FUZZY STRONG PRE-SEMICONTINUITY

Ye Xiao-Hong

Department of Mathematics, Yanan University, Yanan, China

ABSTRACT

In this paper, we introduce and study the fuzzy strongly pre-semicontinuous mapping in fuzzy topological spaces.

Key words: Fuzzy topological spaces; fuzzy pre-semiopen sets; fuzzy preopen sets; fuzzy strongly pre-semicontinuous mappings.

1. PRELIMINARIES

In this paper, non-empty sets will be denoted by X, Y, etc. (X,δ) and (Y,τ) will denoted fuzzy topological spaces with the topology δ and τ respectively. A°, A^{-}, $A_{\circ}$, A_{-} and A' will denote respectively the interior, closure, semi-interior, semi-closure and complement of the fuzzy set A.

Definition 1.1[2]: Let A be a fuzzy set of (X,δ). Then A is called

(1) a fuzzy pre-semiopen set of X iff $A \leqslant (A^{-})_{\circ}$;

(2) a fuzzy pre-semiclosed set of X iff $A \geqslant (A^{\circ})_{-}$;

Definition 1.2[2]: Let A be a fuzzy set of (X,δ). Then the union of all fuzzy pre-semiopen sets containing in A will be called the fuzzy pre-semi-interior of A and denoted $A_{\blacktriangle}$. The intersection of all fuzzy pre-semiclosed sets containing A will be called the fuzzy pre-semi-closure of A and denoted $A_{\sim}$.

2. FUZZY STRONGLY PRE-SEMICONTINUOUS MAPPINGS

Definition 2.1. Let $f:(X,\delta)\to(Y,\tau)$ be a mapping. f is called a fuzzy strongly pre-semicontinuous mapping if $f^{-1}(B)$ is a fuzzy pre-semiopen set of X for each fuzzy preopen set B of Y.

Definition 2.2. Let $f:(X,\delta)\rightarrow(Y,\tau)$ be a mapping. f is said to be fuzzy strongly pre-semicontinuous at a fuzzy point p in X, if fuzzy preopen set B of Y and $f(p)\leqslant B$, there exists a fuzzy pre-semiopen set A of X such that $p\leqslant A$ and $f(A)\leqslant B$.

Theorem 2.3. Let $f: (X,\delta)\rightarrow(Y,\tau)$ be a mapping. Then the following are equivalent:

(1) f is fuzzy strongly pre-semicontinuous.

(2) $f^{-1}(B)$ is a fuzzy pre-semiclosed set of X for each fuzzy preclosed set B of Y.

(3) $f((A^{\circ})_{-})\leqslant(f(A))_{\sim}$ for each fuzzy set A of X.

(4) $((f^{-1}(B))^{\circ})_{-}\leqslant f^{-1}(B_{\sim})$ for each fuzzy set B of Y.

(5) $f^{-1}(B_{\Delta})\leqslant((f^{-1}(B))^{-})_{\circ}$ for each fuzzy set B of Y.

Theorem 2.4. Let $f:(X,\delta)\rightarrow(Y,\tau)$ be a mapping. Then f is fuzzy strongly pre-semicontinuous iff f is fuzzy srtongly pre-semicontinuous for each fuzzy point p in X.

Proof. Let f be fuzzy strongly pre-semicontinuous, p be a fuzzy point in X and B be a fuzzy preopen set of Y such that $f(P)\leqslant B$. Then $p\leqslant f^{-1}(B)$. Let $A=f^{-1}(B)$, then A is fuzzy pre-semiopen set of X, and so $f(A)=ff^{-1}(B)\leqslant B$. Thus f is fuzzy strongly pre-semicontinuous for each fuzzy point p in X.

Conversely, let B be a fuzzy preopen set of Y and p be a fuzzy point in $f^{-1}(B)$. Then $p\leqslant f^{-1}(B)$, i.e., $f(p)\leqslant B$. From hypothesis there is a fuzzy pre-semiopen set A of X such that $p\leqslant A$ and $f(A)\leqslant B$, hence

$$p\leqslant A\leqslant f^{-1}f(A)\leqslant f^{-1}(B)$$

and

$$p\leqslant A\leqslant (A^{-})_{\circ}\leqslant((f^{-1}(B))^{-})_{\circ}.$$

Since p is arbitrary and $f^{-1}(B)$ is the union of all fuzzy points in $f^{-1}(B)$,

$$f^{-1}(B)\leqslant((f^{-1}(B))^{-})_{\circ}.$$

i.e.,$f^{-1}(B)$ is a fuzzy pre-semiopen set of X. Thus f is fuzzy strongly pre-semicontinuous.

Theorem 2.5. Let $f:(X,\delta)\to(Y,\tau)$ be one-to-one and onto, where X and Y are fuzzy spaces. f is a fuzzy strongly pre-semicontinuous mapping iff $(f(A))_{\Delta}\leqslant f((A^{-})_{\circ})$ for each fuzzy set A of X.

Theorem 2.6. Let X_1, X_2, Y_1 and Y_2 be fuzzy spaces such that X_1 is product related to X_2. Then the product $f_1\times f_2:X_1\times X_2\to Y_1\times Y_2$ of fuzzy strongly pre-semicontinuous mappings $f_1:X_1\to Y_1$ and $f_2:X_2\to Y_2$ is fuzzy strongly pre-semicontinuous.

Remark 2.7. For the mapping $f:X\to Y$ the following statements are valid:

f is fuzzy strongly pre-semicontinuous

=> f is fuzzy S_1-pre-semicontinuous[5]

=> f is fuzzy pre-semicontinuous[2]

=> f is fuzzy W-pre-semicontinuous[7].

None is reversible.

Proposition 2.8. Let $f:X\to Y$ be fuzzy strongly pre-semicontinuous and $g:Y\to Z$ be fuzzy almost precontinuous[3]. Then $g\circ f$ is fuzzy W-pre-semicontinuous.

Proposition 2.9. Let $f:X\to Y$ be fuzzy strongly pre-semicontinuous and $g:Y\to Z$ be fuzzy weakly S-irresolute[4]. Then $g\circ f$ is fuzzy S_1-pre-semicontinuous.

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