Fuzziness of Incompleteness and Information Distribution Method[1]

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ABSTRACT

This paper aims to state the essentials of fuzziness of incompleteness which is far away subjective fuzziness that has being studied a lot of and is main topic of today's fuzzy engineering in which experts experience is necessary or very important that would constrain us only to analyse known systems. In this paper, Information Distribution Method is systematically introduced to deal with incomplete fuzzy information and 1-dimensional linear information distribution is discussed. The applications show that information distribution methods have obvious advantages and application future.

Keywords: fuzzy information, incompleteness, information distribution, fuzzy relation matrix, fuzzy approximate reasoning

1. Fuzzy Information

1.1 Information and Science of Information

Long ago, under the influence of Laplace's determinism, almost all researchers believe that everything's motion must be controlled by a particular mechanical law. In the wake of developments in science and technology, people gradually know that there are uncertainties in the world. Randomness is discovered firstly and which elicits Shannon's narrow theory of information. Here, information is defined the uncertainness which is cleared out, and regarded as negentropy. The definition of entropy in statistical thermodynamics:

$$H(x_i) = \sum_{i=1}^{n} p(x_i)h(x_i) = -\sum_{i=1}^{n} p(x_i)\log p(x_i)$$

becomes the foundation of the edifice of narrow theory of information.

Many problems in communication field are solved by using narrow theory of information. However, this classical theory is useful only we employ

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it to study the problems of information transfer, and it can do nothing for seeking for the structure about information that would be very important for recognizing relationships among factors. Therefore, general theory of information is studied by many people. The theory of fuzzy information is an important part of that.

1.2 Properties of Fuzzy Information

At the beginning, the people who are going to set up the theory of fuzzy information only take an interest in using the definition of the entropy of fuzzy event to study quantification index of a fuzzy set and the problems of decision. Obviously, along the train of thought, narrow theory of fuzzy information is the only one which would be established. It can not describe general fuzzy information touched by the common people.

In order to established general theory of fuzzy information, we ought to remove the restriction that information is relative to communications. We define that **information is the reflection of motion state and existential fashion of objective reality. this reflection is revealed in the form of material or energy, and is perceived by human sense organs directly or indirectly.** For examples, indication of a thermometer, flight speed of an airplane, a sentence, a letter, a cipher, a seismogram, a train timetable, a mathematical formula, and a cardiogram are information.

The range of information is so wide that we have to set a limit to it when we do analyse information practically. In this thesis, information of our discussion is limited just to ones that can be accumulated to become experience or knowledge. Our main interest is not to study measurement of information, but to analyse structure of information from which we would know what it tell us, and by which we would discover some useful nature laws.

Any information which is not so exact or is a bit vague can be call fuzzy information. However, in the past, people only deal with the fuzzy information that be in connects with the equipment used to measure the objects. There, fuzziness is produced by using the vague measuring tool or ambiguous classifier. A lot of information of human language possess this kind of fuzziness, where human brain is the measuring tool. Sometimes, there is quantity of physics that carries this kind of fuzziness too, earthquake intensity is an instance.

The property of this kind of fuzzy information is that it is easy to catch sight of their vagueness, or ambiguousness. Why? Because every piece of them carries fuzziness.

In fact, in the world, there is another kind of fuzzy information. Their fuzziness is not produced by the measuring tool, but incompleteness. Its property is that every piece may be precise and clear but their aggregation or collective has uncertainty when we use it to recognize something. Incomplete knowledge sample possesses this kind of fuzziness, where every observed value may be precise but the relationship among factors recognized by using the set of observed values is fuzzy relationship, it is very difficult to get a precise function relationship if we use only incomplete knowledge sample.

Although the work of Zadeh[1-3] and others[4-13] has led to an universal acceptance of the belief that fuzzy systems must be dealt with by the methods provided by fuzzy sets theory, most of researchers always considered that the fuzziness of fuzzy systems must be relevant to the equipment used to measure the objects, in most cases, the equipment is relevant to the human brain. That is the reason why the fuzziness which has been discussing usually is associated with the subjectivity.

Unquestionably the subjective fuzziness point of view has contributed deep insight into the fundamental processes involved in the fields of the "soft" science, and played a key role in the control field which is relevant to experiment rules.

In the "hard" science fields, however, it is necessary to avoid subjectivity affection. It is widely agreed at this juncture that physicists, chemists, engineers, and other "hard" scientists need fuzzy theories and method to deal with the imprecision data, but they wish the fuzzy means would give the results which can pass tests by other researchers. If different researchers obtain different results, people can not know where to turn.

Being aimed at this question, Liu Zhengrong and author suggested CONCEPT OF INFORMATION DISTRIBUTION[14-16], and a few of important applications[17-20] have come to the fore in which the major issues center not on the subjective fuzziness but on the incompletion fuzziness.

