

## REMARK ON INTUITIONISTIC FUZZY EXPERT SYSTEMS

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The idea for unification of the concepts "fuzzy sets" and "Expert Systems" (ESs) is discussed in different papers (see, e.g., [1-13]). Here we shall make the analogical "union" between the concepts "Intuitionistic Fuzzy Sets (IFSs, see [14-16]) and Logics (IFLs, see [17-21]) and "ESs" (see, e.g. [22-24]), defining the objects "Intuitionistic Fuzzy Expert Systems" (IFESs).

A group of Bulgarian specialists, with the author's participation, had made an attempt to describe in [25-31] different types of ESs (existing in the literature or possible in principle) by the means of Generalized Nets (GNs, see [32]). There, some types of ESs are described by GNs, for which the authors do not know other publications. Here, we shall use one of the most general type of these ESs' definitions and over its basis the new type of ESs will be defined.

On the other hand, we shall use the ideas on intuitionistic fuzzy Prolog [33-35], intuitionistic fuzzy constraint logic programming [36, 37] and the first author's attempt to define the concept "IFES" [38, 39], which is not detailed enough.

The main components of a production system are: the Data Base (DB) containing the facts about the problem which are to be solved, the Knowledge Base (KB) containing the rules which are to be used in the reasoning process, and the inference engine which operates through the KB using the DB for proving or rejecting a given hypothesis. ES's facts with new components - priorities are described in [29]. Here, as in [28-31], we shall add new ESs' components - the degrees  $\mu$  and  $\gamma$  of validity and non-validity (correctness and non-correctness) of the facts. Therefore, every fact  $A$  of the DB will have the form:  $[A, p_A, \mu_A, \gamma_A]$ , where  $\mu_A, \gamma_A \in [0, 1]$  and  $\mu_A + \gamma_A \leq 1$  are the above mentioned degrees,  $p_A$  (a natural number) is a priority of  $A$  and  $A$  is a standard ES-fact.

Let the fact  $A$  with the three above components there exist in the DB. Let the new fact  $B$  with a priority  $p_B$  be generated by some way in a certain time-moment when the ES functions. If both facts are not related, then the new fact enters the DB. In the

ordinary ESs, the new fact B substitutes the old fact A, when B coincides with, or contradicts A. Now the ES will function in another way, basing on the new component. When the facts A and B coincide, their representative (in particular - A or B) stays in the DB, but with a new priority - the maximum of  $p_A$  and  $p_B$ . On the other hand, the fact with the maximum priority between  $p_A$  and  $p_B$  stays in the DB when the facts A and B are in a contradiction.

Immediately the question for the relation between  $\mu_A$ ,  $\gamma_A$  and  $p_A$  arises. These three components of the fact can be interpreted in such a way, that they to be independent. For example, when we describe different facts and their estimations generated by a group with n experts, every one of which estimates some information, the i-th of them can estimate the fact A by the values  $\mu_{A,i}$  and  $\gamma_{A,i}$  ( $1 \leq i \leq n$ ), but every fact will have its priority  $p_A$ . The last ES-parameter can be related as to the priority of the corresponding expert, as well as with other factors which are not related to the experts. In the last case, the three parameters will be independent. If every one of the experts estimates some fact, in the DB can be collected facts  $[A, \mu_{A,1}, \gamma_{A,1}, p_A]$ ,  $[A, \mu_{A,2}, \gamma_{A,2}, p_A]$ , ...,  $[A, \mu_{A,n}, \gamma_{A,n}, p_A]$ . After this, in the DB can be kept the fact A with these  $(\mu, \gamma)$ -parameters  $\mu_{i^*}$  and  $\gamma_{i^*}$ , for which  $p_{i^*} = \max_{1 \leq i \leq n} p_i$ , or these  $(\mu, \gamma)$ -parameters for which  $\mu_{i^*} \cdot p_{i^*} = \max_{1 \leq i \leq n} \mu_i \cdot p_i$  (if there are some values of i for which the maximum is obtained, then the value of i, for which  $\gamma_i$  is minimum along the other  $\gamma$ -values, is determined).

The rules of KB in the IFES have one of the following forms:

$$1. \left[ \langle M_H, N_H \rangle H :- e(B_1, B_2, \dots, B_n) \langle M_B, N_B \rangle \right],$$

where  $M_H, N_H, M_B, N_B \in [0, 1]$  and

$$\sup M_H + \sup N_H \leq 1 \text{ and } \sup M_B + \sup N_B \leq 1,$$

and  $e(B_1, B_2, \dots, B_n)$  is a logical expression for the variables

(some of which can be ES's facts)  $B_1, B_2, \dots, B_n$ . The expression

$e(B_1, B_2, \dots, B_n)$  can contain operations "&", "v", "C", "¬", qu-  
 antors "∀", "∃", standard modal ("□", "◇"), extending modal ( $D_\alpha$ ,  
 $F_{\alpha, \beta}$ ,  $H_{\alpha, \beta}$ ,  $J_{\alpha, \beta}$ ,  $H_{\alpha, \beta}^*$ ,  $J_{\alpha, \beta}^*$ ,  $X_{a, b, c, d, e, f}$ ), temporal (P, F, G,  
 H) and level ( $P_{\alpha, \beta}$ ,  $Q_{\alpha, \beta}$ ) operators. Therefore,  $e(B_1, B_2, \dots, B_n)$   
 can have very complex form (cf. [36, 37]).

