Fuzzy Mathematical Model of Urban Natural Hazard & Risk Assessment

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Abstract —— The paper not only analyses the random and fuzzy uncertainty in the assessment of urban natural hazard and risk, but also puts forward the ideal of assessing risk with the help of fuzzy mathematics as well as the mathematics model. The model given in this paper takes account of the comprehensive analyses on single hazard factor, hazard—effected body, urban whole system, and total hazard of factors. It is believed that the method can enhance the feasibility and reliability of risk evaluation due to the overall analyses on the relevant information in the paper.

Keywords and phrases: Uncertainty, possibility distribution, hazard factor, hazard—effected body, fuzzy approximate reasoning.

I . Introduction

Risk, as it is shown in the word, refers to the something uncertain. Urban natural disaster are extremely uncertain with respect to its time and intensity. This uncertainty, however, contains both chance and unknown which reflect the quality of random and fuzziness accordingly. For example, the occurrence of earthquake is a chance event of random[1] , while the intensity evaluation of earthquake is fuzzy[2,3,4]. In the process of risk evaluation, the random phenomena of the concerned object is due to a large amount of unknown factors existing in the object. Moreover, the degree of influence exerted by each factor is undecided[5]. The random here has something to do with people' s cognitive level. It is the existence of unknown factors that give birth to random. Similarly, fuzziness also has something to do with people' s cognition. During the process of recognizing the natural disaster, both the way of simplifying in the terms of macroscopic grad and the way of employing incomplete knowledge sample[6] will lead to the fuzziness of the object.

In the usual approach of risk evaluation[7,8], only the randomness is taken into consideration, while the method of probability statistics serves as the main technique for analyses. The advantages of the method lie in the fact that the theory used is relatively ripe and can be do easily. However, it doesn' t give any consideration to the fuzzy uncertain, therefore, there are some problems concerning its reliability as well as its feasibility. For example, the method can not do when only a few qualitative data and macroscopic description are available; Moreover, the result achieved by the classic statistic method will loss its reliability, whenever it is concerned with a few of samples.

It is known that usual statistic samples can be considered as a single point set in the theory[9] of falling shadows of random sets, and a fuzzy set can appear as a falling shadow of random sets[10,11,12]. Besides this, the theorem of great numbers of fall—shadow[13] is also available. therefore, in order to give a comprehensive evaluation or hazard and risk, the fuzzy mathematics method can be employed. This paper will focus on the method of possibility distribution to establish the new model.

Ⅱ. Definition of Natural Disaster and Risk

Let L is the discourse universe of natural disaster. In most cases, the distribution of the excess probability of L within T years is defined as disaster risk. For example, let the economic losses is the discourse universe of natural disaster, suppose:

$L=\{l_1,l_2,\cdots,l_n\}$

and the distribution of the excess probability is:

$P=\{p_1,p_2,\cdots,p_n\}$

P is called loss risk. More generally, let:

$L=\{l\}$

$P_T=\{p(l)\mid l\in L\}$

We call P_T a risk function within T years about L. $p(l)$ is called probability risk.

If we use P as the result of risk assessment and further as the basis for decision, it implicates that we have gotten a better and clearer understanding about the city′ s disaster patterns with respect to statistics. However, this is not true with all the cases, it is not easy to achieve a reliable understanding of a specific city′ s statistics pattern.

In most cases, there exists a large gap between the cognition and the reality The main reason lies in the fact that we are still unclear about the birth and pattern of many kink of natural disaster, and many predictions on the losses facing the hazard—effected bodies are unreliable too. When P is itself unreliable, it is dangerous to use P as a basis for decision—making.

P_T is unreliable, that is to say, it is possible for $p(l)$ to vary in a certain range. Because $0\leqslant p(l)\leqslant 1$, and there is fuzzy uncertainness, it is possible for $p(l)$ to vary in interval $[0,1]$. $\forall\ x\in[0,1]$, we able to use $\pi(l,x)$to denote the possibility of that $p(l)$ will be x.

In this case, there is an essential difference between the possibility distribution and the probability distribution.

Suppose $U=\{x_1,x_2,\cdots,x_n\}$ is the discourse universe. Probability $p(x_i)$ means that event x will appear in the limit value of frequency when the number of i. i. d samples trends to infinitely great. In other words, probability is a result of data analysis relating to random experiments for several times.

Take the same discourse universe U. Possibility $\pi(x_i)$ expresses the degree that event x will appear. It would not relate to any random experiments.

According to that, we suggest a more common description about risk as the following definition:

Definition Let $L=\{l\}$ is the discourse universe of hazard. $x\in[0,1]$ is a probability value. Suppose the possibility that hazard will surpass l in x is $\pi(l,x)$. If there is x_0, and $\pi(l,x_0)=1$, we call

$R_r=\{\pi(l,x)\mid l\in L,\ x\in[0,1]\}$

a hazard possibility distribution in T years. $\pi(l,x)$ is called possibility risk.

