ON THE GEOMETRICAL INTERPRETATIONS OF THE INTUITIONISTIC FUZZY LOGICAL OBJECTS. Part 2.

Krassimir T. Atanassov

Math. Research Lab. - IPACT, P.O.Box 12, Sofia-1113, BULGARIA

In [1] is given the first geometrical interpretation of the Intuitionistic Fuzzy Set (IFS-) objects (see e.g. [2]). It is transformed there for the Intuitionistic Fuzzy Logical (IFL-) objects (see e.g. [3-5]), too. In [6] is given the second geometrical interpretation of the IFS-objects, which is transformed for the IFL-objects in [7]. Here we shall describe a third geometrical interpretation of the IFL-objects.

Let a set S of propositions be fixed. Let the truth-valued function V is defined as follows. For $p \in S$:

$$V(p) = \langle \mu(p), \gamma(p) \rangle,$$

where the functions $\mu: S \to [0, 1]$ and $\gamma: S \to [0, 1]$ define the degrees of validity and of non-validity and

$$0 \le \mu(p) + \gamma(p) \le 1. \qquad (*)$$

Obviously, in the case of the ordinary fuzzy logic it is valid that:

$$V(p) = \langle \mu(p), 1 - \mu(p) \rangle.$$

If

$$\pi(x) = 1 - \mu(x) - \gamma(x) ,$$

then $\pi(x)$ is the degree of indeterminacy of the proposition p.

In the case of the ordinary fuzzy logic, $\pi(p) = 0$ for every $p \in S$.

For the newly generated function V, some of the defined in [3-5] operations and operators will be valid. Here we shall discuss only their geometrical interpretations.

Contrary to the geometrical interpretation of the ordinary and of the second type of IFL-objects, the new geometrical interpretation has the form from Fig. 1. Here the angles $\alpha(p)$ and $\beta(p)$ are equal to $\pi.\mu(p)$ and $\pi.\gamma(p)$, respectively. The condition (*) ensures the validity of:

$$\alpha(p) + \beta(p) \le \pi$$

and therefore these angles generate a triangle.

We hope that the colision of symbols "π" as the mathematical constant "pi" and IFS-function π is not dangerous.

When $\pi(p) = 0$, the IFL-object is degenerated to a fuzzy one

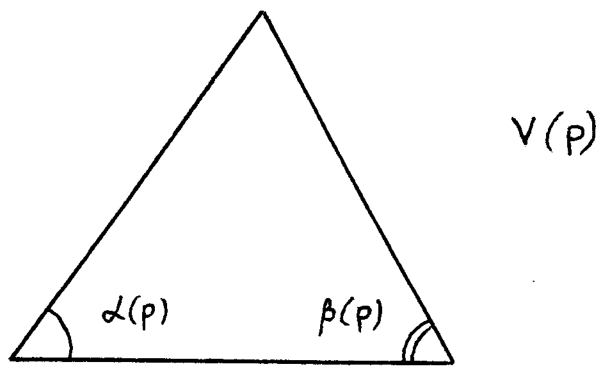

Fig. 1

and its interpretation is only two parallel lines, i.e. the triangle is degenerated.

Let the propositions p and q have the geometrical interpretations in Fig. 2. Then the propositions ¬p (a negation of p), p & q (a conjunction of p and q), p ⊻ q (a disjunction of p and q), p ⊃ q (a (max-min)-implication of p and q) have the froms from Fig. 3-6, respectively.