The intervals have the forms

$$M_H = [\mu_i^H, \mu_s^H], N_H = [\gamma_i^H, \gamma_s^H], M_B = [\mu_i^B, \mu_s^B], N_B = [\gamma_i^B, \gamma_s^B].$$

To them it can be given the following interpretation. For each  
 assignment of each variable occurring in the rule, if  $B_i$  are all  
 true with degrees within the intervals  $[\mu_i^B, \mu_s^B]$  (for the degree  
 of validity) and  $[\gamma_i^B, \gamma_s^B]$  (for the degree of non-validity) the  
 consequent H has values  $\mu_H$  and  $\gamma_H$  within the intervals  $[\mu_i^H, \mu_s^H]$   
 and  $[\gamma_i^H, \gamma_s^H]$  respectively. Naturally, the calculated degrees  $\mu_H$   
 and  $\gamma_H$  satisfy the constraint

$$0 \leq \mu_H + \gamma_H \leq 1.$$

Let  $\mu_B$  and  $\gamma_B$  be the already calculated validity and non-vali-  
 dity degrees of the rule. The degrees' calculation of the conse-  
 quent H  $\mu_H$  and  $\gamma_H$  in terms of the interval rule is the following

$$\mu_H = \mu_i^H + \alpha_\mu \cdot (\mu_s^H - \mu_i^H),$$

$$\gamma_H = \gamma_i^H + \alpha_\gamma \cdot (\gamma_s^H - \gamma_i^H),$$

where:

$$\alpha_\mu = \begin{cases} \frac{\mu_B - \mu_i^B}{\mu_s^B - \mu_i^B}, & \text{if } \mu_s^B > \mu_i^B \\ 1/2, & \text{otherwise} \end{cases}$$

$$\alpha_\gamma = \begin{cases} \frac{\gamma_B - \gamma_i^B}{\gamma_s^B - \gamma_i^B}, & \text{if } \gamma_s^B < \gamma_i^B \\ 1/2, & \text{otherwise} \end{cases}$$

$$2. [\langle M_H, N_H \rangle H :- e(B_1, B_2, \dots, B_n) \langle \mu_B, \gamma_B \rangle],$$

where  $M_H, N_H, \mu_B$  and  $\gamma_B$  are as above. The interpretation is as above, too, except that the validity and non-validity degrees of  $B_1, B_2, \dots, B_n$  must be greater than or equal to  $\mu_B$  and less than or equal to  $\gamma_B$  respectively. The calculation is as follows:

$$\alpha_\mu = \begin{cases} \frac{\mu_B - b_\mu}{1 - b_\mu}, & \text{if } b_\mu < 1 \\ 1/2 & , \text{ otherwise} \end{cases}$$

$$\alpha_\gamma = \begin{cases} \frac{\gamma_B - b_\gamma}{b_\gamma}, & \text{if } b_\gamma > 0 \\ 1/2 & , \text{ otherwise} \end{cases}$$

where degrees of  $e(B_1, B_2, \dots, B_n)$  are  $b_\mu$  and  $b_\gamma$ .

$\mu_H$  and  $\gamma_H$  are calculated in the same way (cf. [35]).

The next two cases are modifications of the first ones:

$$3. [Y_{\alpha, \beta, \dots}; H :- e(B_1, B_2, \dots, B_n) \langle M_B, N_B \rangle],$$

$$4. [Y_{\alpha, \beta, \dots}; H :- e(B_1, B_2, \dots, B_n) \langle \mu_B, \gamma_B \rangle],$$

where all components without the last ones in both types of rules are equal and the last components are as the corresponding ones above.

The sense of the first components in both types of rules (the next components are as the corresponding above) is, that  $Y$  is the identifier of an operator, i.e.  $Y \in \{\square, \diamond, D, F_{\alpha, \beta}, \dots\}$  and  $\alpha, \beta, \dots$  are its necessary components (0, 1, 2 or 6 in number, to a relation with the identifier).

For example, the calculation of the degrees of the clause head is based on operator  $F_{\alpha, \beta}$ , for  $0 \leq \alpha + \beta \leq 1$  and is as follows:

$$\langle \mu_H, \gamma_H \rangle = F_{\alpha, \beta}(\mu_B, \gamma_B) = \langle \mu_B + \alpha \cdot \pi_B, \gamma_B + \beta \cdot \pi_B \rangle,$$

where  $\pi_B = (1 - \mu_B - \gamma_B)$  can be interpreted as a certainty factor.

The process of calculation of the truth-values (degrees of validity and non-validity) of the expression  $e(B_1, B_2, \dots, B_n)$  is made as it is described in [17-21].

Therefore, we define an ES which can describe more detailed processes than the classical ESs.

Moreover, this ES can contain the other elements, which are described in [30, 31], as metafacts and apparatus for the changing of the rules.

On the other hand, the new type of ESs can be simplified with the omission of the priority-component of the facts. In this case, this component can be interpreted by both truth-valued components  $\mu$  and  $\gamma$ . But, as we have shown above, there are situations for which the three components have themselves independent interpretations.

We must note, that the functioning and the results of the work of the above defined IFESs can be described by a GN, but this will be an object of different our research. The functioning and the results of the work of the first type of IFESs [38] is described by a GN (see [39]). Now, the form of the GN will be more complex, but it will contain a GN-interpretation of the shown above new ES-components (i.e.  $\mu$  and  $\gamma$ ).

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