COINCIDENCE DEGREE THEORY FOR FUZZY MAPPINGS

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Abstract:

In this paper the concept of coincidence degree for fuzzy mappings and set-valued mappings is discussed. The results presented in this paper are the improvement and generalization of relevant results of [1].

Keywords: fuzzy mapping, coincidence degree for fuzzy mappings and set-valued mappings, Kakutani-Ky Fan's theorem.

1. Introduction and preliminaries

In [1] the concept of coincidence degree has been introduced and the existence problems of coincidence point for fuzzy mappings and single-valued mappings have been studied by Chang Shih-sen. Since the concept of coincidence degree for fuzzy mappings is the unification and expansion of the concepts of fixed point for set-valued mapping, coincidence point for mappings and fixed degree for fuzzy mappings etc. [2,3]. Therefore the study about the theory of coincidence degree for fuzzy mappings will certainly have an important influence on the development of the theory and application of fuzzy mapping.

In this paper the concept of coincidence degree for fuzzy mapping and set-valued mapping is discussed. The results presented in this paper are the improvement and generalization of relevant results of [1].

For ease of relation we first introduce the following concepts:

Definition 1. Let X be a nonempty set, M a linear topological space, $T: X \to 2^M$ a set-valued mapping, $F: X \to \mathcal{F}(M)$ a fuzzy mapping ($F(x)$ is denoted by F_x in the sequel), $\mathcal{F}(M)$ a collection of all fuzzy sets over M. The number $\max_{y \in T_x} F_x(y) \in [0,1]$ is called to be the coincidence degree of x for T and F, and we denote it by

$$D_{coin}(x; T, F).$$

Specially, when

$$\max_{y \in T_x} F_x(y) = D_{coin}(x; T, F) = \max_{u \in M} F_x(u),$$

we say that x is a coincidence point for T and F.

Definition 2. Let X be a topological space, M a linear topological space. The fuzzy mapping $F: X \to \mathcal{F}(M)$ is called convex, if for each $x \in X$ the fuzzy set F_x on M is a fuzzy convex set, i.e. for any $y, z \in M$ the following is true:

$$F_x(ty+(1-t)z) \geqslant \min\{F_x(y), F_x(z)\}, \quad \forall t \in [0, 1].$$

F is called closed, if $F_x(y) = F(x,y)$ is upper semi-continuous (as a two element function on $X \times M$).

In the sequel we denote by

$$(A)_\alpha = \{x \in M;\ A(x) \geqslant \alpha\}, \quad \alpha \in [0.1],$$

the α-cut set of $A \in \mathcal{F}(M)$.

2. Coincidence theorems for fuzzy mappings in locally convex linear topological spaces

Theorem 1. Let $X\neq\emptyset$, M a locally convex Hausdorff linear topological space, $T: X\to 2^M$ a set-valued mapping, and $Y=T(X)=\bigcup_{x\in X} Tx$ a compact convex set of M. Suppose that $\alpha(x): X\to[\gamma,1]$, $0<\gamma\leq 1$, is a functional and $F: X\to\mathcal{F}(Y)$ a fuzzy mapping satisfying the following conditions:

(i) for each $u\in Y$, the set

$$\bigcap_{x\in T^{-1}u}\{y\in Y;\ F_x(y)\geq\alpha(x),\ \forall x\in T^{-1}u\}$$

is a nonempty set of Y;

(ii) $\tilde{F}: Y\to\mathcal{F}(Y)$ is a convex closed fuzzy mapping which is defined by

$$\tilde{F}_u(y)=\begin{cases}\max_{z\in T^{-1}u} F_z(y) & \text{if } y\in\bigcap_{z\in T^{-1}u}(F\ (y))\\ 0 & \text{otherwise;}\end{cases}$$

(iii) $F_x(y)\leq\alpha(x)$, $\forall x\in X$, $y\in Y$.

Then there exists $x_*\in X$ such that

$$D_{coin}(x_*;\ T,\ F)=\alpha(x_*)$$

and x_* is a coincidence point for T and F.

