# Fuzzy almost semicontinous functions

Guo Shuangbing Dang Faning and Liu Wangjin
Department of Applied Mathematics, Southwest Jiaotong University, chengdu, Sichuan, China, 610031

Abstract: In this paper, fuzzy almost semicontinuons functions, a new class of functions, is introduced. Some Characteristic properties of this class of the funotious is investigated. The composition of fuzzy almost semicontinuous functious and other function is studied.
Keywords: Fuzzy semi-open, Fuzzy $\delta$-open, Fuzzy regular open, Fuzzy almost continuous, Fuzzy almost semi-continuous.

## 1 Introduoe

Zadeh in [1] introduced the concept of fuzzy set. Weaker forms of continuity in fuzzy topological spaces have been studied in [2-12].

In this paper, we introduce a new class of functions, called fuzzy almost semicontinuous functions, as a generalization of fuzzy almost continuous functions and fuzzy semicontinuous functions. The characters of fuzzy almost semi-continuous functions are investigated. The property of fuzzy almost semicontinuous functions and other functions is studied .

For general terminologies and the concepts not explained here, we refer to [2,3,12]. Some definitions and results which will be needed are recalled here.

In this paper X and Y mean fuzzy topological spaces.

Definition 1.1[2] Let A be a fuzzy set in a fuzzy topological space X, A is called

(i) fuzzy semi-open if there is a open set B such that $B \leq A \leq clB$.

(ii) fuzzy semiclosed if there is a closed set B such that $intB \leq A \leq B$.

(iii) fuzzy regular open if $int(cl(A))=A$.

(iv) fuzzy regular closed if $cl(int(A))=A$.

Theorem 1.2[13] For a fuzzy set A in a fuzzy space X, the following are equivalent .

(i) A is a fuzzy semiclosed set.

(ii) $A'$ is a fuzzy semiopen set.

(iii) $int(cl(A)) \leq A$

(iv) $cl(int(A')) \geq A'$

Definition 1.3 [3] Let A be a fuzzy set in X and define the following sets:

$s\text{-}clA = \bigcap \{ B \mid A \leq B,\ B \text{ is fuzzy semiclosed}\}$

s-intA = $\bigcup$ { B|B $<$ A, B is fuzzy semiopen}

A is a fuzzy semi-open set iff A=s-intA, A is a fuzzy semi-closed set iff A = s-clA

Definition 1.4 [13] A fuzzy set A on a fuzzy topological space X is said to be $\delta$-open if for each fuzzy point $x_\alpha \in A$, there exist a regular open fuzzy set B in X, such that $X_\alpha \in B < A$.

It follows the frout definition that a $\delta$-open fuzzy set is a union of fuzzy regular open sets and a fuzzy regular open set is a $\delta$-open fuzzy set.

Definition 1.5 [13] A fuzzy point $x_\alpha$ in a fuzzy topological space X is said to be a $\delta$-adherent point of a fuzzy set A in X if every regular open quasi-neighbourhood of $x_\alpha$ is quasi-coincident with A.

The union of all $\delta$-adherent fuzzy points of a fuzzy set A in a fuzzy topological space X is called the $\delta$-closure of A and is denoted by $\delta$-cl(A). If A=$\delta$-cl(A) then A is called fuzzy $\delta$-closed.

Lemma 1.6 [13] The $\delta$-closure of a fuzzy set in a fuzzy topological space is $\delta$-closed.

Definition 1.7 [5] Let A be a fuzzy set in a fuzzy topological space X, the fuzzy semi $\theta$-closure of A, denoted by cls-$\theta$ (A) is defined as {x $\in$ X | for every fuzzy semi-open semi qnasi-neighbourhood B of $x_\alpha$, s-clB Q A}, and A is fuzzy semi $\theta$-closed iff A = cls-$\theta$ (A)

Definition 1.8 Let $f: X \rightarrow Y$ be a function between two fuzzy topological spaces, then f is called

(i) fuzzy semicontinuous function [2] iff $f^{-1}(A)$ is a fuzzy semi-open set of X for each fuzzy open set A in Y.

