# SELECTED MODIFICATIONS OF FUZZY CONTROLLERS AND THEIR APPLICATION TO THE CONTROL OF DYNAMIC SYSTEMS

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**Abstract**

Selected modifications of fuzzy controllers with application to the control of dynamic systems have been described in this paper. The first type of modification of fuzzy controllers consists in combining PI and PD fuzzy controllers (a hybrid fuzzy PID controller). The other two types of modification concern improving the fuzzy PI controller. The simulation results show that the performance of such modified fuzzy controllers in the control of dynamic systems is satisfactory.

**Keywords:** fuzzy set, fuzzy control, modified fuzzy controller, dynamic system

## 1. Introduction

After Mamdani described the first application of a fuzzy controller (FC) to the control of ill-defined (mathematically difficult to encompass) complex process [10,11], fuzzy controllers (basically fuzzy PD and PI controllers) have been gaining popularity. Fuzzy controllers as such synthesize their outputs from a collection of qualitative "rules of thumb" based on human experience or a collection of rules coming from the knowledge of an expert by means of the compositional rule of inference together with fuzzification and defuzzification procedures.

Intuitive design reflecting the behaviour of a human operator, the fact that the model of the controlled process is not necessary (an important feature when ill-defined processes are to be controlled), and good control quality (not worse than that of classical controllers [2]) are the most important advantages of fuzzy controllers, whereas the main disadvantages of fuzzy controllers are: the necessity of the acquisition and preprocessing of the human operator's knowledge about the controlled process, sequential search through rule bases, and time consuming defuzzification methods [8].

Numerous papers deal with two types of fuzzy controllers mentioned above which have been applied to the control of various complex processes. The first type is a position-type controller which generates controller output (u) from error (e) and change in error ($\dot{e}$) (error rate) called proportional-derivative fuzzy controller (PD FC), while the second type is a velocity- type fuzzy controller generating incremental controller output ($\Delta u$) from error and change in error called proportional-integral fuzzy controller (PI FC). The PD controller delivers steady state error for a large class of systems, whereas the PI controller is known to

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perform poorly in transient response due to the internal integrating operation, which is not easy to improve, e.g. for systems of higher (more than one) order. It can be observed that out of these two controllers, PI FC is more frequently used because it seems to be more practical than PD FC.

Due to the fact that PD and PI fuzzy controllers have similar disadvantages as classical PD and PI controllers a fuzzy type of PID controller should be designed. The number of rules to cover all possible combinations of fuzzy sets for two input variables is the number of sets for error multiplied by the number of sets for change in error (error rate) or integral error. The addition of another control variable significantly increases the number of rules (the number of rules for fuzzy PD or PI controllers is additionally multiplied). The design of a rule base in such a way would be a tedious task and the obtained rule base may occur very large.

Selected methods for improving the performance of above mentioned fuzzy PD and PI controllers have been collected and described here. As the first method we describe a proposal of the construction of a hybrid fuzzy PID controller introduced in [2] which is a connection of fuzzy PI and PD controllers.

In paper [9] two methods for improving the performance of PI FC have been introduced. The simulation results presented in that paper concern the control of linear high order (more than one) systems. In this paper we show that the above mentioned methods for improving PI FC can be extended to the control of nonlinear systems, as e.g. an inverted pendulum-car system.

## 2. The basic structure of a non-modified fuzzy controller

In this section we will recall a rule-based approach to an approximate reasoning process based on the compositional rule of inference [13,14], which preserves a maximal amount of information contained in the rules and observations and forms a common basis of the PD and PI fuzzy controllers. The design of a fuzzy controller includes the specification of the collection of control rules consisting of linguistic statements that link the controller inputs with appropriate outputs. Such knowledge can be collected and delivered by a human expert (e.g. an operator of an industrial complex process). This knowledge, expressed by a finite number (i=1,2,..,n) of heuristic rules of the MISO type (two inputs single output), may be written in the form:

$$R^i: \quad \textbf{if } x \textbf{ is } E^{(i)} \textbf{ and } y \textbf{ is } DE^{(i)} \textbf{ then } u \textbf{ is } U^{(i)} \tag{1}$$

where $E^{(i)}$, $DE^{(i)}$ denote values of linguistic variables x,y representing error and change in error (conditions) defined in the universes of discourse **X**, **Y**, and $U^{(i)}$ stands for the value of linguistic variable of action (conclusion) in the universe of discourse **U**.