Proof. We define a set-valued mapping as follows:

$$P: Y\to 2^Y,\qquad u\longmapsto\bigcap_{z\in T^{-1}u}(F_z)_{\alpha(z)}$$

First, we prove that for each $u\in Y$, P(u) is a nonempty compact convex set of Y.

In fact, it follows from condition (i) that $P(u)\neq\emptyset$. Suppose that $\{y_\beta\}_{\beta\in J}\subset P(u)$ (where J is an index set) which converges to

$y_0 \in Y$, then for each $z \in T^{-1}u$, $F_z(y) \geqslant \alpha(z)$. Since $\tilde{F}$ is a closed fuzzy mapping,

$$\tilde{F}_u(y_0) \geqslant \overline{\lim}\, \tilde{F}_u(y_\beta) = \overline{\lim} \max_{z \in T^{-1}u} F_z(y_\beta) \geqslant \max_{z \in T^{-1}u} \alpha(z) \geqslant r > 0.$$

It follows from definition of $\tilde{F}_u(y)$ that $y_0 \in P(u)$. This shows that $P(u)$ is a closed set of Y, and hence it is compact.

Let y_1, $y_2 \in P(u)$, $t \in (0,1)$, since $\tilde{F}$ is a convex fuzzy mapping, we have

$$\begin{aligned}\tilde{F}_u(ty_1 + (1-t)y_2) &\geqslant \min\{\tilde{F}_u(y_1), \tilde{F}_u(y_2)\}\\ &= \min\{\max_{z \in T^{-1}u} F_z(y_1), \max_{z \in T^{-1}u} F_z(y_2)\}\\ &\geqslant \min\{\max_{z \in T^{-1}u} \alpha(z), \max_{z \in T^{-1}u} \alpha(z)\}\\ &= \max_{z \in T^{-1}u} \alpha(z) \geqslant r > 0.\end{aligned}$$

i.e. $ty_1 + (1-t)y_2 \in \bigcap_{z \in T^{-1}u} (F_z)_{\alpha(z)} = P(u)$. Hence $P(u)$ is a nonempty compact convex set.

Next we prove the graph of P

$$\text{Graph}(P) = \bigcup_{u \in Y} \{(u,y):\ y \in P(u)\}$$

is a closed set of $M \times M$. Since $\tilde{F}$ is a closed fuzzy mapping, the set

$$\Omega = \{(u,y):\ \tilde{F}_u(y) \geqslant r,\quad u,\ y \in Y\}$$

is a closed set of $Y \times Y$. We shall prove that $\Omega = \text{Graph}(P)$.

Suppose that $(u,y) \in \text{Graph}(P)$, Then $u \in Y$, $y \in P(u)$, $y \in \bigcap_{z \in T^{-1}u} (F_z)_{\alpha(z)}$ This implies that

$$\tilde{F}_u(y) = \max_{z \in T^{-1}u} F_z(y) \geqslant \max_{z \in T^{-1}u} \alpha(z) \geqslant r.$$

i.e. $(u,y) \in \Omega$

Conversely, if $(u,y) \in \Omega$ then we have $\tilde{F}_u(y) \geqslant r > 0$, this is implies that $y \in P(u)$, i.e. $(u,y) \in \text{Graph}(P)$. We have thus shown that the Graph(P) is a closed set of $Y \times Y$. By Ky Fan's theorem (cf. [4])

there exists $u_* \in Y$, such that $u_* \in P(u_*)$, hence there exists $x_* \in X$ such that $x_* \in T^{-1}u_*$. Since $u_* \in P(u_*)$, for each $z \in T^{-1}u_*$, we have $F_z(u_*) \geq \alpha(z)$. Hence $F_{x_*}(u_*) \geq \alpha(x_*)$. so that

$$\max_{y \in Tx_*} F_{x_*}(y) \geq F_{x_*}(u_*) \geq \alpha(x_*). \tag{1}$$

By condition (iii) and (1) we have $\max_{y \in Tx_*} F_{x_*}(y) = \alpha(x_*)$ i.e.

$$D_{coin}(x_*;\ T,\ F) = \alpha(x_*), \tag{2}$$

and

$$\max_{y \in Tx_*} F_{x_*}(y) = \alpha(x_*) = D_{coin}(x_*;\ T,\ F).$$

Thus x_* is a coincidence point for T and F. This completes the proof of theorem.

