuzzy half-space described by $\underset{\sim}{a}x+\underset{\sim}{b}y+\underset{\sim}{c}z+\underset{\sim}{d}\leqslant 0$ ($\underset{\sim}{a},\underset{\sim}{b},\underset{\sim}{c}$ are fuzzy numbers) is a fuzzy set $H\subset R^3$, which satisfies:

$$\forall\ (x,y,z)\in R^3,$$

$$\mu_H(x,y,z)=(\text{area } \underset{\sim}{T}\cap I)/(\text{area } \underset{\sim}{T}) \qquad (2)$$

where

$$\underset{\sim}{T}=\underset{\sim}{a}x+\underset{\sim}{b}y+\underset{\sim}{c}z+\underset{\sim}{d}$$

$$I(u)=\begin{cases}1 & u\leqslant 0\\ 0 & u>0\end{cases}$$

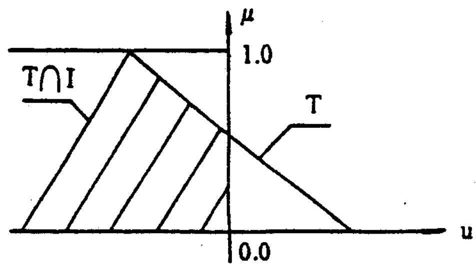

Fig. 1 Agreement index for fuzzy number with regard to an upper bound

Using fuzzy half-spaces, an m_i-faced obstacle O_i can be represented as:

$$O_i: \qquad A_i\cdot(x,y,z)^T+D_i\leqslant 0 \qquad (3)$$

where

$$A_i=\begin{bmatrix}\underset{\sim}{a}_i(1,1),\underset{\sim}{a}_i(1,2),\underset{\sim}{a}_i(1,3)\\ \underset{\sim}{a}_i(2,1),\underset{\sim}{a}_i(2,2),\underset{\sim}{a}_i(2,3)\\ \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\\ \underset{\sim}{a}_i(m_i,1),\underset{\sim}{a}_i(m_i,2),\underset{\sim}{a}_i(m_i,3)\end{bmatrix} \qquad D_i=\begin{bmatrix}\underset{\sim}{d}_i(1)\\ \underset{\sim}{d}_i(2)\\ \cdot\quad\cdot\\ \underset{\sim}{d}_i(m_i)\end{bmatrix}$$

For any $j=1..m_i$, the inequality $\underset{\sim}{a}_i(j,1)\cdot x+\underset{\sim}{a}_i(j,2)\cdot y+\underset{\sim}{a}_i(j,3)\cdot z+\underset{\sim}{d}_i(j)\leqslant 0$ defines a fuzzy half-space H_{ij}. Considering the interior area of this obstacle is a fuzzy set in 3D space, denoted by R_i, we can write:

$$R_i=\bigcap_{j=1\cdots m_i}(H_{ij}) \qquad (4)$$

$$\mu_{R_i}(x,y,z)=\min_{j=1\cdots m_i}[\mu_{H_{ij}}(x,y,z)] \qquad (5)$$

Finally, imagine the obstructing region R in the workspace. It is the union of the interior areas (R_i) of all obstacles in the environment. Thus

$$R=R_1\cup R_2\cup\cdots\cdots\cup R_p \qquad (6)$$

$$\mu_R(x,y,z)=\max_{i=1\cdots p}[\mu_{R_i}(x,y,z)] \qquad (7)$$

With Equation 5, Equation 7 can be rewritten as:

$$\mu_R(x,y,z)=\max_{i=1\cdots p}\{\min_{j=1\cdots m_i}[\mu_{H_{ij}}(x,y,z)]\} \qquad (8)$$

3. Path Planning under Uncertainty

3.1 Formulation of Path Planning Task

The path planning task for the robot is modelled as a decision-making problem by considering two objectives: reach the destination and avoid collisions with fuzzy obstacles. Without any obstacles, the best trajectory is depicted by plotting a 3D line from starting point to destination point. However, with one or more obstacles presenting in this path, an acceptable path is the trajectory which avoids colliding with obstacles but still reaches the destination. The above two goals are considered as the goals of the decision-making problem, whose variables are x, y, and z coordinates. At any point in the 3D space, satisfaction of the first goal is examined by computing the distance from this point to the destination. This objective is satisfied when the distance is zero or small enough. The second objective is to prevent the robot from interfering with any obstacles. The path planning algorithm should select points on the trajectory such that any collision with fuzzy obstacles may be avoided. Depending on the distribution of obstacles, some points in the workspace might belong to the obstructing region with certain membership values, but they are closer to the destination point. In this situation the two planning goals conflict with each other, and an aggregation operation is needed to combine the influence of the two together. With this idea, the decision function is constructed and expressed as follows:

$$W(x,y,z)=C_1\cdot\mu(x,y,z)+C_2\sqrt{(x-x_g)^2+(y-y_g)^2+(z-z_g)^2} \qquad (9)$$

where $\mu(x,y,z)\in[0,1]$ is the level of a point belonging to the obstructing region, and (x_g, y_g, z_g) are coordinates of the destination. The weights C_1, C_2 are used to indicate the preference relationship between two planning goals. The modelling of a preference relationship involves decision-maker's intuition, and for simplification, precise numerical weights are used here. In the cases of C_1 dominating over C_2, a trajectory that is longer but free from possible collisions is preferable. But in the cases of C_2 dominating over C_1, risk of collision along a selected path is understood and accepted in return for a shorter length. Thus under the two distinct path planning philosophies of conservative vs. aggressive, different combinations of weights define interesting trade-offs.