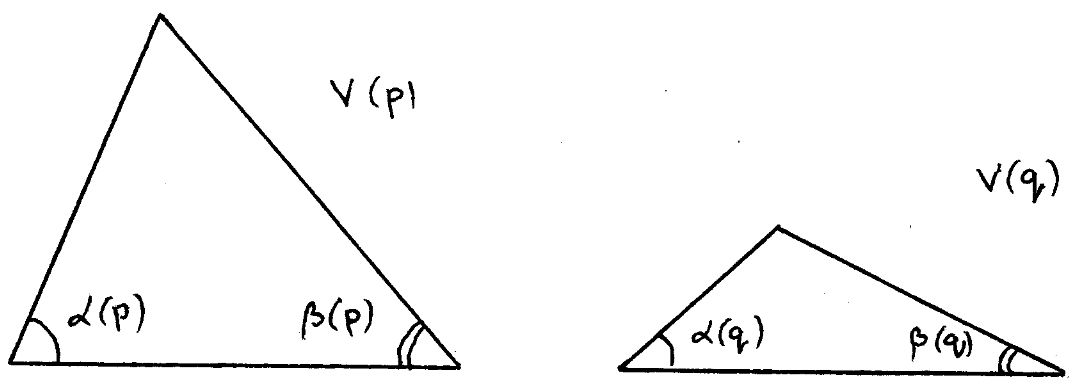

Fig. 2

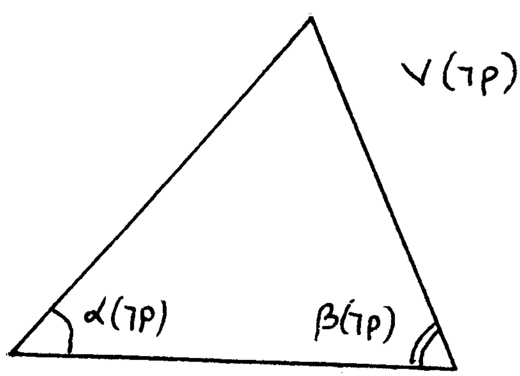

Fig. 3

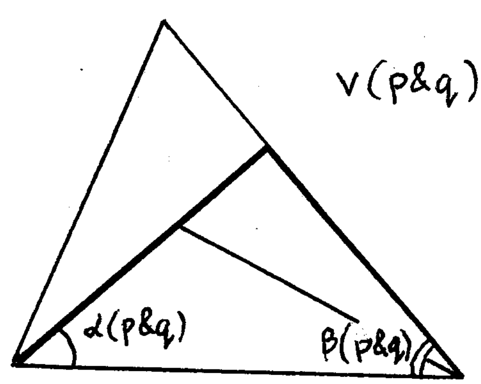

Fig. 4

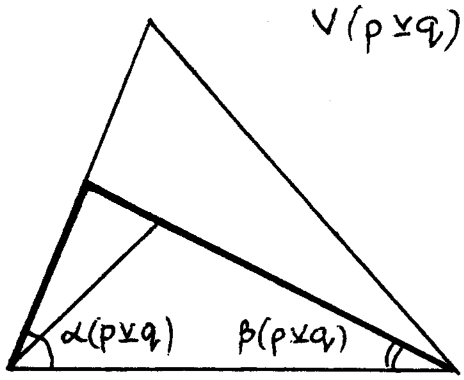

Fig. 5

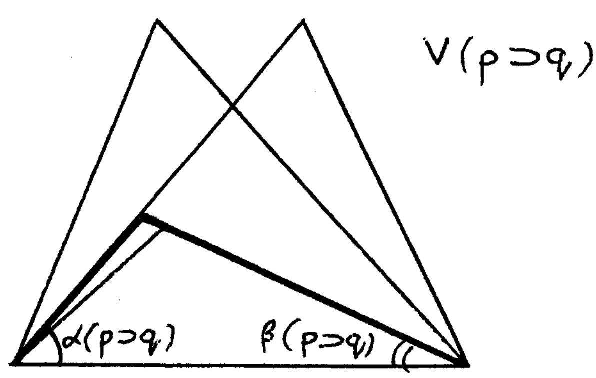

Fig. 6

We must note that these geometrical interpretations can be drawn by a ruler and compass alone, if the angles $\alpha(p)$ and $\beta(p)$ are already constructed. Unfortunatelly, the geometrical interpretations of the sg-variant of implication and more of the IFL-operations and operators are not drawn only by these two instruments. And what is more, the geometrical interpretations of operators □ and ◊ is in the form of two parallel lines. The geometrical interpretations of operators D_{α}, $F_{\alpha,\beta}$, $G_{\alpha,\beta}$, $H_{\alpha,\beta}$, $H^{*}_{\alpha,\beta}$, $J_{\alpha,\beta}$ and $J^{*}_{\alpha,\beta}$ (see [4]) are analogical to this from Fig. 1 and they can be drawn, e.g. by a protractor, after calculation of the valies of the angles.

