A SIMPLE COMPUTATIONAL METHOD OF β_{max} AND β_{min} IN MULTILEVEL FUZZY LINEAR WEIGHTED ANALYSIS

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Abstract: On the basis of [1], this paper presents a simple method to find β_{max} and β_{min} in multilevel fuzzy linear weighted analysis. This method has the computational quantities of finding β_{max} and β_{min} diminished greatly.

Keywords: Multilevel fuzzy linear weighted analysis; Orthogonal layout;Orthogonal design; Combination; Range.

1. INTRODUCTION

Multilevel fuzzy linear weighted analysis method can settle some problems that are difficult to solve by linear regression analysis. This method need apply fuzzy matrix

$$R = \begin{bmatrix} r_{111} & \cdots & r_{11m} \\ \vdots & & \vdots \\ r_{1n1} & \cdots & r_{1nm} \\ \vdots & & \vdots \\ r_{k11} & \cdots & r_{k1m} \\ \vdots & & \vdots \\ r_{kn1} & \cdots & r_{knm} \end{bmatrix}$$

to compute some $\beta_{p_1p_2\cdots p_k}$ values.When m is a even number,we need compute $\beta_{p_1p_2\cdots p_k}$ values according to the two formulas followed

$$\beta^{(j)}_{p_1p_2\ldots p_k} = \frac{\sum_{i=1}^{k}\sum_{u=\frac{m}{2}+1}^{m} r_{ip_iu} - \sum_{i=1}^{k}\sum_{u=1}^{\frac{m}{2}} r_{ip_iu} - \left(\frac{m-2}{2}\right)\sum_{i=1}^{k} r_{ip_ij}}{\left(\sum_{i=1}^{k}\sum_{u=\frac{m}{2}+1}^{m} r_{ip_iu}\right)^2 + \left(\frac{1}{4}\sum_{i=1}^{k}\sum_{u=1}^{m} r_{ip_iu}\right)^2} \qquad (1)$$

Here $j \leq \frac{m}{2}$.

$$\beta^{(j)}_{p_1p_2\ldots p_k} = \frac{\sum_{i=1}^{k}\sum_{u=\frac{m}{2}+1}^{m} r_{ip_iu} - \sum_{i=1}^{k}\sum_{u=1}^{\frac{m}{2}} r_{ip_iu} - (\frac{m-2}{2})\sum_{i=1}^{k} r_{ip_ij}}{(\sum_{i=1}^{k}\sum_{u=1}^{\frac{m}{2}} r_{ip_iu})^2 + (\frac{1}{4}\sum_{i=1}^{k}\sum_{u=1}^{m} r_{ip_iu})^2} \qquad (2)$$

Here $j > \frac{m}{2}$.

Then we find

$$\beta^{(j)}_{min} = \min_{(p_1,p_2,\ldots,p_k)} \beta^{(j)}_{p_1p_2\ldots p_k}, \quad j \leq \frac{m}{2}, \qquad (3)$$

$$\beta^{(j)}_{max} = \max_{(p_1,p_2,\ldots,p_k)} \beta^{(j)}_{p_1p_2\ldots p_k}, \quad j > \frac{m}{2}, \qquad (4)$$

Here we take all the combination of $(p_1,p_2,\ldots,p_k)$ there are n^k altogether. If n=5,k=5,we need compute 5^5=3125 $\beta^{(j)}_{p_1p_2\ldots p_k}$ values. So the computational quantities are very large when n and k are larger. Then there is some difficulty in applying this method. In order to diminish the computational quantities.We design a part of combinations of $p_1,p_2,\cdots,p_k$ using orthogonal layout,and find $\beta^{(j)}_{p_1p_2\ldots p_k}$ to the part of combinations. Then according to the characteristic of orthogonal layout β_{max} and β_{min} are deduced.

2. ORTHOSONAL DESIGN METHOD TO FIND β_{max} AND β_{min}

Let $a_1,a_2,\cdots,a_k$ express k factors, A_i is universe of discourse of a_i, i = 1,2,...,k. n is the number of the subsets divided with A_i, b is predicted quantity, B is universe of discourse of b,m is the number of the subsets divided with B.

The following is the orthosonal design method getting β_{max} and β_{min}.