If we employ a knowledge base of a MISO system, the compositional rule of inference may be written symbolically as:

$$U' = (DE' \times E') \circ R \tag{2}$$

In the last formula $R$ stands for the global relation aggregating all the rules, i.e.

$$R = \mathbf{also}_i (R^i) \tag{3}$$

where an implicit sentence connective "also" denotes any $t$- or $s$-norms (e.g. **min**, **max** operators) or averages [3,4]. Symbol $\circ$ stands for the operators of a compositional rule of inference (e.g. **sup-min**, **sup-prod** etc.). Similar operations have to be taken for implication and explicit sentence connective "and".

An output of the fuzzy logic controller (MISO), which has a knowledge base containing a finite number of rules connected by means of the implicit rule connective "also" interpreted as a union (**max** operator), takes the following form:

$$U' = (DE' \times E') \circ \bigcup_i (E^{(i)} \times DE^{(i)} \rightarrow U^{(i)}) = \bigcup_i U'^{(i)} \tag{4}$$

where $\times$ stands here for the explicit sentence connective "and".
Applying **sup-min** operations to the compositional rule of inference, the membership function of the output fuzzy set may by expressed as follows:

$$U'(u) = \sup_{x,y} \min\left[\min(DE'(y), E'(x)),\ \max_i (E^{(i)} \times DE^{(i)} \rightarrow U^{(i)})(x,y,u)\right] \tag{5}$$

If we take fuzzy sets $E'$, $DE'$ as singletons (measurements), i.e. $E'(x) = \delta_{x,x0}$ and $DE'(y) = \delta_{y,y0}$ where

$$\delta_{z,z0} = \begin{cases} 1 & \text{for } z = z_0 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

the membership function of the output may be simplified:

$$U'(u) = \max_i \left[(E^{(i)} \times DE^{(i)} \rightarrow U^{(i)})(x_0, y_0, u)\right] \tag{7}$$

Different defuzzification methods determining the 'crisp output' based on 'fuzzy output' resulting from applicable rules may be used, i.e.:
1. Centre of gravity (COG)

$$u' = \frac{\sum_{i=1}^{m} U'(u_i) \cdot u_i}{\sum_{i=1}^{m} U'(u_i)}$$

2. Mean of maxima (MOM)

$$u' = \sum_{j=n}^{n} \frac{u_{max_j}}{n}, \quad where \left\{ u_{max_j} \in \mathbf{R} \mid U'(u_{max_j}) = \sup_{u}(U'(u)) \right\} \tag{9}$$

3. Modified centre of gravity (MCOG)
(Height method)

$$u' = \frac{\sum_{i=1}^{p} \bar{u}_i \cdot h_i}{\sum_{i=1}^{p} h_i} \tag{10}$$

where $u_i$ stands for centres of gravity of activated $U'^{(i)}$, $h_i$ denotes the respective level of activation of $U'^{(i)}$

The formulas written above constitute the essentials of a conventional fuzzy controller.

## 3. A hybrid PID fuzzy controller.

Neither a fuzzy PD controller nor a classical PD controller can eliminate steady-state error. The fuzzy PI controller has a slower response (a slower rise time) due to the integral error control variable, but it eliminates steady-state error. Due to the disadvantages of PD and PI controllers the PID controller is often used. Let us assume that it is possible to divide the action of the PID controller into two separate control actions: the action of the PD controller for faster response and the action of the PI controller in order to eliminate the steady-state error. This can be realized by means of the so called switching logic (Fig. 1). The fuzzy PD controller is active when the error is large and should be reduced quickly. Still, the fuzzy PI controller is activated only when the PD part reduces the error and change in error to a nearly-zero range. Both PD and PI parts are separately designed for the hybrid fuzzy PID controller. Such a controller was used to simulate the control of a linear plant of second order [2]. The simulation results show that the performance of a hybrid fuzzy PID controller is satisfactory.