Definition 3. Suppose that X is a topological space, (Y,d) is a metric space, $S: X \to 2^Y$ is a set-valued mapping and A is any subset of Y. Let

$$A_\varepsilon = \bigcup_{a \in A} \{x \in Y;\ d(x,a) < \varepsilon,\ \ a \in A\}.$$

Suppose that $\{A_n\}$ is any sequence of subset in Y. A is a subset of Y. We call that $\{A_n\}$ converges to A, if

(i) for each $a \in A$, there are $a_n \in A_n$, $n=1,2,\cdots$, such that $a_n \to a$;

(ii) for every $\varepsilon > 0$, there is a $N > 0$, such that $n > N$ implies that $A_n \subset A_\varepsilon$

$S: X \to 2^Y$ is called to be continuous at x, if $S(x_n) \to S(x)$ as $x_n \to x$. If S is continuous at every point of X, then S is called to be continuous on X.

Theorem 2. Let X be a nonempty compact convex set of a locally convex Hausdorff linear topological space, M a metric space,

$T: X \longrightarrow 2^M$ a continuous mapping (we write $Y = T(X)$), $F: X \longrightarrow \mathscr{F}(Y)$ a fuzzy mapping and $\alpha: X \longrightarrow (0,1]$ a continuous functional. Suppose that $\tilde{F}$ is a fuzzy mapping which is defined by

$$\tilde{F}: Y \longrightarrow \mathscr{F}(X), \quad \tilde{F}_y(x) = F_x(y),$$

and satisfying the following condition:

(i) for each $x \in X$, the set

$$\bigcap_{y \in Tx} \{ z \in X; \quad \tilde{F}_y(z) \geq \alpha(x), \quad y \in Tx \}$$

is a nonempty closed convex set;

(ii) $\tilde{F}$ is a closed fuzzy mapping;

(iii) $\tilde{F}_y(x) \leq \alpha(x), \quad \forall x \in X, \quad \forall y \in Y.$

Then there exists $x \in X$ such that

$$D_{coin}(x_* ; \ T, \ F) = \alpha(x_*),$$

and x_* is a coincidence point of T and F.

Proof. we define a set-valued mapping as follows:

$$S: X \longrightarrow 2^X \quad S(x) = \bigcap_{y \in Tx} \{ z \in X: \quad \tilde{F}_y(z) \geq \alpha(x), \quad y \in Tx \}.$$

Then $S(x)$ is a nonempty closed convex set. Since X is compact, therefore $S(x)$ is a nonempty compact convex set.

We now prove the graph of S

$$\mathrm{Graph}(S) = \bigcup_{x \in X} \{ (x,y); \quad y \in S(x) \} \subset X \times X$$

is a closed set. In fact, suppose that $\{ (x_\beta, y_\beta) \}_{\beta \in J}$ is any net of graph(S)(where J is an index set) and $(x_\beta, y_\beta) \longrightarrow (x_0, y_0)$. Since $y_\beta \in S(x_\beta)$, hence for each $y \in Tx_\beta$, we have

$$\tilde{F}_y(y_\beta) \geq \alpha(x_\beta).$$

Since T and α are continuous and $\tilde{F}$ is a closed fuzzy mapping, it follows that

$$\tilde{F}_y(y_0) \geqslant \alpha(x_0), \qquad \forall y \in Tx_0 .$$

Hence $(x_0, y_0) \in \text{graph}(S)$. By Ky Fan's theorem [4] there exists $x_* \in X$ such that $x_* \in S(x_*)$. Thus for each $y \in Tx_*$, we have $\tilde{F}_y(x_*) \geqslant \alpha(x_*)$, i.e.

$$F_{x_*}(y) \geqslant \alpha(x_*), \qquad \forall y \in Tx_* .$$

Therefore we have

$$\max_{y \in Tx_*} F_{x_*}(y) \geqslant \alpha(x_*). \tag{3}$$

it follows from condition (iii) and (3) that

$$D_{coin}(x_*;\ T,\ F) = \alpha(x_*) = \max_{y \in Y} F_{x_*}(u).$$

Hence x_* is a coincidence point of T and F. This completes the proof.