(ii) fuzzy almost continuous function [2] iff $f^{-1}(A)$ is a fuzzy open set of X for each regular open set A in Y.

(iii) fuzzy weakly continuous function [2] iff $f^{-1}(A) < \text{int} f^{-1}$ (clA) for each open set A in Y.

(iv) a fuzzy weakly semi-continuous function [10] iff $f^{-1}(A) <$ s-int $f^{-1}$ (s-clA) for each open set A in Y.

(v) a fuzzy R-map [14] iff $f^{-1}(A)$ is a fuzzy regular open set of X for each fuzzy regular open set A in Y

(vi) a fuzzy semi irresolute function [5] iff $f^{-1}(A)$ is a fuzzy semi-open of X for each fuzzy semi $\theta$-open set A in Y.

(vii) a fuzzy irresolute function [8] iff $f^{-1}(A)$ is fuzzy semi-open set of X for each fuzzy semi-open set A in Y.

(viii) a fuzzy completely irresolute fuuotion [11] if $f^{-1}(A)$ is a fuzzy regular open set of X for each fuzzy semi-open set A in Y.

(IX) a fuzzy completely weakly irresolute function iff $f^{-1}(A)$ is a regular open set of X for each semi-θ open set A in Y.

Definition 1.9 [2] A fuzzy topological space ( X, δ ) is called a fuzzy semiregular space iff the collection of all fuzzy regular open sets of X form a base for fuzzy topology δ .

A fuzzy topological space X is called a fuzzy regular space iff each fuzzy open sets A of X is a union of fuzzy open sets λi of X such that cl λi ≤ λ .

Definition 1.10 [13] A fuzzy topological space X is normal if for every closed fuzzy set C in X and fuzzy open set A in X containing C, there exists a fuzzy open set B in X such that C≤B≤CLB≤A.

2 Fuzzy almost semicontinuous functions

Definition 2.1 A fuzzy function f:X→Y from a fuzzy topological space X to another fuzzy topological space Y is said to be a fuzzy almost semicontinuous function if $f^{-1}(A)$ is a fuzzy semi-open set of X for each fuzzy regular open set A in Y.

Theorem 2.2 If f:X→Y is a fuzzy almost continuous function , then f is a fuzzy almost semicontinuous .

Proof: obvious.

The converse of theorem 2.2 need not be true which is shown by the following Example 2.3.

Example 2.3 Let X={a,b},Y={a,b,c} and f:X→Y be defined as f(a)=a, f(b)=b, let us define fuzzy sets A in X and B in Y as follows, A(a) =0.3 ,A(b)=0.4, B(a)=0.3, B(b)=0.4, B(c)=0.5, Then $\{0, A, 1_X\}$ is a fuzzy topology on X and $\{0, B, 1_Y\}$ is a fuzzy topology on Y. It can be verified that f is fuzzy almost semicontinous, but the inverse image of a fuzzy regular open set B is not a fuzzy open set in X.

Theorem 2.4 If f:x→Y is a fuzzy almost semicontinuous function from a fuzzy topological space X to a fuzzy semiregular space , then f is almost continous.

Proof: The proof is straightforward from the definition 1.9 and 2.1 .

Theorem 2.5 Fuzzy weakly semicontinuous functions and fuzzy almost semicontinuous functions is independent notions.

This is shown by Example 2.6 and Example 2.7 .

Example 2.6 Refer to Example 2.4 f is a fuzzy almost semicontinuous function. then $f^{-1}(B) \le s\text{-}intf^{-1}(s\text{-}cl(B)) = s\text{-}intf^{-1}(B)$. Thus f is not a fuzzy weakly semicontinuous function.