## 4. Modifications of the fuzzy PI controller.

A control output of the fuzzy PI controller (a control input to the process) is accumulated by the following equation:

$$u(k+1) = u(k) + \Delta u(k) \tag{11}$$

As we can see, it is not easy to determine the maximum variation of the incremental controller output ($\Delta u$) that gives satisfactory rise time and overshoot in step response. An approach which requires the selection of the maximum absolute value of incremental controller output was proposed in [9]. If the response of the system is to move faster, a large value of incremental control input is necessary, while a smaller maximum absolute value is required in order to damp the system properly.

In the paper [9] two simple, intuitively obvious methods for the fuzzy resetting of the controller output accumulated by the integrating operation as necessary in the control situation, are proposed. According to the authors of [9], the following equation should be considered

$$u(k+1) = \left[1 - (r(k))^p\right] u(k) + \Delta u(k) \tag{12}$$

where $k$ is a sampling instance, $r(k)$ stands for resetting rate, $p$ is a respective value of an exponent that determines the nonlinearity of the effect of $r$ in resetting operation [8]. Let us notice that if $r(k)=1$, there is no integration action (PD type of a fuzzy logic controller is employed) and if $r(k)=0$, equation (12) describes a conventional fuzzy PI controller. The resetting rate can be determined by fuzzy methods which have some advantages as mentioned in [9]. One of them is that the rules for determining $r$ are easily constructed by intuition or experiments.

Let us recall the two methods of determining the resetting rate as introduced by [9]. The first method enables us to determine the resetting rate $r$ from the rules defined on $(e, \dot{e})$ input space. These rules may be written in the form:

$$R^j : \quad \textbf{if } x \textbf{ is } E^{(j)} \textbf{ and } y \textbf{ is } DE^{(j)} \quad \textbf{then } r \textbf{ is } R^{(j)} \tag{13}$$

A detailed list of such a collection of rules together with the characterization of respective fuzzy sets is given in [9]. Inference and defuzzification procedures complete the description of the first method.

The second method of determining the resetting rate is derived from rules defined on $(e, u)$ space. They take the form:

$$R^l : \quad \textbf{if } x \textbf{ is } E^{(l)} \textbf{ and } u \textbf{ is } U^{(l)} \quad \textbf{then } r \textbf{ is } R^{(l)} \tag{14}$$

The reasons for introducing the controller output $u$ instead of e.g. rate of error rate $\ddot{e}$ come from the following observations:

- The expert may know the fuzzy output of the controller rather than the crisp controller input,
- The acceleration of the system is related to the force exerted on the system.

The above mentioned set of rules and the characterization of respective fuzzy sets are also given in [9]. Like in the first method, inference and defuzzification procedures are indispensable.

The diagram scheme illustrating both of these methods is shown in Fig. 2.

## 5. The mathematical model of the inverted pendulum in a car system.

The pendulum-car system [4], shown in Fig. 3, consists of

- a car moving along a line on two rails of limited length,
- a pendulum hinged in the car by means of ball bearings, rotating freely in the plane containing the line,

- a car driving device containing a dc motor, a dc amplifier, and a pulley-belt transmission system.

Such a system is characterized by an unstable equilibrium point in upright position of the pendulum, a stable equilibrium point in pendant position, as well as two uncontrollable points when the pendulum is in horizontal position.

Now let us give a simplified mathematical description of the system. Assuming that the pendulum is a rigid body, both friction and damping forces are neglected in the system. Thus we obtain differential equations describing the system by projecting respective forces onto corresponding axes. These differential equations describe the dynamic behaviour of the pendulum-car system:

$$(M+m)\ddot{x} - \frac{1}{2}ml\dot{\theta}^2\sin\theta + \frac{1}{2}ml\ddot{\theta}\cos\theta = f \qquad (15)$$

$$\frac{1}{2}ml\ddot{x}\cos\theta + \frac{1}{3}ml^2\ddot{\theta} + \frac{1}{2}mgl\sin\theta = 0 \qquad (16)$$

Rearranging equations (15) and (16) we get:

$$\ddot{x} = -g\tan\theta - \frac{2}{3}\frac{l}{\cos\theta}\ddot{\theta} \qquad (17)$$

$$\ddot{\theta} = \frac{\left[(M+m)g\cdot\tan\theta + \frac{1}{2}ml\sin\theta\cdot\dot{\theta}^2\right] + f}{-\frac{2}{3}(M+m)\frac{l}{\cos\theta} + \frac{1}{2}ml\cos\theta} \qquad (18)$$

For the sake of simplicity, only the last equation is used for the stabilization of the system in two positions: upright and slightly deflected from vertical.