Theorem 3. Let X be a nonempty compact convex set of a locally convex Hausdorff linear topological space, M a locally convex metrizable linear topological space. Suppose that $T: X \to 2^M$ is continuous, $Y = T(X)$ a compact convex set, $\alpha: X \to (0,1]$ a continuous functional and $F: X \to \mathcal{F}(Y)$ is a closed convex fuzzy mapping satisfying condition: for each $u \in Y$, the set

$$\bigcap_{z \in T^{-1}u} \{ y \in Y:\ F_z(y) \geqslant \alpha(z),\quad z \in T^{-1}u \} \neq \emptyset . \tag{4}$$

Then there exists $x_* \in X$ such that

$$D_{coin}(x_*;\ T,\ F) \geqslant \alpha(x_*).$$

Proof. We define a set-valued mapping as follows:

$$P:\ Y \to 2^Y \quad P(u) = \bigcap_{z \in T^{-1}u} \{ y \in Y;\ F_z(y) \geqslant \alpha(z),\quad z \in T^{-1}u \} \tag{5}$$

By above conditions, we easily obtain that P(u) is a nonempty closed convex set, thus $P(u)$ is a nonempty compact convex set and the set

$$Graph(P)=\bigcup_{u\in Y}\{(u,y):\quad y\in P(u)\}$$

is a closed set of $Y\times Y$. By Ky Fan's theorem there exists $u_*\in Y$, such that $u_*\in P(u_*)$. Since $T(X)=Y$, there exists $x_*\in X$, such that $x_*\in T^{-1}u_*$ (or $u_*\in Tx_*$). Since $u_*\in P(u_*)$, for each $z\in T^{-1}u_*$ we have $F_z(u_*)\geqslant\alpha(z)$, therefore $F_{x_*}(u_*)\geqslant\alpha(x_*)$. Then

$$\max_{y\in Tx_*}F_{x_*}(y)\geqslant F_{x_*}(u_*)\geqslant\alpha(x_*),$$

i.e.

$$D_{coin}(x_*;\ T,\ F)\geqslant\alpha(x_*).$$

This completes the proof.

Corollary. Under the conditions of the theorem 3, if we take that $\alpha(x)=\max_{u\in Y}F_x(u)$, then x_* is a coincidence point for T and F.

Theorem 4. Let X be a nonempty convex set of a locally convex Hausdorff linear topological space, M a locally convex Hausdorff linear topological space, $T: X\to 2^M$ a set-valued mapping, $Y=T(X)$ a compact convex set, $\alpha: X\to[r,1]$, $r\in(0,1]$, a lower semi-continuous functional and $F: X\to\mathcal{F}(Y)$ a closed convex fuzzy mapping satisfying the following conditions:

(i) for each $u\in Y$ the set

$$\bigcap_{z\in T^{-1}u}\{y\in Y:\quad F_z(y)\geqslant\alpha(z),\quad z\in T^{-1}u\}\neq\phi\ ;$$

(ii) $\max_{z\in T^{-1}u}F_z(y)$ is a upper semi-continuous on $Y\times Y$;

(iii) $F_x(y)\leqslant\alpha(x)$, $\forall x\in X$, $\forall y\in Y$.

Then there exists $x_*\in X$, such that

$$D_{coin}(x_* ;\ T,\ F) = \alpha(x_*).$$

Proof. We define a fuzzy mapping as follows:

$$\tilde{F}: Y \to \mathscr{F}(Y), \quad \tilde{F}_u(y) = \begin{cases} \max\limits_{z \in T^{-1}u} F_z(y), & \text{if } y \in \bigcap\limits_{z \in T^{-1}u} (F_z)_{\alpha(z)}, \\ 0 & \text{otherwise.} \end{cases}$$