$\pi(l,x)$ can be simply named as risk as long as it doesn′ t cause confusion. In the following part, sometimes, $p(l)$ is denoted as p in short, $\pi(l,x)$ as $\pi(x)$, moreover, $\pi(x)$ can be denoted as π.

Suppose probability risk p is known, it can be turned to possibility risk $\pi(x)$. In fact,

$$\pi(x)\begin{cases}1, & x=p;\\ 0, & \text{others.}\end{cases} \qquad (1)$$

That is to say, the probability risk mentioned before is a special case of the possibility risk in this paper.

In the practice, we should analyse further about hazard factors and hazard—effected bodies before $\pi(x)$ is defined. This paper will first take into account how to synthetic the whole hazard when single factor hazards are already known.

Ⅲ. The Synthesizing Model for Risk Assessment

Assume that a city is going to face N kinds of natural disaster in T years, we say that there are N hazard factors which construct set Z as:

$Z=\{z_i \mid i=1,2,\cdots,N\}$

Suppose the discourse universe of hazard is L as before. $\forall\ z\in Z$, we call

$R_i=\{\pi_i(l,x)\mid l\in L,\ x\in[o,1]\}$

the single factor risk.

Let $\pi_i \triangleq \pi_i(x) \triangleq \pi_i(l,x)$, and f is a synthesizing approach, we call

$R=\{\pi(l,x)\mid l\in L,\ x\in[o,1]\}$

$\pi(l,x)=f(\pi_1,\pi_2),\cdots,\pi_N)(x)$

the synthesizing possibility risk of urban natural hazard.

Then, we are going to analyse how to choose f.

In order to focus on main topics, we will regard the extra hazard of z as one of disaster brought by z . That means z ,z ,…,z are independent each other.

Model I Suppose $\pi_i(i=1,2,\cdots,n)$ are defined by probability risk as Eq. (1), we can synthetic them directly. Let P_i is the probability risk of z, that is

$P_i=\{p_i(l)\mid l\in L\},\ i=1,2,\cdots,N$

We want to get:

$p(l)=f(p_1(l),p_2(l),\cdots,p_N(l))$

For writing easily, $p_i(l)$ is denoted as p_i, $i=1,2,\cdots,N$.

We will first analyse the cases when $N=2$. Thi s time, in some sense, we are dealing with an object which will be struck by a force in probability p_1 from left side, and other force in probability p_2 from right side. What we want to know is the probability $f(p_1,p_2)$ for more than once attack. Then we further assume that event A happens if the left is struck, and event B happens if the right is struck. Obviously, $P(A)=p_1$, $P(B)=p_2$, and $f(p_1,p_2)=P(A\cup B)$.

Because A and B are independent each other and would appear at the same time, according to the theorems of addition and multiplication, we obtain:

$$f(p_1,p_2)=P(A)+P(B)-P(A)P(B)=p_1+p_2-p_1p_2 \tag{2}$$

To any N, we obtain:

$$f(p_1,p_2,\cdots,p_N) =\sum_i p_i+\sum_{j>i}p_ip_j+\sum_{k>j>i}p_ip_jp_k-\sum_{l>k>j>i}p_ip_jp_kp_l+\cdots$$

That is:

$$f(p,p,\cdots,p)=1-(1-p)(1-p)\cdots(1-p)$$

Model Ⅱ Assume that π_i stands for a general possibility risk. Firstly, we analyse the situation when $N=2$. In some sense, we are similarly dealing with an object which will be struck by a force form left side in an imprecise probability which is in $[0,1]$ and may be x in possibility $\pi_1(x)$, and other force in an imprecise probability of possibility $\pi_2(x)$ form right side. What we want to know is the possibility distribution $f(\pi_1,\pi_2)(x)$ about probability x for more than once attack.

Suppose that π_1 and π_2 are fuzzy number, similarly Eq. (2), we obtain:

$f(\pi_1,\pi_2)=\pi_1+\pi_2-\pi_1\pi_2$

According to the rule of operation about fuzzy numbers,

$f(\pi_1, \pi_2)(x) = sup_{s+t-st=x} min(\pi_1(s), \pi_2(t)), x \in [0,1]$.

In the following, we are going to discuss how the risk R_i relating to π_i is achieved. The model will be called the second level of urban natural disaster. Moreover, we are going to analyse the single hazard—causing factor and hazard—effected body.

Ⅳ. Analysis of Urban Hazard Caused by Single Factor

Let z is a single natural hazard factor, and its value can be represented by y, For example, when z stands for the hazard causing factor of earthquake , y can be understand as magnitude M, or intensity I, or the acceleration a in the field. One of three can be chosen according to the accuracy requirement for analysis and the data provided. One more example, when z is used to represent the hazard—causing factor of flood, y can be water lever, or the rainfall during a period. the choice can be made according to the geographical environment of the city.

Let Y is the discourse universe of y. $\mu_z(y,x)$ represents the possibility that hazard relevant to z will surpass y with probability x in T years. We call

$$r_z = \{\mu_z(y,x) \mid y \in Y,\ x \in [0,1]\} \quad (3)$$

the risk of hazard—causing factor z.

the process of getting $\mu_z(y,x)$ is usually one of physical analysis, or one of historical data analysis. Sometimes the process involves both of them. It is a task for experts in relevant special departments to achieve the value of $\mu_z(y,x)$. This paper focuses on how to get R when $\mu_z(y,x)$ is known.