## 6. Simulation results.

Numerical results obtained by simulating the control of the pendulum using modified PI fuzzy controller will be presented here. As in [9], two forty-nine knowledge bases were used in each experiment.

The control objective was:

a) to stabilize the pendulum in upright (180°) position and

b) to stabilize it in a position that would be slightly deflected from vertical, i.e. 185°.

The parameters of the model were taken as follows [3]:

$M = 2.8$ kg, $m = 0.2$ kg, $l = 0.75$ m, $g = 9.81$ m/s$^2$

The initial position of the pendulum was 170° and the initial control value was 0.

The deflection angle $\theta = X1$, its derivative $d\theta/dt = X2$, control value $f = Dr$ and control error $\theta_i$-$SP_i = Err$ as functions of time for both types of controllers are shown in Figs. 4, 5, and 6. The resetting rate $r^p$ = Rp is shown in Figs 5 and 6 istead of the derivative of the deflection angle.

For the purpose of comparative study a quality index (*QI*) was defined as below

$$QI = \frac{1}{N+1} \sum_{i=0}^{N} (\theta_i - SP_i)^2 \tag{19}$$

where $SP_i$ is the set point and $N+1$ is the total number of observation points.

The results obtained by means of the modified PI fuzzy logic controllers were compared with the results obtained using conventional PI fuzzy logic controllers and the same scaling coefficients were used. Although the quality indices QI differ for all three controllers (cf. Figs 4,5 and 6), control results obtained by modified PI controllers seem to be smoother than those obtained by means of a conventional fuzzy controller.

## 7. Concluding remarks.

Three in our opinion important methods of modifying the conventional fuzzy controllers have been described in this paper. The first method consists in connecting fuzzy PD and PI controllers thus creating the so called hybrid fuzzy PID controller [2]. It makes it possible to avoid the disadvantages of fuzzy PD and PI controllers. The other two methods concern the improvement of a fuzzy PI controller [9]. Such an improved controller performs better than a conventional fuzzy PI controller due to the fact that the controller output accumulated by the integrating operation according to formula (12) is fuzzily reset. However, it should be pointed out that in spite of simplifications used in modification all the modified fuzzy controllers are considerably slower than conventional fuzzy controllers. Therefore modified controllers may require hardware solutions. Autonomous hardware solutions based on embedded microprocessors are being developed by the authors.

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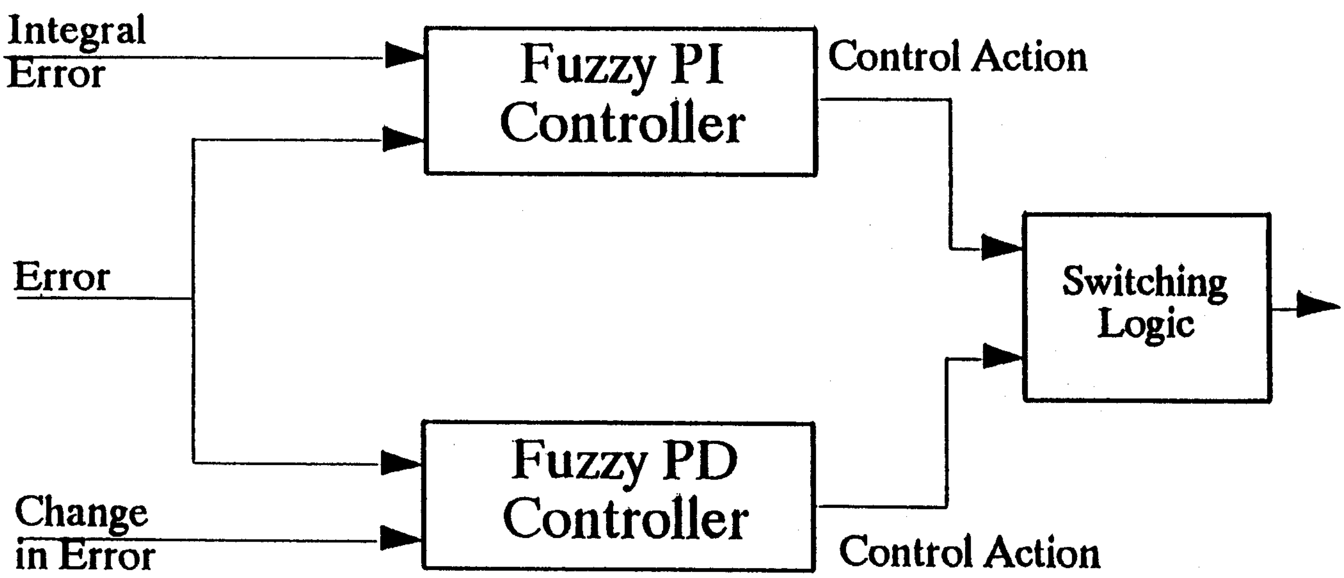