We shall now prove that $\tilde{F}$ is a closed convex fuzzy mapping. In fact, suppose that $y_1, y_2 \in \bigcap\limits_{z \in T^{-1}u} (F_z)_{\alpha(z)}$, then for each $z \in T^{-1}u$, $t \in [0,1]$, we have

$$F_z(ty_1 + (1-t)y_2) \geq \min\{F_z(y_1),\ F_z(y_2)\} \geq \alpha(z) \geq r > o,$$

hence $ty_1 + (1-t)y_2 \in \bigcap\limits_{z \in T^{-1}u} (F_z)_{\alpha(z)}$, and thus

$$\tilde{F}_u(ty_1 + (1-t)y_2) = \max_{z \in T^{-1}u} F_z(ty_1 + (1-t)y_2)$$
$$\geq \max_{z \in T^{-1}u} (\min\{F_z(y_1),\ F_z(y_2)\}) = \min\{\tilde{F}_u(y_1),\ \tilde{F}_u(y_2)\}.$$

This shows that $\tilde{F}$ is a convex fuzzy mapping.

Next we prove that for each $\alpha \in (0,1]$ the set

$$\{(u,y):\ \tilde{F}_u(y) \geq \alpha,\ u,\ y \in Y\} \tag{6}$$

is a closed set of $Y \times Y$. In fact, by definition of $\tilde{F}$, we have that

$$\{(u,y):\ \tilde{F}_u(y) \geq \alpha,\ u,y \in Y\} = \{(u,y):\ \max_{z \in T^{-1}u} F_z(y) \geq \alpha,\ y \in \bigcap_{z \in T^{-1}u} (F_z)_{\alpha(z)},\ u \in Y\}$$
$$= \{(u,y):\ \max_{z \in T^{-1}u} F_z(y) \geq \alpha,\ u,y \in Y\} \cap \{(u,y):\ y \in \bigcap_{z \in T^{-1}u} (F_z)_{\alpha(z)},\ u \in Y\}.$$

Since $\max\limits_{z \in T^{-1}u} F_z(y)$ is upper semi-continuous, hence the set

$$\{(u,y):\ \max_{z \in T^{-1}u} F_z(y) \geq \alpha,\ u,\ y \in Y\}.$$

is a closed set of $Y \times Y$. Since $F_z(y) - \alpha(z)$ is upper semi-continuous, the set

$$\{(u,y):\ y \in \bigcap_{z \in T^{-1}u} (F_z)_{\alpha(z)},\ u \in Y\}$$

is a closed set. Therefore the set $\{(u,y) \in Y \times Y:\ \tilde{F}_u(y) \geq \alpha\}$ is a closed set, this implies that $\tilde{F}$ is a closed fuzzy mapping, thus all conditions oftheorem 1 are satisfied, and there exists a point $x_* \in X$ such that

$$D_{coin}(x_*;\ T,\ F)=\alpha(x_*).$$

The theorem follows.

Definition 4. Suppos X, Y are topological spaces, S, T: $X \to 2^Y$ are set-valued mappings. We call there is the coincidence between S and T, if there exists $(x_0, y_0) \in X \times Y$ such that $y_0 \in S(x_0) \times T(x_0)$.

Theorem 5. Let X be a nonempty set, M a locally convex Hausdorff linear topological space. Let T: $X \to 2^M$ be a set-valued mapping, and Y=T(X) a compact convex set of M, P: $X \to 2^Y$ a set-valued mapping satisfying the following conditions:

(i) for each $u \in Y$ the set $\bigcap_{z \in T^{-1}u} P(z)$ is a nonempty closed convex set of Y;

(ii) the set $\bigcup_{u \in Y} \{(u,y):\ y \in \bigcap_{z \in T^{-1}u} P(z),\ z \in T^{-1}u\}$ is a closed set of $Y \times Y$.

Then there is the coincidence between P and T.

Proof. We take that $\alpha(x) \equiv 1$, $\forall x \in X$, F: $X \to \mathcal{F}(Y)$, $x \to \chi_{P(x)}$ and $\tilde{F}$: $Y \to \mathcal{F}(Y)$, as follows:

$$\tilde{F}_u(y)=\begin{cases} 1, & \text{if } y \in \bigcap_{z \in T^{-1}u} P(z); \\ 0, & \text{otherwise.} \end{cases}$$

Then all conditions of theorem 1 are satisfied. By theorem 1 there exists $u_* \in Y$, $x_* \in T^{-1}u_*$ such that

$$\max \tilde{F}_x(y) \geq F_{x_*}(u_*) \geq \alpha(x_*)=1.$$

Hence $F_{x_*}(u_*)=1$, i.e. $u_* \in P(x_*)$, so that $u_* \in P(x_*) \cap T(x_*)$. Then there is the coincidence between T and P. This completes the proof.