Let A_j is a hazard — effected body which may be a building or other. Suppose city C consists of m parts as:

$$C = \{A_1, A_2, \cdots, A_m\}$$

The hazard of A_j by z is called single body hazard which is denoted by d_j. The hazard of C by z is called urban hazard which is denoted by D. In fact, D is the second level risk of city C.

Ⅴ. Analysis Model for Single Hazard—Effected Body

L represents the hazard discourse universe of the single body. When an earthquake happens, L usually consists of the damage level of building. In the case of flood, L usually consists of the level of economic loss. L will be chosen by experts on the basis of analysis requirement. In most cases, experts can succeed in analyzing hazard degree for n kinds of typical hazard—effected bodies according to the value of y.

To k—th typical body, we assume that experts can give single body hazard as:

$$d_k(y) = \{POSS_k(l,y) \mid l \in L,\ y \in Y\}$$

where $POSS_k$ represents that the body will be destroyed in possibility $POSS_k$ for l, $l \in L$, when the force of hazard—causing is y.

Then, we consider a general hazard — effected body to estimate its possibility distribution. Suppose that a general body A can be classified by a fuzzy set $\mu_A(k)$ in discourse universe $K = \{1, 2, \cdots, n\}$. Let

$$s = \sum_{k=1}^{n} \mu_A(k)$$

$$\lambda(k) = \mu_A(k)/s$$

$\lambda(k)$ is called the weight assignment parameter of A. The hazard possibility distribution of A is:

$POSS_A(l,y) = s = \sum_{k=1}^{n} \mu_A(k) POSS_k(l,y)$

In the following, we are going to analyse the hazard risk of single hazard—effected body when y occurs in the way represented by Eq. (3), that is to say, we are going to get composition output $\pi_A(l,x)$ by input $\mu_z(y,x)$.

As we know, if y_0 acts on A and causes hazard l_0, the force which is over y_0 will cause the hazard that will be over l_0. Therefore, when the probability of the force over y_0 is x, the probability of the hazard over l_0 is x too. Let$\pi_A(l,x)$ is the possibility of the hazard over l in probability to A, it can be obtained by using $\mu_z(y,x)$ and $POSS_A(l,y)$ directly.

Let:

$$\begin{cases} R_z = \{r_1(x,y) \mid x \in [0,1],\ y \in Y\} \\ r_1(x,y) = \pi_z(y,x) \end{cases}$$

$$\begin{cases} R_A = \{r_2(y,l) \mid y \in Yl,\ \in L\} \\ r_2(y,l) = POSS_A(l,y) \end{cases}$$

R_z represents the fuzzy relationship between y and x, R_A for l and y. The fuzzy relationship R between l and x can be combined by using R_z and R_A.

$$R = \{r(x,l) \mid x \in [0,1],\ l \in L\} = R_Z \cdot R_A$$

$$r(x,l) = \sup_{\in Y} \{\min\{r_1(x,y), r_2(y,l)\}\}$$

Let: $\pi(l,x) = r(x,l), l \in L,\ x \in [0,1]$, then:

$$\pi_A(l,x) = \sup_{\in Y}$$

$$\{\min\{\pi_z(y,x), POSS_A(l,y)\}\}, lL,\ x \in [0,1]$$

Ⅵ. Hazard analysis of Urban System

Using the formulae in the last section, we can get the hazard possibility distribution $\pi_{Aj}(l,x)$ for any A_j. Now, let us analyse the hazard possibility distribution $\pi_C(l,x)$ of the whole urban system.

Because L many vary according to different factor or body, we had better come to loss field before calculating $\pi_C(l,x)$. This task should be performed by relevant experts according to the character of A_j. Here, we suppose L has been changed into loss field.

Let us first analyse two simple bodies. "Two simple bodies" refers to the fact their loss does not effect each other. In this paper, we always assume that in loss there is no any relationship among bodies. If there is relationship, these bodies will be analysed as a single body.

Let A and B are two simple body, their loss risk are$\pi_A(l,x)$ and $\pi_C(l,x)$. The total loss risk of them can be calculated by:

$$\pi_{A+B}(l,x) = \sup_{l_1+l_2=l} \{\min\{\pi_A(l_1,x),\ (l_2,x)\}\},\ l \in L,\ x \in [0,1]$$

Obviously, the total loss risk of city C is:

$$\pi_C(l,x) = \sup_{l_1+l_2+\cdots+l_m=l} \{\min_{Aj \in C}\{\pi_{Aj}(l,x)\}\},\ l \in L,\ x \in [0,1]$$

$\pi_C(l,x)$ is possibility assessment for hazard risk causing by single factor z. Various z can lead to different distribution. Using the synthesizing model in the 3rd section, we can obtain synthetic possibility distribution $\pi(l,x)$ for all hazard factors.

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