First according to n,k, we select orthogond layout $L_s(t^q)$. The principle of selection is : t=n,q≥k. Then factors $a_1,a_2,\ldots,a_k$ are arranged into any k columns in $L_s(t^q)$,and accoding to the combination arranged and using formulas (1),(2),we compute $\beta^{(j)}_{p_1p_2\ldots p_k}$. Then according to the characteristic of $L_s(t^q)$ we deduce β_{max} and β_{min}. Specitic step is the following:

(I) Compute $I\,f^{(j)}_i, II\,f^{(j)}_i, \ldots, t f^{(j)}_i$,where $I\,f^{(j)}_i$ is the sum of the $\beta^{(j)}_{p_1p_2\ldots p_k}$ appropriated level number i in the column number i in the $L_s(t^q)$, $II\,f^{(j)}_i$ is the sum of the $\beta^{(j)}_{p_1p_2\ldots p_k}$ appropriated level number 2 in the column number i in the $L_s(t^q)$,...,$t f^{(j)}_i$ is the sum of the $\beta^{(j)}_{p_1p_2\ldots p_k}$ appropriated level number t in the column number i in the $L_s(t^q)$.

(II) Find the biggest (smallest) value in the $I\,f^{(j)}_i, II\,f^{(j)}_i, \ldots, t f^{(j)}_i$ in every column. Let's suppose that levels that the biggest (smallest) value corresbond in every column are $t_1,t_2,\ldots,t_k$ respectively,then

$\beta^{(j)}_{t_1t_2\ldots t_k}$ (if there is no $\beta^{(j)}_{t_1t_2\ldots t_k}$ in the $L_m(t^q)$, it will be filled) is called the initial biggest (smallest) value . Take off the biggest(smallest)value in every column.

(III) First find the biggest (smallest) value in the rest values in every column,then find the biggest (smallest) value in the biggest (smallest) values. Let level that the biggest (smallest) value corresbond in the column number i be t'_i , then footnote t_i of the $\beta^{(j)}_{t_1t_2\ldots t_k}$ is changed into t'_i,nemely $\beta^{(j)}_{t_1t_2\ldots t'_i\ldots t_k}$ (if there is no $\beta^{(j)}_{t_1t_2\ldots t_k}$ in the $L_m(t^q)$,it will be filled) . The biggest (smallest) value of the $\beta^{(j)}_{t_1t_2\ldots t_k}$ and $\beta^{(j)}_{t_1t_2\ldots t'_i\ldots t_k}$ is considered as the temporary biggest (smallest) value . Let's suppose that the temporary biggest (smallest) value is $\beta^{(j)}_{t_1t_2\ldots t_k}$, take off the biggest (smallest) value that t'_i corresbond.

(IV) Work on 3 again. If there are the same two biggest (smallest) values in a certain column,and the levels they correspond are t'_r,t''_r, then the footnote t_i of the $\beta^{(j)}_{t_1t_2\ldots t_k}$ is changed into t'_r, t''_r respectively,get $\beta^{(j)}_{t_1t_2\ldots t'_r\ldots t_k}$, $\beta^{(j)}_{t_1t_2\ldots t''_r\ldots t_k}$(if there are no one's,they will be filled).The biggest (smallest) value of the $\beta^{(j)}_{t_1t_2\ldots t_k}$, $\beta^{(j)}_{t_1t_2\ldots t'_r\ldots t_k}$ and $\beta^{(j)}_{t_1t_2\ldots t''_r\ldots t_k}$ is considered as the temporary biggest (smallest) value. If the biggest (smallest) values in the two columns are the same,we get the biggest (smallest) value in the column that range is smaller.

Do it until the values in the $L_m(t^q)$ are finished, so the β_{max} and β_{min} are determined.

3. EXAMPLE

Let n=k=3, m=2, and

$$R = \begin{bmatrix} 0.192 & 0.050 \\ 0.308 & 0.283 \\ 0 & 0.167 \\ 0.283 & 0 \\ 0.175 & 0.175 \\ 0.042 & 0.325 \\ 0.258 & 0.042 \\ 0.242 & 0.325 \\ 0 & 0.133 \end{bmatrix}$$

m=2 is a even,we have formulas

$$\beta^{(1)}_{p_1p_2p_3} = \frac{\sum_{i=1}^{3} r_{ip_i2} - \sum_{i=1}^{3} r_{ip_i1}}{(\sum_{i=1}^{3} r_{ip_i2})^2 + (\frac{1}{4}\sum_{i=1}^{3}\sum_{u=1}^{2} r_{ip_iu})^2} \qquad (5)$$