In the following, we shall discuss the relationship between the coincidence and coincidence point in special conditions.

Theorem 6. Let X be a nonempty set, Y a topological space, $\alpha: X \longrightarrow (0,1]$, a functional, $F: X \longrightarrow \mathcal{F}(Y)$ a fuzzy mapping, and $T: X \longrightarrow 2^Y$ a set-valued mapping satisfying the following conditions:

(i) for each $x \in X$, $(F_x)_{\alpha(x)} \neq \emptyset$.

(ii) for each $x \in X$, Tx is a compact set and F_x is a continuous function on Y.

We now define set-valued mapping as follows:

$$G: X \longrightarrow 2^Y, \quad x \longrightarrow (F_x)_{\alpha(x)} .$$

If there is the coincidence point between T and F, then there exists the coincidence between T and F.

Proof. Let x be a coincidence point for T and F, i.e.

$$\max_{y \in Tx_0} F_{x_0}(y) = \max_{u \in Y} F_{x_0}(u),$$

Since the set $T(x_0)$ is a compact set and F_x is continuous, hence there exists $y_0 \in Tx_0$ such that

$$F_{x_0}(y_0) = \max_{u \in Y} F_{x_0}(u).$$

Since $(F_x)_{\alpha(x)} \neq \emptyset$ hence $\alpha(x) \leq \max_{u \in Y} F_x(u)$, thus

$$F_{x_0}(y_0) = \max_{u \in Y} F_{x_0}(u) \geq \alpha(x_0).$$

Therefore $y_0 \in (F_{x_0})_{\alpha(x_0)} = G(x_0)$, so that $y_0 \in T(x_0) \cap G(x_0)$, i.e. there exists the coincidence between T and F. The theorem follows.

Theorem 7. Let X, Y satisfy the same conditions as in theorem 6. Suppose that T: $X \longrightarrow 2^Y$ is a set-valued mapping, F: $X \longrightarrow \mathcal{F}(Y)$, is a fuzzy mapping and $\alpha(x) = \max_{u \in Y} F_x(u)$. If for each $x \in X$, $(F_x)_{\alpha(x)} \neq \phi$, we define the mapping G as follows:

$$G: X \longrightarrow 2^Y, \quad x \longrightarrow (F_x)_{\alpha(x)}.$$

Then when there is the coincidence between T and G implies there exists the coincidence point between T and F.

Proof. Suppose that there is the coincidence between T and G, hence there exists $(x_0, y_0) \in X \times Y$, such that $y_0 \in T(x_0) \cap G(x_0)$, thus $F_{x_0}(y_0) \geq \alpha(x_0) = \max_{u \in Y} F_x(u)$, i.e. $F_{x_0}(y_0) = \max_{u \in Y} F_{x_0}(u)$. Therefore

$$\max_{y \in Tx_0} F_{x_0}(y) = \max_{u \in Y} F_{x_0}(u),$$

i.e. x_0 is a coincidence point for T and F. The theorem follows.

Similarly, we can prove that the following results are true.

Theorem 8. Let X, Y satisfy the same conditions as theorem 6. Suppose that S, T are set-valued mappings from X into 2^Y, F: $X \longrightarrow \mathcal{F}(Y)$ is a fuzzy mapping satisfying $F_x = \chi_{S(x)}$. Then

(i) if there is the coincidence between S and T, i.e. there exists $(x_0, y_0) \in X \times Y$, implies there exists the coincidence point between T and F;

(ii) if T is a compact-valued mapping, i.e. for eaxch $x \in X$, T(x) is a compact subset in Y and there is a coincidence point between

T and F implies there exists the coincidence between T and S, i.e. when $D_{Coin}(x_0;\ T,\ F)=1$, then $Tx_0 \cap Sx_0 \neq \emptyset$.

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