Example 2.7 A fuzzy weakly semicontinuous function need not be a *fuzzy almost semicontinuous function.*

Let X={a,b,c} δ = {0,A,B,1} L = {0,D,1}, where A(a)=0.4, A(b)=0.2, A(c)=0.1, B(a)=0.5, B(b)=0.5, B(c)=0.5, D(a)=0.5, D(b)=0.5, D(c)=0.6. Consider the identify function f:(X, δ)→(Y,L), simple computations give $D=f^{-1}(D)<s\text{-int } f^{-1}(s\text{-cl}(D))=s\text{-int } f^{-1}(1)=1$

Hence f is a fuzzy weakly semicoutinuous function, Also by easy computations it follows that the inverse image of the regular open set D is not semi-open in X. Thus f is not a fuzzy almost semicontinuous.

From [10] we know that fuzzy almost continuous function and fuzzy weakly semicontinuous function are independent notions .Since a fuzzy almost continuous function is a fuzzy almost semicontinuous function and a fuzzy almost semicontinuous function need not be a fuzzy weakly continuous function.

The following Example 2.8 shows that a fuzzy weakly continuous function need not be a fuzzy almost semicontinuous function.

Example 2.8 Let X={a,b,c}, δ={0,B,1} and τ={0,A,1}, where A(a)=0.3, A(b)=0.1, A(c)=0.4, B(a)=0.6, B(b)=0.7, B(c)=0.5, consider the identity function f:(X, δ)→(Y, τ), simple computations give $A=f^{-1}(A) < \text{int } f^{-1}(\text{cl-}A)=\text{int } A'=B$.

Hence f is a fuzzy weakly continuous function. Also by computations, it follows that the inverse image of the regular open set A is not semi-open, thus f is not a fuzzy almost semicontinuous functions, so we obtain theorem 2.9.

Theorem 2.9 fuzzy weakly continuous functions and fuzzy almost semicoutinuons function is indepent notons.

Theorem 2.10 If f:X→Y is a semicontinuons function, then f is a fuzzy almost semi-continuous.

Proof: Noting a open set is semi-open, a regular open set is open.

The inverse of this theorem need not be true is shown by Example 2.8.

Example 2.11 let X = {a, b, c} , δ = {0, D, 1} and τ = {0, A, B, 1} , where A (a) =0.4, A (b) =0.2, A (c) =0.1, B (a) =0.5, B (b) =0.5, B (c) =0.5, D (a) =0.3, D (b) =0.2, D (c) =0.2, Consider the identity function f: (X, δ ) → (X, τ) , by computations it follows that B is regular open set and A is not.

The inverse of each regular open set is semi-open. However, the inverse of the open set A is not semi-open,therefore f is almost semicontinuous.

Definition 2.12 A function f:X→Y from a fuzzy topological space X to a fuzzy topological space Y is said to be fuzzy almost semicontinuous at a fuzzy point $X_\alpha$ in X if for each regular open set of Y containing $f(x_\alpha)$ there exists a fuzzy semi-open set B

Containing $x_\alpha$ such that $f(B) \subseteq A$

Theorem 2.12 let $f: X \to Y$ be a fuzzy function. then the following are equivalent.

(i) f is a fuzzy almost semicontinuous function

(ii) f is a fuzzy almost semicontinuous at each fuzzy point in X

(iii) $f^{-1}(A)$ is a fuzzy semi-closed set for each regular closed set A in Y

(vi) $f^{-1}(A)$ is a fuzzy semi-closed set for each $\delta$-closed fuzzy set A in Y.

(v) $f^{-1}(A)$ is a fuzzy semi-open set for each $\delta$-open fuzzy set A in Y.

(vi) $\text{int cl}(f^{-1}(A)) \leq f^{-1}(\delta\text{-cL}(A))$, for all fuzzy sets A in Y

(vii) $f(\text{in tcl}A) \leq \delta\text{-cl } f(A)$ for all sets A in X.

(viii) $f^{-1}(A) \leq \text{cl int } f^{-1}(\text{int cl}(A))$ for each open set A in X

(Xi) $f^{-1}(A) \geq \text{int cl } f^{-1}(\text{cl int}(A))$ for each closed set A in Y .

Proof: (i) <=> (ii) It is easy from the definitions .

(i) <=> (iii) Noting that $f^{-1}(A') = (f^{-1}(A))'$ for any fuzzy set A of Y, this is obvious.