$$\beta^{(2)}_{p_1p_2p_3} = \frac{\sum_{i=1}^{3} r_{ip_i2} - \sum_{i=1}^{3} r_{ip_i1}}{(\sum_{i=1}^{3} r_{ip_i1})^2 + (\frac{1}{4}\sum_{i=1}^{3}\sum_{u=1}^{2} r_{ip_iu})^2} \qquad (6)$$

$$\beta^{(1)}_{min} = \min_{(p_1,p_2,p_3)} \beta^{(1)}_{p_1p_2p_3} \qquad (7)$$

$$\beta^{(2)}_{max} = \max_{(p_1,p_2,p_3)} \beta^{(2)}_{p_1p_2p_3} \qquad (8)$$

The number of all the combination of (p_1,p_2,p_3) is $3^3 = 27$. Before we find $\beta_{min}^{(1)}$ and $\beta_{max}^{(2)}$ with original method, we must compute 54 β values.

Now we find β values with orthosonal design method. Since n=k=3, we select orthogonal layout $L_9(3^4)$. Then factors a_1,a_2,a_3 are arranged in the first three columns in the $L_9(3^4)$. According to level combination arranged in the $L_9(3^4)$ and using formulas (5),(6),we compute $\beta^{(1)}_{p_1p_2p_3}$ and $\beta^{(2)}_{p_1p_2p_3}$.See Table 1.

Table 1 $L_9(3^4)$

	1 a_1	2 a_2	3 a_3	4	$\beta^{(1)}_{p_1p_2p_3}$	$\beta^{(2)}_{p_1p_2p_3}$
1	1	1	1	1	$\beta_{111}^{(1)}=-12.596$	$\beta_{111}^{(2)}=-1.106$
2	1	2	2	2	$\beta_{122}^{(1)}=-0.151$	$\beta_{122}^{(2)}=-0.129$
3	1	3	3	3	$\beta_{133}^{(1)}=0.939$	$\beta_{133}^{(2)}=3.096$
4	2	1	2	3	$\beta_{212}^{(1)}=-0.450$	$\beta_{212}^{(2)}=-0.273$
5	2	2	3	1	$\beta_{223}^{(1)}=0.257$	$\beta_{223}^{(2)}=0.354$
6	2	3	1	2	$\beta_{231}^{(1)}=0.080$	$\beta_{231}^{(2)}=0.089$
7	3	1	3	2	$\beta_{313}^{(1)}=0.150$	$\beta_{313}^{(2)}=0.164$
8	3	2	1	3	$\beta_{321}^{(1)}=-0.265$	$\beta_{321}^{(2)}=-0.218$
9	3	3	2	1	$\beta_{332}^{(1)}=0.718$	$\beta_{332}^{(2)}=3.421$
$\text{I}_i^{(1)}$	-11.808	-12.898	-12.781			
$\text{II}_i^{(1)}$	-0.113	-0.159	0.117			
$\text{III}_i^{(1)}$	0.603	1.737	1.340			
$\text{I}_i^{(2)}$	1.861	-1.215	-1.235			
$\text{II}_i^{(2)}$	0.170	0.007	3.019			
$\text{III}_i^{(2)}$	3.367	6.606	3.614			

Now we analyse β values in Table 1. For the first three columns in the $L_9(3^4)$,we compute $\text{I}_i^{(j)},\text{II}_i^{(j)},\text{III}_i^{(j)}$,i=1,2,3;j=1,2(See Table 1)

First we determine $\beta_{min}^{(1)}$. The smallest value of the I $f^{(1)}$, II $f^{(1)}$, III $f^{(1)}$ is -11.808. The smallest value of the I $s^{(1)}$, II $s^{(1)}$, III $s^{(1)}$ is -12.898. The smallest value of the I $s^{(1)}$, II $s^{(1)}$, III $s^{(1)}$ is - 12. 781. Levels they correspond are 1,1,1 respectively, thus $\beta_{111}^{(1)}$ is initial smallest value. Take off -11.808, -12.898, -12.781. The smallest values of the rest values in every column are -0.113, -0.159, -0.117 respectively . The smallest value of the -0.113, -0.159, -0.117 is the -o.159. Level that -0.159 corresbond is 2, thus the footnote number 2 of $\beta_{111}^{(1)}$ is changed to 2, get $\beta_{121}^{(1)}$, there is no $\beta_{121}^{(1)}$ in the Table 1, we fill $\beta_{121}^{(1)}$ = -2.966. Since $\beta_{121}^{(1)} = -2.966 > -12.596 = \beta_{111}^{(1)}$, hence $\beta_{111}^{(1)}$ is considered as the temporary smallest value, take off - 0. 159. By analogy, at last we deduce $\beta_{min}^{(1)} = \beta_{111}^{(1)} = -12.596$.