Thus, in Fig. 7 and 8 only the geometrical interpretations of $P_{a,b}$ and $Q_{a,b}$ (see [5]) are given, where the angles A and B are: $A = a.\pi$ and $B = b.\pi$.

Finally, we shall note that this geometrical interpretation gives a possibility to illustrate the two important concepts (see [6]).

The concepts of an Intuitionistic Fuzzy Tautology (IFT) and Intuitionistic fuzzy Sure (IS) are defined through:

"p is an IFT" iff "if $V(p) = \langle a, b \rangle$, then $a \geq b$",

and

"p is an IS" iff "if $V(p) = \langle a, b \rangle$, then $a \geq 1/2$".

Let the midperpendicular of the basic side of the triangle is constructed. When the above vertex of the triangle is in the left hand of the midperpendicular, then the proposition p is an IFT, and when the left angle of the triangle is no more than $\pi/2$, the proposition is an IS one.

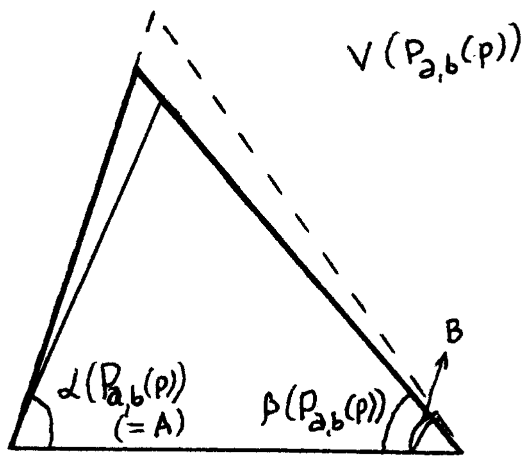

Fig. 7

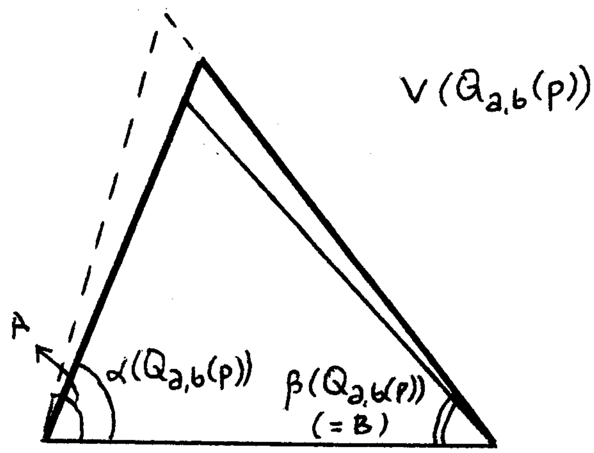

Fig. 8

REFERENCES:

[1] Atanassov K., Geometrical interpretations of the elements of the intuitionistic fuzzy objects, Preprint IM-MFAIS-1-89, Sofia, 1989.

[2] Atanassov K., A second type of intuitionistic fuzzy sets, BUSEFAL, Vol. 56 (1993) (in press).

[3] Atanassov K., Two variants of intuitionistic fuzzy modal logic. Preprint IM-MFAIS-3-89, Sofia, 1989.

[4] Atanassov K., Some modal type of operators in intuitionistic fuzzy modal logic. Part I, BUSEFAL, Vol. 58, 1994 (in press).

[5] Atanassov K., Some modal type of operators in intuitionistic fuzzy modal logic. Part II, submitted to BUSEFAL.

[6] Atanassov K., Two variants of intuitonistic fuzzy propositional calculus. Preprint IM-MFAIS-5-88, Sofia, 1988.