(i) => (v) let A be a $\delta$-open fuzzy set in y, there exist fuzzy regular open set $B_i$ (i=I, is an index set) such that $A = \bigcup_{i \in I} B_i$

Now $f^{-1}(A) = f^{-1}(\bigcup_{i \in I} B_i) = \bigcup_{i \in I} f^{-1}(B_i)$ for each $B_i$(i=I), $f^{-1}(B_i)$ is a fuzzy semi-open set, so $f^{-1}(A)$ is a semi-open set.

(vi) <=> (v) This is obvious being a camplement of each other.

(iv) => (vi) Since the $\delta$-closure of the fuzzy set A in y is $\delta$-closed, $f^{-1}(\delta\text{-CL}(A))$ is a fuzzy semi-open set. Hence $f^{-1}(\delta\text{-CL}(A)) \geq \text{int cl}(f^{-1}\delta\text{-CLA}) \geq \text{int cl}(f^{-1}(A))$

(vi) => (vii) Let $f(x_\alpha) \nleq \delta\text{-cl } f(A)$ be a fuzzy point in Y, then $x_\alpha \nleq f^{-1}(\delta\text{-cL}f(A))$. Since $f^{-1}(\delta\text{-cl}f(A)) \geq \text{int cl}f^{-1}(f(A)) \geq \text{int cl}(A)$, from (vi) it follows that $x_\alpha \nleq \text{int cl}(A)$, which implies that $f(x_\alpha) \nleq f(\text{int cl}(A))$. Hence $f(\text{int cl}(A)) \leq \delta\text{-cl}f(A)$.

(vii) => vi It is easy proved.

(i) => (Viii) Since int cL(A) is a fuzzy regular open set when A is any open set in Y , $f^{-1}(\text{int cl}(A))$ is a fuzzy semi-open set, Hence $f^{-1}(\text{int cl}(A)) \leq \text{cl int}f^{-1}(\text{int cl}(A))$. Now A is fuzzy open set , so $\text{int cl}(A) \geq \text{int } A = A$ . Hence $f^{-1}(A) \leq \text{cl int}f^{-1}(\text{int cl}(A))$.

(Viii)=>(i) Let A be any regular open set in Y, Then A = int cl(A), by (Viii) $cl\ intf^{-1}(int\ cl(A)) = cl\ intf^{-1}(A) \geq f^{-1}(A)$, which shows that $f^{-1}(A)$ is a fuzzy semi-open set.

(iii)<=>(iX) It is analogical to the proof of (i)<=>(viii)

Theorem 2.14 Every fuzzy R_map is a fuzzy almost semicontinuous function

proof: obvious

The converse of the above is not true by Example 2.3.

3. Composition of fuzzy almost semicontinuous functions

Theorem 3.1 If $f:X\to Y$ is fuzzy almost semicontinuous function and $g: Y\to z$ is fuzzy R-map, then $g \circ f : X\to Z$ is fuzzy almost semicontinuous.

proof: The theorem follows from the definitions .

Theorem 3.2 If $f:X\to Y$ is fuzzy irresolute and $g: Y\to Z$ is fuzzy almost semicontinuous, then $g \circ f : X\to Z$ is fuzzy almost semicontinuous.

proof: The theorem follows from the definitions.

Theorem 3.3 If $f:X\to Y$ is completely irresolute and $g:Y\to z$ is fuzzy almost semicontinuous, then $g \circ f: X\to z$ is R-map.

proof: let B be a fuzzy regular open set of z, the $g^{-1}(B)$ is a fuzzy semi-open set in y. Now $f^{-1}(g^{-1}(B))=(g \circ f)^{-1}(B)$ is a fuzzy regular open set in X, since f is a fuzzy completely irresolute function. Hence the theorem correct.

Theorem 3.4. If $f:X\to Y$ is fuzzy almost semicontinuous and $g:y\to z$ is weakly completely irresolute, then $g \circ f: x\to z$ is semi-irresolute.

proof: obvious.

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