As above, we analyse I $f^{(2)}$, II $f^{(2)}$, III $f^{(2)}$ (i=1,2,3), deduce $\beta_{max}^{(2)} = \beta_{333}^{(2)} = 19.765$. In order to verify whether the result is exact, we compute all β values, there are 54 altogether, see Table 2.

Table 2

$\beta_{111}^{(1)}$=-12.596	$\beta_{132}^{(1)}$=0.390	$\beta_{122}^{(1)}$=-0.151	$\beta_{211}^{(1)}$=-2.736
$\beta_{133}^{(1)}$=0.939	$\beta_{213}^{(1)}$=-0.738	$\beta_{212}^{(1)}$=-0.450	$\beta_{221}^{(1)}$=0.698
$\beta_{223}^{(1)}$=0.257	$\beta_{222}^{(1)}$=0.077	$\beta_{231}^{(1)}$-0.080	$\beta_{232}^{(1)}$=0.336
$\beta_{313}^{(1)}$=0.150	$\beta_{233}^{(1)}$=0.627	$\beta_{321}^{(1)}$=-0.265	$\beta_{311}^{(1)}$=-4.243
$\beta_{332}^{(1)}$=0.718	$\beta_{312}^{(1)}$=-0.109	$\beta_{121}^{(1)}$=-2.966	$\beta_{322}^{(1)}$=0.483
$\beta_{112}^{(1)}$=-1.588	$\beta_{323}^{(1)}$=1.190	$\beta_{113}^{(1)}$=-4.805	$\beta_{331}^{(1)}$=0.712
$\beta_{123}^{(1)}$=-0.052	$\beta_{333}^{(2)}$=1.394	$\beta_{131}^{(1)}$=-0.333	$\beta_{111}^{(2)}$=-1.106
$\beta_{112}^{(2)}$=-0.581	$\beta_{132}^{(2)}$=0.721	$\beta_{113}^{(2)}$=-1.154	$\beta_{133}^{(2)}$=3.096
$\beta_{121}^{(2)}$=-0.814	$\beta_{211}^{(2)}$=-0.649	$\beta_{122}^{(2)}$=-0.129	$\beta_{212}^{(2)}$=-0.273
$\beta_{123}^{(2)}$=-0.050	$\beta_{213}^{(2)}$=-0.423	$\beta_{131}^{(2)}$=-0.256	$\beta_{221}^{(2)}$=-0.374
$\beta_{222}^{(2)}$=0.087	$\beta_{223}^{(2)}$=0.354	$\beta_{231}^{(2)}$=0.089	$\beta_{232}^{(2)}$=0.690
$\beta_{233}^{(2)}$=1.988	$\beta_{311}^{(2)}$=-1.015	$\beta_{312}^{(2)}$=-0.098	$\beta_{313}^{(2)}$=0.164
$\beta_{322}^{(2)}$=1.012	$\beta_{321}^{(2)}$=-0.218	$\beta_{323}^{(2)}$=5.280	$\beta_{331}^{(2)}$=1.749
$\beta_{332}^{(2)}$=3.421	$\beta_{333}^{(2)}$=19.765		

Obviously $\beta_{min}^{(1)} = \min_{(p_1,p_2,p_3)} \beta_{p_1p_2p_3}^{(1)} = \beta_{111}^{(1)} = -12.596$.

$\beta_{max}^{(2)} = \max_{(p_1,p_2,p_3)} \beta_{p_1p_2p_3}^{(2)} = \beta_{333}^{(2)} = 19.765$. The results are all the same with above. As long as we compute 28 β-values we can surely get $\beta_{max}^{(2)}$ and $\beta_{min}^{(1)}$ with orthogonal design method.

REFERENCE

[1] Guo Dawi, Multilevel fuzzy linear weighted analysis method, Fuzzy mathematics,2(1986)32-41.