In fact, fuzziness is not equal to human brain subjectivity. When we use the data information to recognize relationships among factors, sometimes, the information may be fuzzy information. In this thesis, in an all-round way, we will discus when the data information carries fuzziness and how to use its fuzzy information to improve recognition.

2. Information Distribution

2.1 Origin of the Concept of Information Distribution

Generally, the true probability distribution for the happening of an event has been built up from a great number of tests and we want to approximate this condition by two different approaches, the classical statistics and the information distribution method suggested in papers [14-16], with a relative small number of tests. Advantages of the new method would be established if relative errors induced at each point in the case considered are small than that from classical method. Let us have a look at the meaning of the new method by the example of the problem of intensity zoning of an earthquake.

We have a bach of strong earthquake data with includes 134 seismic records observed in China from 1900 to 1975 with magnitudes in the range 4.3-8.5 and intensity in the range VI-XII degrees.

In the first step, the controlling points, which are in the universe of discourse concerned and each numerical data would contribute its information in some fashion to the neighboring controlling points with total amount equal to one, should be determined. This is generally done with equal spacing in the universe of discourse. Here, for earthquake magnitude M, 14 points starting from 4.6 and spanning 0.3 have been gauged, i.e.

$$M = \{m_i \mid i = 1, 2, \cdots, 14\} = \{4.6, 4.9, \cdots, 8.5\}$$

Elements in the universe of discourse of earthquake intensity I remain unchanged, i.e.

$$D = \{I_j \mid j = 1, 2, \cdots, 7\} = \{VI, VII, \cdots, XII\}$$

The second step is to construct information matrix $Q(14 \times 7)$ using information gains from 134 basic data of earthquake records at all controlling points. Suppose we have an earthquake record (m, I) where the magnitude m satisfies

$$m_i \leq m \leq m_{i+1}, m_i, m_{i+1} \in M.$$

and

$$\exists I_j \in D, \;\; s.t. \; I = I_j$$

Suppose again that information distribution here is conducted in linear form. Thus information gain at Q_{tj} due to (m, I) can be expressed as

$$Q_{tj} = 1 - |m - m_t| / |m_{i+1} - m_i| = 1 - |m - m_t| / 0.3, t = i, i + 1$$

If an earthquake data is read as $(m, I_j) = (5.75, VII)$, then $m_i = m_4 = 5.5$, $m_{i+1} = m_5 = 5.8$, and $I_j = I_2 = VII$. therefore

$$Q_{42} = 1 - |5.75 - 5.5| / 0.3 = 1 - 0.83 = 0.17,$$

$$Q_{52} = 1 - |5.75 - 5.8| / 0.3 = 1 - 0.17 = 0.83.$$

After 134 earthquake data have been treated with this simple process and information gains at each controlling point have been summed up, an information matrix will turn out. The third step is to establish fuzzy relationship

R between D and M which can be done by normalizing the intensity column of the information distribution matrix Q. The last step is to recognize earthquake intensity i_0 by using the fuzzy inference formula as the following when the magnitude m_0 is given.

$$I_0 = M_0 \odot R$$

where the operator $\odot$ can be $(\vee, \wedge)$ model,and when $m_i \leq m_0 \leq m_{i+1}$, the membership function of M_0 is

$$\mu_{M_0}(m) = \begin{cases} 1 - |m_0 - m|/|m_{i+1} - m_i|, & \text{for } m = m_i \text{ or } m = m_{i+1} \\ 0, & \text{otherwise} \end{cases}$$

and the membership function of I_0 is

$$\mu_{I_0}(I_j) = \max_{1 \leq i \leq 14}(\mu_{M_0}(m_i) \wedge r(m_i, I_j))$$

If $\mu_{I_0}(I') = \max\{\mu_{I_0}(I_1), \mu_{I_0}(I_2), \cdots, \mu_{I_0}(I_7)\}$, then $i_0 = I'$.

The applications[17−20] show that the information distribution method possesses the advantage of keeping the data structure of the original information, so better a result can be obtained by the inference rule.

In fact, the seismic records is a knowledge set which can be denoted as

$$X = \{x_i \mid i = 1, 2, \cdots, 134\} = \{(m_i, I_i) | i = 1, 2, \cdots, 134\}$$

each sample x_i provides us information to recognize the population from which 134-size knowledge set X is drawn.

Let $X = \{x_1, \cdots, x_n\}$ be a knowledge set, and U is the discrete universe of X, which consists of the controlling points. The general definition of information distribution is

Definition 2.1 A mapping from $X \times U$ to [0,1]

$$\mu : X \times U \to [0, 1]$$

is called information distribution, if it satisfies:

(1) $\forall x \in X$, if $\exists u \in U$, s.t. $x = u$, then $\mu(x, u) = 1$;

(2) $\forall u \in U$, $\mu(x, u)$ is continual about x, and reducing when $\| x - u \|$ is increasing;

(3) Let $u_1 = inf\{\| u \| \mid u \in U\}, u_n = sup\{\| u \| \mid u \in U\}$, and $I = [u_1, u_n)$. $\forall x \in X$, if $x \notin U$, then

(i) When $\| x \| \in I$, there must and only are two elements u', u'' in U, s.t.