**Fig. 1. Block diagram of a hybrid PID fuzzy controller**

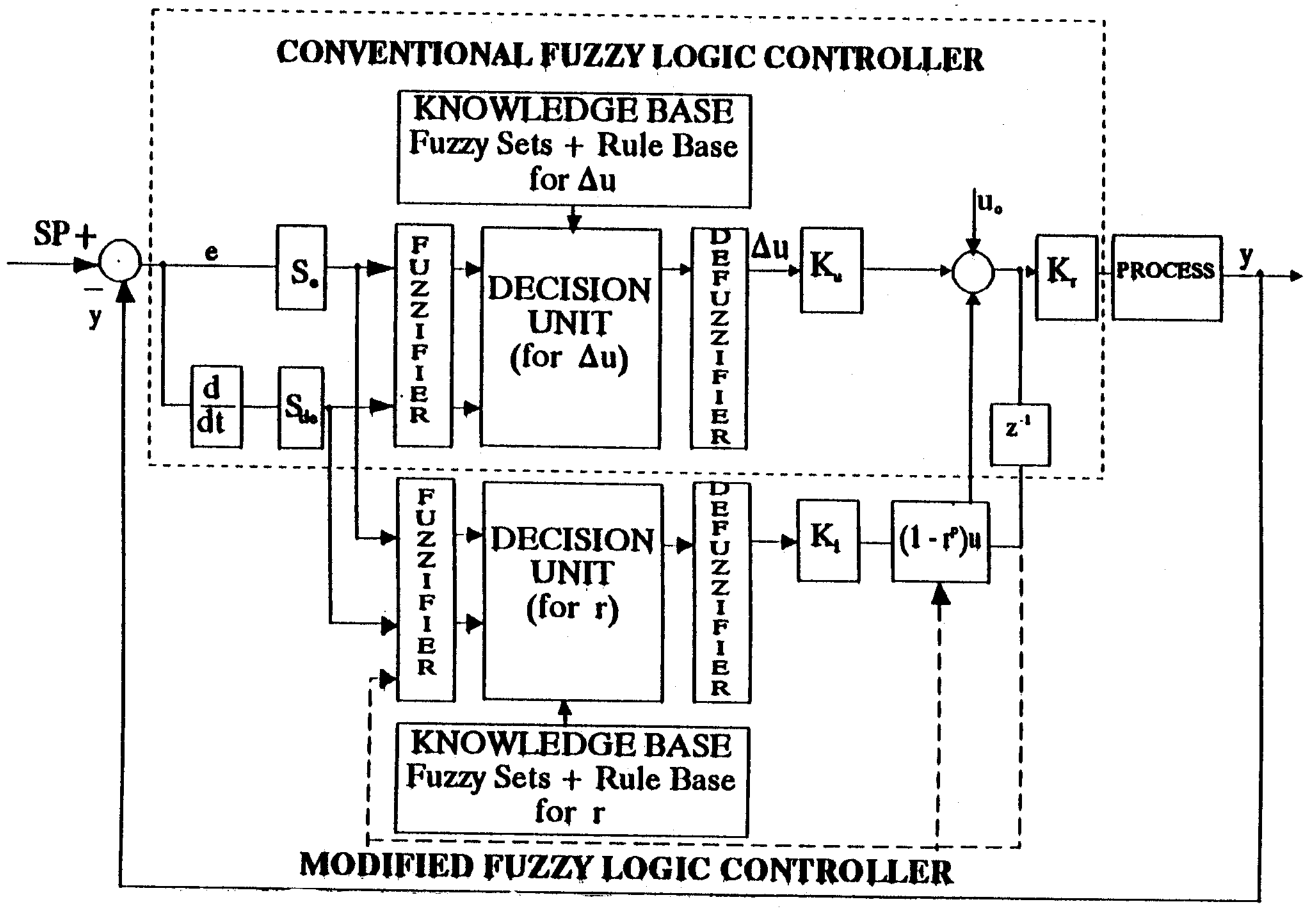



**Fig. 2.** **Block diagram of the control system with