Workshop: Mathematical Foundations of Fuzzy Set Theory

Representation of Lattices by Fuzzy Sets

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From algebraic point of view it seems that Fuzzy Set Theory is unable to give anything new (for instance, level functions are isotone functions on a lattice; hence, instead of a fuzzy set one can consider the corresponding structure of the collection of level functions). But it is possible to use methods of that theory to obtain some well known results of classical theory in an easier way and also to give new representation theorems for some classical mathematical objects in therms of fuzzy sets.

In the following, among other propositions, a representation theorem for lattices will be stated, a consequence of which is the well known Birkhoff's representation theorem for distributive lattices.

1. Level functions are isotone functions when considered as functions from the set of function values to $\{0,1\}$.

2. Necessary and sufficient conditions under which all level subsets differ are that the set of all meet irreducible elements

The Fuzzy Set Theory is a branch of mathematics connected with the majority of mathematical theories. Most of mathematical objects can be observed in the fuzzy manner and many mathematical theorems can be generalized using fuzzy therms in order to obtain new theorems. One of the mathematical justifications of the theory of fuzzy sets could be in applying this theory to obtain results in classical mathematical theories.

Fuzzy set can be considered as a mapping from a nonempty set to the [0,1] real interval, to a Boolean algebra, lattice, or to some other algebraic (mostly ordered) structure. In the following, lattice valued fuzzy sets will be considered. of the lattice is a subset of the set of function values.

3. For every complete lattice there is a fuzzy set such that the set of level subsets of that fuzzy set under the set inclusion is a lattice anti-isomorphic with that lattice.

4. For a lattice with finite chains there is a lattice valued fuzzy set on the partially ordered set of all meet irreducible elements of that lattice, such that the lattice of all level functions (which are isotone functions) under the reverse order is isomorphic with that lattice.

As a consequence of 4. in the distributive case it follows:

5. Every finite distributive lattice is isomorphic with the set of all isotone functions on the set of all meet-irreducible elements of that lattice under the reverse order.

The last theorem is the well known Birkhoff's theorem obtained here by means of fuzzy sets, and 3 and 4 are new theorems which are generalizations of 5 i.e. which give the similar representation for an arbitrary lattice.

References

[1] B. Šešelja, A. Tepavčević, Representation of Lattices by Fuzzy Sets, Information Sciences, (to appear).