$\mu(x,u') > 0, \mu(x,u'') > 0$, and $\mu(x,u') + \mu(x,u'') = 1$;

(ii) When $\| x \| \notin I$, there is only one element u, s.t. $\mu(x,u) > 0$.

Definition 2.2 μ is called an information distribution function about U if it can distribute a sample information to two controlling points. If $X = \{x_i \mid i = 1, 2, \cdots, n\}$, $U = \{u_j \mid j = 1, 2, \cdots, m\}$, let

$$q_{ij} = \mu(x_i, u_j)$$

We say that sample x_i gives controlling point u_j information gain in q_{ij} by information distribution function μ.

Let $Q_j = \sum_{i=1}^{n} q_{ij}, j = 1, 2, \cdots, m$. We say that knowledge set X can provide information in the total gain Q_j to controlling point u_j. $Q = (Q_1, Q_2, \cdots, Q_m)$ is called the primary information distribution matrix of X in U.

Definition 2.3 If the dimension of the universe U is N, and μ is information distribution function, we call μ a N-dimension distribution function, Q a N-dimension primary information distribution matrix.

2.2 1-dimensional Linear Information Distribution and Numerical Proof

Definition 2.4 Let $X = \{x_1, \cdots, x_n\}$ be a sample(where x_i, $i = 1, \cdots, n$, are called sample point), R^1 is the universe of discourse of X, and $U = \{u_1, \cdots, u_m\}$ is the discrete universe of X. For any $x_i \in X$, and $u_j \in U$, the following formula is called 1-dimensional linear information distribution:

$$\mu(x_i, u_j) = \begin{cases} 0 \vee (1 - \frac{u_j - x_i}{u_j - u_{j-1}}), & \text{for } x_i \le u_j \\ 0 \vee (1 - \frac{x_i - u_j}{u_{j+1} - u_j}), & \text{for } x_i > u_j \end{cases}$$

where $u_0 = u_1 - (u_2 - u_1)$, and $u_{m+1} = u_m + (u_m - u_{m-1})$.

Particularly, if $u_j - u_{j-1} \equiv \Delta$, $j = 2, 3, \cdots, m$, and for $\forall x_i \in X, u_1 \le x_i \le u_m$, then 1-dimensional linear information distribution is:

$$\mu(x_i, u_j) = 1 - \frac{| x_i - u_j |}{\Delta} \tag{2.1}$$

In paper [16], the study of comparison of errors due to classical statistic method and 1-dimensional linear information distribution approaches shows that the superiority of this new one over the classical one in approximation the true probability distribution. The error can be reduced more than 11 per cent.

2.3 The Fuzzy Relation Matrix R Based on Information Distribution

Let $X = \{x_1, \cdots, x_n\}$, and $x_i \in R^N$, i.e.

$$x_i = (x_{1i}, x_{2i}, \cdots, x_{Ni}), x_{ki} \in R^1$$

Denote $W_k = \{x_{k1}, \cdots, x_{kn}\}$, $k = 1, \cdots, N$. Let: $U_k = \{u_{k1}, \cdots, u_{km_k}\}$ is the discrete universe of W_k. $U = \Pi_{1 \leq k \leq N} U_k$ is called the discrete universe of X. For $u_j \in U$, we denote it as $u_j = (u_{1j_1}, u_{2j_2}, \cdots, u_{Nj_N})$, where $u_{kj_k} \in U_k$, $k = 1, \cdots, N$, and $j_k \in \{1, \cdots, m_k\}$. Distribution function of W_k in U_k can be written as $\mu_{(k)}(x_{ki}, u_{kj_k})$.

Definition 2.5 $\forall x_i \in X$, $\forall u_j \in U$

$$\mu(x_i, u_j) = \mu_{(1)}(x_{1i}, u_{1j_1})\mu_{(2)}(x_{2i}, u_{2j_2}) \cdots \mu_{(N)}(x_{Ni}, u_{Nj_N})$$

is called N-dimensional information distribution function.

For example, let $d_k = d_k(x_i, u_j) = \mid x_{ki} - u_{kj_k} \mid, k = 1, 2, 3$, then

$$\mu(x_i, u_j) = \begin{cases} (1 - \frac{d_1}{\Delta_1})(1 - \frac{d_2}{\Delta_2})(1 - \frac{d_3}{\Delta_3}), & \text{for } d_k \leq \Delta_k (k = 1, 2, 3) \\ 0, & \text{otherwise} \end{cases} \tag{2.2}$$

is called 3-dimensional linear information distribution function. Where, we suppose $u_{kj_k} - u_{k(j_k-1)} \equiv \Delta_k$, and for $\forall x_{ki} \in W_k, u_{k1} \leq x_{ki} \leq u_{km_k}$, $k = 1, 2, 3$.