the modified PI fuzzy controller**

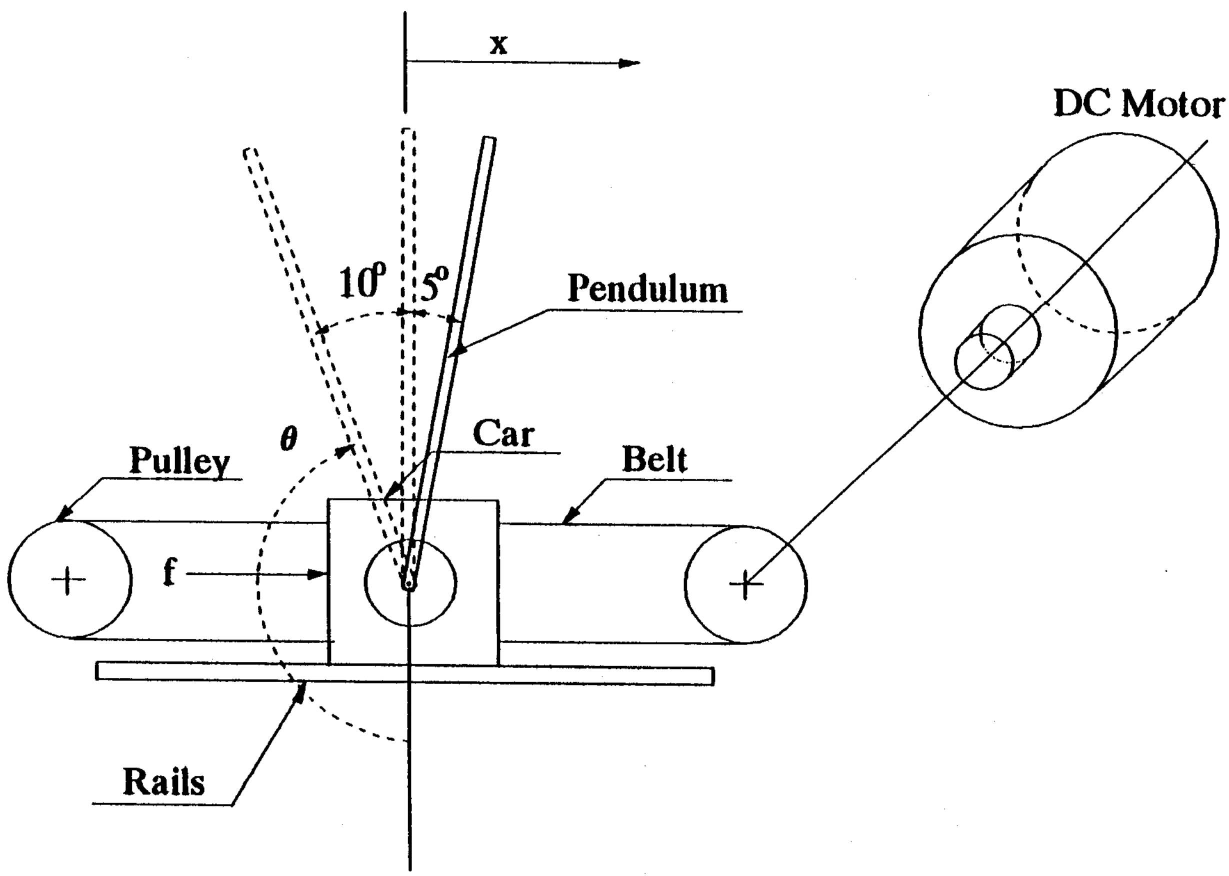
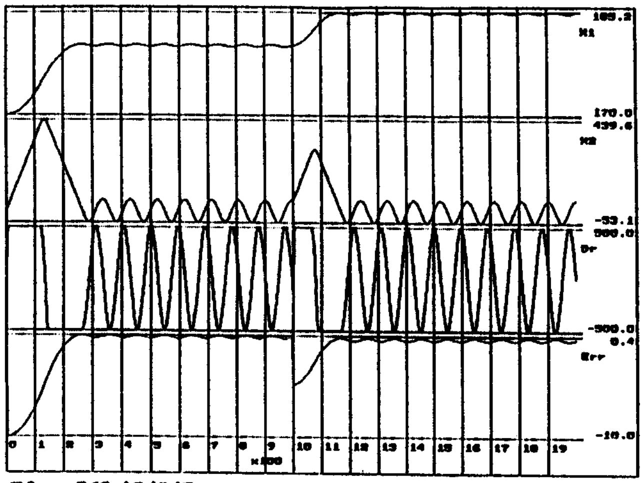
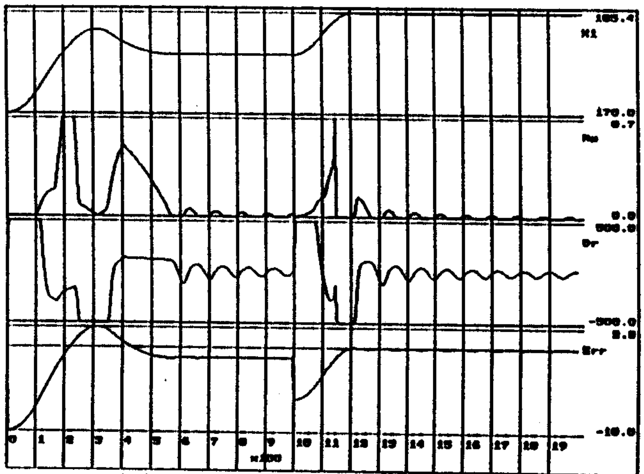