3.2 Path Planning Algorithm

The path planning strategy selects points in the 3D space by using the minus gradient information of the decision function to generate an acceptable trajectory. An component of the gradient vector is computed by differentiating W(x, y, z) by variable x, y,

and z. So the corresponding minus gradient vector is written as:

$$-\nabla W=\left[-\frac{\partial W(x,y,z)}{\partial x},-\frac{\partial W(x,y,z)}{\partial y},-\frac{\partial W(x,y,z)}{\partial z}\right]^T \tag{10}$$

where

$$-\frac{\partial W(x,y,z)}{\partial x}=-C_1\cdot\frac{\partial\mu(x,y,z)}{\partial x}+C_2\cdot\frac{x_g-x}{\sqrt{(x-x_g)^2+(y-y_g)^2+(z-z_g)^2}} \tag{11a}$$

$$-\frac{\partial W(x,y,z)}{\partial y}=-C_1\cdot\frac{\partial\mu(x,y,z)}{\partial y}+C_2\cdot\frac{y_g-y}{\sqrt{(x-x_g)^2+(y-y_g)^2+(z-z_g)^2}} \tag{11b}$$

$$-\frac{\partial W(x,y,z)}{\partial z}=-C_1\cdot\frac{\partial\mu(x,y,z)}{\partial z}+C_2\cdot\frac{z_g-z}{\sqrt{(x-x_g)^2+(y-y_g)^2+(z-z_g)^2}} \tag{11c}$$

The partial differentiation of membership function $\mu(x,y,z)$ **is approximated in the algorithm by employing a simple forward differencing technique as given by:**

$$\frac{\partial\mu(x,y,z)}{\partial x}=\frac{\mu(x+\Delta x,y,z)-\mu(x,y,z)}{\Delta x} \tag{12a}$$

$$\frac{\partial\mu(x,y,z)}{\partial y}=\frac{\mu(x,y+\Delta y,z)-\mu(x,y,z)}{\Delta y} \tag{12b}$$

$$\frac{\partial\mu(x,y,z)}{\partial z}=\frac{\mu(x,y,z+\Delta z)-\mu(x,y,z)}{\Delta z} \tag{12c}$$

Using Equation 10, 11, and 12, the minus gradient vector of the decision function is constructed. A local search direction at a point is determined by identifying this information. At any point in the 3D space, the minus gradient vector is used to make a move (i.e. to select the next point on the trajectory) . The algorithm terminates when the destination is reached, that is, when the value of the decision function is of minimum.

The search procedure for new points on the path is given below:

1. Assign the weights of the decision function, the starting point $X_s=(x_s,y_s,z_s)$ and the destination point $X_g=(x_g,y_g,z_g)$.
2. Let $X=X_s=(x_s,y_s,z_s)$.
3. If $\|X-X_g\|\leqslant\varepsilon$ then exit with success, else go to the next step.
4. Compute the gradient vector ∇W (of the decision function)
5. Let $X=X-\alpha\cdot\frac{\nabla W(X)}{\|\nabla W(X)\|}$, go to step 3.

4. Test and Results

Simple test of the proposed work has been implemented on PC-386 in Turbo Pascal. The test bed is chosen to be a 20×20×30 cuboid, with four fuzzy obstacles, which are represented by $A_i \cdot (x,y,z)^T + D_i \leqslant 0$ $(i=1..4)$, as described in section 2.

In order to indentify the behavior of the planned path, we define the measure of risk as follows:

$$\text{risk} = \int_{path} \mu(x,y,z)ds \Big/ \int_{path} ds \qquad (13)$$

Suppose the path is composed of n steps with equal size, then Equation 13 can be approximated by:

$$\text{risk} = \sum_{i=1}^{n} \mu(x_i,y_i,z_i)/n \qquad (14)$$

In the example illustrated in the following table, the start and goal location are (0, 0, 0), (17, 20, 23) respectively, and the size of every step to move is chosen to be 0.2. This table depicts the various performances of the path planning algorithm under different combinations of the weights of the decision function. When c_1/c_2 is higher, the path is longer but of less risk. Otherwise the path is shorter but more risky.

C_1/C_2	Path Length	Risk
0.5	35.2	0.236
0.8	36.8	0.172
1.0	37.4	0.133
1.2	37.8	0.103
1.5	38.2	0.071
2.0	38.8	0.044

5. Conclusion

Uncertain geometry is an important issue in the current robot research. In this paper, we developed a fuzzy approach to modelling the imprecise description of 3D workspace for the robot. The concept of fuzzy half-spaces is established to describe a fuzzy obstacle in 3D environment. And this fuzzy representation is incorporated into the path planning methodology by defining a decision function. Compared with the previous approaches, our work has the advantage of flexible representation and increased robustness. We hope this is a contribution to the development of 3D robot systems capable of dealing with uncertain geometry.

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