Let $U_x = \Pi_{1 \leq k \leq N-1} U_k$, $U_y = U_N$. $\forall u_x \in U, \forall u_y \in U_y$, we write them as the following

$$\begin{cases} u_x = (u_1, u_2, \cdots, u_{N-1}), & u_k \in U_k, k = 1, \cdots, N-1 \\ u_y = (u_N), & u_N \in U_N \end{cases} \tag{2.3}$$

Let

$$q_i(u_x, u_y) = \mu(x_i, u), \quad \text{where } x_i \in X, u = (u_x, u_y)$$

$$q(u_x, u_y) = \sum_{i=1}^{n} q_i(u_x, u_y)$$

$$Q = \{q(u_x, u_y)\} \tag{2.4}$$

Obviously, Q is a primary information distribution matrix of X in U.

According to the character of Q, there are three types of fuzzy relation matrices can be obtained.

Type I (When U_y is a universe of discourse of some fuzzy concepts)

Let

$$s(u_y) = \max_{u_x \in U_x} \{q(u_x, u_y)\}$$

$$R_s = \{r_s(u_x, u_y)\} = \{q(u_x, u_y)/s(u_y)\} \tag{2.5}$$

R_s is called factor space fuzzy relation.

Type II (When there are a lot of nonzero elements in Q)

Let A is a fuzzy set of $U = \{u_1, \cdots, u_m\}$. If

$$A = \frac{a_1}{u_1} + \frac{a_2}{u_2} + \cdots + \frac{a_m}{u_m}$$

A is written as $A = [a_i] = [a_1, a_2, \cdots, a_m]$ for short.

Denote

$$\begin{cases} U_x = \{z_1, z_2, \cdots, z_T\}, & T = m_1 m_2 \cdots m_{N-1} \\ U_y = \{y_1, y_2, \cdots, y_L\}, & L = m_N \\ q_{ij} = \sum_{k=1}^{n} \mu(x, z_i, y_j), & x_k \in X, z_i \in U_x, y_j \in U_y \end{cases}$$

Construct fuzzy sets A_j in U_x and B_i in U_y as the following

$$A_j = A(y_j) = [q_{ij}/s(y_j)], \text{ where } s(y_j) = \max_{1 \le i \le T}\{q_{ij}\}, \text{ and } j = 1, 2, \cdots, L$$

$$B_i = B(z_i) = [q_{ij}/s(z_i)], \text{ where } s(z_i) = \max_{1 \le j \le L}\{q_{ij}\}, \text{ and } i = 1, 2, \cdots, T$$

We define

$$g(A_j, B_i) = \max_{z \in U_x, y \in U_y} \{\mu_{A_j}(z) \wedge \mu_{B_i}(y)\}$$

If

$$g(A_o, B_i) \ge \max_{1 \le i \le L}\{g(A_j, B_i)\}$$

we say $A_o \to B_i$ is true. Let

$$R_i(z, y) = \mu_{A_o \to B_i}(z, y) = \mu_{A_o}(y) \wedge \mu_{B_i}(z), \text{ where } x \in U_x, y \in U_y$$

$$R_m = \{r_m(z, y)\} = \cup_{i=1}^{T} R_i(z, y) \tag{2.6}$$

R_m is called Mamdani fuzzy relation.

Type III (When n is sufficient big and X nearly is complete)

Let

$$\mu_{\eta|\xi}(u_y \mid u_x) = q(u_x, u_y) / \sum_{u_y \in U_y} q(u_x, u_y)$$

$$R_s = \{r_s(u_x, u_y)\} = \{\mu_{\eta|\xi}(u_y \mid u_x)\} \tag{2.7}$$

R_s is called falling shadow fuzzy relation.

2.4 Fuzzy Approximate Reasoning

General, if a fuzzy set A in U_x is known, we can obtain B in U_y by using fuzzy approximate reasoning formula as the following

$$B = A \circ R \tag{2.8}$$

The operator "$\circ$" is chosen according to the character of R.

When $R = R_f$ or R_m,

$$\mu_B(u_y) = \max_{u_x \in U_x} \{\mu_a(u_x) \wedge r(u_x, u_y)\} \tag{2.9}$$

If $R = R_s$,

$$\mu_B(u_y) = \frac{\sum_{u_x \in U_x} \mu_A(u_x) r_s(u_x, u_y) du_x}{\sum_{u_x \in U_x} \mu_A(u_x) du_x} \tag{2.10}$$

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