**Fig. 3.** **The inverted pendulum-car system**



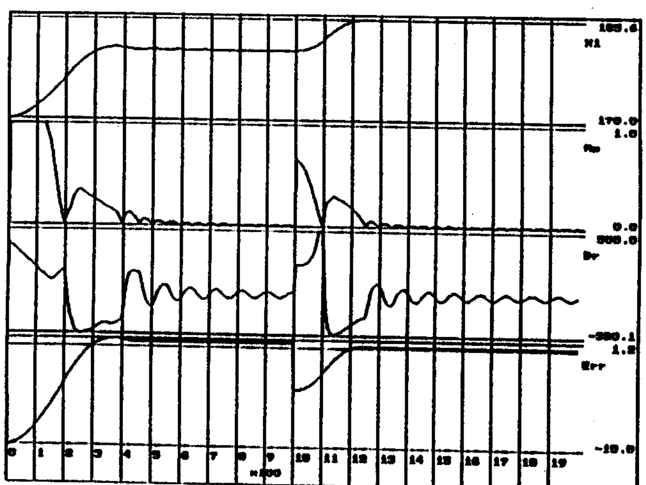
QI = 563.194047

Fig. 4. Results of control; conventional PI fuzzy controller applied. X1 = deflection angle, X2 = derivative of deflection angle, Dr = control value, Err - control error



QI = 701.561223

Fig. 5. Results of control; modified (e,ė) PI fuzzy controller applied. X1 = deflection angle, Rp = resetting rate, Dr = control value, Err - control error



QI = 752.916810

Fig. 6. Results of control; modified (e,u) PI fuzzy controller applied. X1 = deflection angle, Rp = resetting rate, Dr = control value, Err - control error