INTUITIONISTIC FUZZY SETS AND EXPERT ESTIMATIONS. II

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Using all notations from [1], we shall discuss some ways for changing of experts' estimations of given events (or objects; for brevity, below we shall use only the word "event"), when the experts are made inadmissible mistakes in themselve Intuitionistic Fuzzy Estimations (IFEs).

Let us call the expert E_i ($i \in I$; I is an index set, related to the experts) unconscientious one, if among his estimations $\{<\mu_{i,j}, \gamma_{i,j}>/j \in J_i\}$, where $J = \bigcup_{i \in I} J_i$ is an index set, related to the events estimated by the experts indexed by the elements of the set I, there are such ones for which $\mu_{i,j} \le 1$, $\gamma_{i,j} \le 1$, but $\mu_{i,j} + \gamma_{i,j} > 1$.

In this case, different ways for a re-estimation of his uncorrect estinations can be introduced which an aim to transform these estimations in an IF-form.

It is obvious, that if the estimations must have fuzzy-form, then for the estimation of an expert who assert that a given event is possible 101%, we can think that he jokes and we can assume his estimation as 1. Unfortunately, in the case of the IF-estination, the procedure of the re-estimation is very complex. Below we shall show some different ways for such re-estimation of the estimations of the unconscientious experts.

Let about the estimations of the expert E_i $\{<\mu_{i,j}, \gamma_{i,j}>/j \in J_i\}$ the is a subset $\{<\mu_{i,k}, \gamma_{i,k}>/k \in K_i\}$, where $K_i \subset J_i$ for which $\mu_{i,k} \le 1$, $\gamma_{i,k} \le 1$, but $\mu_{i,k} + \gamma_{i,k} > 1$. Let $\bar{K}_i = J_i - K_i$.

<u>Way 1</u> (trivial): We decrease both IF-degrees of the estimation, removing the degree of undeterminacy, as follows:

$$\bar{\mu}_{i,k} = \mu_{i,k}/(\mu_{i,k} + \gamma_{i,k})$$

$$\bar{\gamma}_{i,k} = \gamma_{i,k}/(\mu_{i,k} + \gamma_{i,k})$$

In this case, the new values do not give a possibility for a hesitation, which can have the expert.

Way 2: We substitute μ- and γ- values by:

$$\bar{\mu}_{i,k} = \mu_{i,k} - \min(\mu_{i,k}, \gamma_{i,k})/2$$

$$\bar{\gamma}_{i,k} = \gamma_{i,k} - \min(\mu_{i,k}, \gamma_{i,k})/2$$

Obviously,

$$\bar{\mu}_{i,k} + \bar{\gamma}_{i,k} = \mu_{i,k} - \min(\mu_{i,k}, \gamma_{i,k})/2 + \gamma_{i,k} - \min(\mu_{i,k}, \gamma_{i,k})/2$$

$$= \mu_{i,k} + \gamma_{i,k} - \min(\mu_{i,k}, \gamma_{i,k})/2 = \max(\mu_{i,k}, \gamma_{i,k})/2 \leq 1.$$

These two ways admit only the values of the expert estimation for the j-th event. More complex are the following ways which admit the values of the other expert estimations, too.

Way 3: When $card(K_i)$ is large enough number, at least greater than card(K), we determine the number

$$p_i = (\sum_{k \in \bar{K}_i} \bar{\pi}_k)/card(\bar{K}_i),$$

which corresponts to the moddle degree of indeterminacy of the correct expert's estimations (in the IF-sense) of the events. Then we can re-estimate his non-correct estimations, as follows:

$$\bar{\mu}_{i,k} = (1 - p_i).\mu_{i,k}/(\mu_{i,k} + \gamma_{i,k})$$

$$\bar{\gamma}_{i,k} = (1 - p_i).\gamma_{i,k}/(\mu_{i,k} + \gamma_{i,k})$$

Obviously,

$$\bar{\mu}_{i,k} + \bar{\gamma}_{i,k} = 1 - p_i,$$

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i.e. $\bar{\pi}_{i,k} = p_i$.

In this case, the "hesitation" of the expert is simulated, but the hesitations are very homogeneous for all elements of the set K_i.

Thus, we can change the individual estimations of the experts, but we do not influence of the oppinions of the other experts about the concrete event. When card($\bar{K}_i$) is not enough large num-number, i.e., when the majority of the experts are unconscientio-us (at least for the concrete event), we must use some of the first two ways (or other similar to them).

Let $L \subset I$ and let for $l \in L$ the l-th expert gives the following estimations $\{<\mu_{l,k}, \gamma_{l,k}>/k \in J_l\}$, where $\mu_{l,k} \leq 1$, $\gamma_{l,k} \leq 1$ and $\mu_{k,l} + \gamma_{k,l} > 1$. Let $\bar{L} = I - L$.

If the majority of the experts are unconscientious for the estimation of k-th event ($k \in K_l$), we must again use some of first two ways. If the number of the unconscientious experts in not large, we can use one of the following ways.

<u>Way 4</u>: By analogy with Way 3 we determine the numbers p_i for all $i \in I - L$ and the number $\bar{p}_l = (\sum_{i \in I-L} p_i)/\text{card}(I - L)$. After this, we use the formulas

$$\bar{\mu}_{l,k} = (1 - \bar{p}_l).\mu_{l,k}/(\mu_{l,k} + \gamma_{l,k})$$

$$\bar{\gamma}_{l,k} = (1 - \bar{p}_l).\gamma_{l,k}/(\mu_{l,k} + \gamma_{l,k})$$

By this way, we intuitionistically fuzzy the estimation of the unconscientious expert "l" in relation of the estimations of the conscientious his colleagues. In this case, the ratings of the different experts are not used. This can be made by the following

<u>Way 5</u>: the numbers p_l are calculated as in Way 4, after which the number

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$$p^*_l = \frac{\sum_{i \in I-L} \partial_i . p_i}{card(I - L).\partial_l}$$

is calculated. Now we can use the formulas from Way 4.

Obviously, the introduced ways are only a part of all the possible ways for correcting of expert estimations.

REFERENCE:

[1] Atanassov K., Intuitionistic fuzzy sets and expert estimations, BUSEFAL, Vol. 55, 1993, 67-71.

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