FUZZY S_2-PRE-SEMICONTINUITY

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ABSTRACT

In this paper, we introduce and study the fuzzy S_2-pre-semicontinuous mapping in fuzzy topological spaces.

Key words: Fuzzy pre-semiopen sets; fuzzy semiopen sets; fuzzy S_2-pre-semicontinuous mappings.

1. PRELIMINARIES

In this paper, A°, A^-, $A_\circ$, A_- and A' will denote respectively the interior, closure, semi-interior, semi-closure and complement of the fuzzy set A. Let A be a fuzzy set of a fuzzy topological space (X,δ). Then A is called (1) a fuzzy pre-semiopen set of X iff $A\leqslant(A^-)_\circ$[3]; (2) a fuzzy pre-semiclosed set of X iff $A\geqslant(A^\circ)_-$[3]; (3) a fuzzy strongly semiopen set of X iff there is a $B\in\delta$ such that $B\leqslant A\leqslant B^{-\circ}$[2]; (4) a fuzzy strongly semiclosed set of X iff there is a fuzzy closed B such that $B^{\circ-}\leqslant A\leqslant B$[2]. Let $f: X\rightarrow Y$ be a mapping from a fuzzy space X to another fuzzy space Y, f is called (1) a fuzzy S_1-pre-semicontinuous mapping if $f^{-1}(B)$ is a fuzzy pre-semiopen set of X for each fuzzy strongly semiopen set B of Y[4]. (2) a fuzzy irresolute mapping if $f^{-1}(B)$ is a fuzzy semiopen set of X for each fuzzy semiopen set B of Y[1].

2. FUZZY S_2-PRE-SEMICONTINUOUS MAPPINGS

Definition 2.1. Let $f:(X,\delta)\rightarrow(Y,\tau)$ be a mapping from a fuzzy space X to another fuzzy space Y, f is called a fuzzy S_2-pre-semicontinuous mapping if $f^{-1}(B)$ is a fuzzy pre-semiopen set of X for each fuzzy semiopen set B of Y.

Definition 2.2. Let $f:X\to Y$ be a mapping from a fuzzy space X to another fuzzy space Y , f is said to be fuzzy S_2-pre-semicontinuous at a fuzzy point p in X, if fuzzy semiopen set B of Y and $f(p)\leqslant B$, there exists a fuzzy pre-semiopen set A of X such that $p\leqslant A$ and $f(A)\leqslant B$.

Theorem 2.3. Let $f:X\to Y$ be a mapping from a fuzzy space X to another fuzzy space Y. Then the following are equivalent:

(1) f is fuzzy S_2-pre-semicontinuous.

(2) $f^{-1}(B)$ is a fuzzy pre-semiclosed set of X for each fuzzy semiclosed set B of Y.

(3) $f((A^{\circ})_{-})\leqslant(f(A))_{-}$ for each fuzzy set A of X.

(4) $((f^{-1}(B))^{\circ})_{-}\leqslant f^{-1}(B_{-})$ for each fuzzy set B of Y.

(5) $f^{-1}(B_{\circ})\leqslant((f^{-1}(B))^{-})_{\circ}$ for each fuzzy set B of Y.

Theorem 2.4. A mapping $f:X\to Y$ is fuzzy S_2-pre-semicontinuous iff f is fuzzy S_2-pre-semicontinuous for each fuzzy point p in X.

Proof. We prove only sufficiency.

Let B be a fuzzy semiopen set of Y and p be a fuzzy point in $f^{-1}(B)$. Then $p\leqslant f^{-1}(B)$, i.e., $f(p)\leqslant B$. From hypothesis there is a fuzzy pre-semiopen set A of X such that $p\leqslant A$ and $f(A)\leqslant B$, hence

$$p\leqslant A\leqslant f^{-1}f(A)\leqslant f^{-1}(B)$$

and

$$p\leqslant A\leqslant(A^{-})_{\circ}\leqslant((f^{-1}(B))^{-})_{\circ}.$$

Since p is arbitrary and $f^{-1}(B)$ is the union of all fuzzy points in $f^{-1}(B)$, $f^{-1}(B)\leqslant((f^{-1}(B))^{-})_{\circ}$. i.e., $f^{-1}(B)$ is a fuzzy pre-semiopen set of X. Thus f is fuzzy S_2-pre-semicontinuous.

Theorem 2.5. Let $f:X\to Y$ be one-to-one and onto, where X and Y are fuzzy spaces. f is a fuzzy S_2-pre-semicontinuous mapping iff $(f(A))_{\circ}\leqslant f((A^{-})_{\circ})$ for each fuzzy set A of X.

Theorem 2.6. Let X_1, X_2, Y_1 and Y_2 be fuzzy spaces such that X_1 is product related to X_2. Then the product $f_1\times f_2:X_1\times X_2\to Y_1\times Y_2$ of fuzzy S_2-pre-semicontinuous mappings $f_1:X_1\to Y_1$ and $f_2:X_2\to Y_2$ is fuzzy S_2-pre-semicontinuous.

3. MUTUAL RELATIONSHIPS

Remark 3.1. For the mapping $f:X\to Y$ the following statements are valid:

f is fuzzy irresolute => f is fuzzy S_2-pre-semicontinuous

=>f is fuzzy S_1-pre-semicontinuous.

None is reversible.

Example 3.2. Let $X=\{a, b, c\}$, and A, B, C be fuzzy sets of X defined as follows:

$A(a)=0.4$, $A(b)=0.5$, $A(c)=0.3$;

$B(a)=0.2$, $B(b)=0.3$, $B(c)=0$;

$C(a)=0.5$, $C(b)=0.5$, $C(c)=0.5$.

(1) Let $\delta=\{0, A, 1\}$, and $\tau=\{0, B, A, 1\}$. Consider the identity mapping $f:(X,\delta)\to(X,\tau)$. Then f is fuzzy S_2-pre-semicontinuous. Clearly f is not fuzzy irresolute mapping.

(2) Let $\delta=\{0, B, C, 1\}$ and $\tau=\{0, A, 1\}$. Consider the identity mapping $f:(X,\delta)\to(X,\tau)$. Then f is fuzzy S_1-pre-semicontinuous. But f is not fuzzy S_2-pre-semicontinuous.

Proposition 3.3. Let $f:X\to Y$ and $g:Y\to Z$ be mappings. Then the following statements are valid:

(1) If f is fuzzy S_2-pre-semicontinuous and g is fuzzy irresolute, then $g\circ f$ is fuzzy S_2-pre-semicontinuous.

(2) If f is fuzzy S_2-pre-semicontinuous and g is fuzzy weakly irresolute, then $g\circ f$ is fuzzy S_1-pre-semicontinuous.

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