FUZZY S_1-PRE-SEMICONTINUITY

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ABSTRACT

Fuzzy continuous and its weaker forms constitute an important area in the field of fuzzy topology(cf.[1-7]). In this paper, we introduce and study the fuzzy S_1-pre-semicontinuous mapping in fuzzy topological spaces.

Key words: Fuzzy pre-semiopen sets; fuzzy strongly semiopen sets; fuzzy S_1-pre-semicontinuous mappings.

1. PRELIMINARIES

In this paper, A°, A^{-}, $A_{\circ}$, A_{-} and A' will denote respectively the interior, closure, semi-interior, semi-closure and complement of the fuzzy set A.

Definition 1.1[2,3]. Let A be a fuzzy set of a fuzzy topological space (X,δ). Then A is called

(1) a fuzzy pre-semiopen set of X iff $A\leqslant(A^{-})_{\circ}$;

(2) a fuzzy pre-semiclosed set of X iff $A\geqslant(A^{\circ})_{-}$;

(3) a fuzzy strongly semiopen set of X iff there is a $B\in\delta$ such that $B\leqslant A\leqslant B^{-\circ}$.

(4) a fuzzy strongly semiclosed set of X iff there is a fuzzy closed set B in X such that $B^{\circ -}\leqslant A\leqslant B$.

Definition 1.2[2]. Let A be a fuzzy set of a fuzzy space (X,δ). Then

$$A^{\Delta}=\bigcup\{B:\ B\leqslant A,\ B \text{ fuzzy strongly semiopen}\}$$

is called the fuzzy strong semi-interior of A and

$A^{\sim}=\bigcap\{B: A\leqslant B,$ B fuzzy strongly semiclosed$\}$

is called the fuzzy strong semi-closure of A.

Definition 1.3[2,3,4]. Let $f:(X,\delta)\rightarrow(Y,\tau)$ be a mapping from a fuzzy space X to another fuzzy space Y, f is called

(1) a fuzzy strongly semicontinuous mapping if $f^{-1}(B)$ is a fuzzy strongly semiopen set of X for each $B\in\tau$.

(2) a fuzzy pre-semicontinuous mapping if $f^{-1}(B)$ is a fuzzy pre-semiopen set of X for each $B\in\tau$.

(3) a fuzzy S-irresolute mapping if $f^{-1}(B)$ is a fuzzy strongly semiopen set of X for each fuzzy strongly semiopen set B of Y.

2. FUZZY S_1-PRE-SEMICONTINUOUS MAPPINGS

Definition 2.1. Let $f:(X,\delta)\rightarrow(Y,\tau)$ be a mapping from a fuzzy space X to another fuzzy space Y, f is called a fuzzy S_1-pre-semicontinuous mapping if $f^{-1}(B)$ is a fuzzy pre-semiopen set of X for each fuzzy strongly semiopen set B of Y.

Definition 2.2. Let $f:(X,\delta)\rightarrow(Y,\tau)$ be a mapping from a fuzzy space X to another fuzzy space Y , f is said to be fuzzy S_1-pre-semicontinuous at a fuzzy point p in X, if fuzzy strongly semiopen set B of Y and $f(p)\leqslant B$, there exists a fuzzy pre-semiopen set A of X such that $p\leqslant A$ and $f(A)\leqslant B$.

Theorem 2.3. Let $f: (X,\delta)\rightarrow(Y,\tau)$ be a mapping from a fuzzy space X to another fuzzy space Y. Then the following are equivalent:

(1) f is fuzzy S_1-pre-semicontinuous.

(2) $f^{-1}(B)$ is a fuzzy pre-semiclosed set of X for each fuzzy strongly semiclosed set B of Y.

(3) $f((A^{\circ})_{-})\leqslant(f(A))^{\sim}$ for each fuzzy set A of X.

(4) $((f^{-1}(B))^{\circ})_{-}\leqslant f^{-1}(B^{\sim})$ for each fuzzy set B of Y.

(5) $f^{-1}(B^{\wedge})\leqslant((f^{-1}(B))^{-})_{\circ}$ for each fuzzy set B of Y.

(6) f is fuzzy S_1-pre-semicontinuous for each fuzzy point p in X.

Proof. We prove only (1)<=>(6).

(1)=>(6): Let f be fuzzy S_1-pre-semicontinuous, p be a fuzzy point in X and B be a fuzzy strongly semiopen set of Y such that $f(P)\leqslant B$. Then $p\leqslant f^{-1}(B)$. Let $A=f^{-1}(B)$, then A is fuzzy pre-semiopen set of X, and so $f(A)=ff^{-1}(B)\leqslant B$. Thus f is fuzzy S_1-pre-semicontinuous for each fuzzy point p in X.

(6)=>(1): Let B be a fuzzy strongly semiopen set of Y and p be a fuzzy point in $f^{-1}(B)$. Then $p\leqslant f^{-1}(B)$, i.e., $f(p)\leqslant B$. From hypothesis there is a fuzzy pre-semiopen set A of X such that $p\leqslant A$ and $f(A)\leqslant B$, hence

$$p\leqslant A\leqslant f^{-1}f(A)\leqslant f^{-1}(B)$$

and

$$p\leqslant A\leqslant (A^{-})_{\circ}\leqslant ((f^{-1}(B))^{-})_{\circ}.$$

Since p is arbitrary and $f^{-1}(B)$ is the union of all fuzzy points in $f^{-1}(B)$,

$$f^{-1}(B)\leqslant ((f^{-1}(B))^{-})_{\circ}.$$

i.e., $f^{-1}(B)$ is a fuzzy pre-semiopen set of X. Thus f is fuzzy S_1-pre-semicontinuous.

Theorem 2.4. Let $f:(X,\delta)\rightarrow(Y,\tau)$ be one-to-one and onto, where X and Y are fuzzy spaces. f is a fuzzy S_1-pre-semicontinuous mapping iff $(f(A))^{\Delta}\leqslant f((A^{-})_{\circ})$ for each fuzzy set A of X.

Theorem 2.5. Let X_1, X_2, Y_1 and Y_2 be fuzzy spaces such that X_1 is product related to X_2. Then the product $f_1\times f_2:X_1\times X_2\rightarrow Y_1\times Y_2$ of fuzzy S_1-pre-semicontinuous mappings $f_1:X_1\rightarrow Y_1$ and $f_2:X_2\rightarrow Y_2$ is fuzzy S_1-pre-semicontinuous.

3. MUTUAL RELATIONSHIPS

Remark 3.1. For the mapping $f:X\rightarrow Y$ the following statements are valid:

f is fuzzy S-irresolute

=> f is fuzzy S_1-pre-semicontinuous

=>f is fuzzy pre-semicontinuous.

None is reversible.

Example 3.2. Let X={a, b, c}, and A, B, C be fuzzy sets of X defined as follows:

$$A(a)=0.4,\quad A(b)=0.5,\quad A(c)=0.3;$$
$$B(a)=0.2,\quad B(b)=0.3,\quad B(c)=0;$$
$$C(a)=0.3,\quad C(b)=0.4,\quad C(c)=0.2.$$

Let $\delta=\{0, A, 1\}$, and $\tau=\{0, B, A, 1\}$. Consider the identity mapping $f:(X,\delta)\to(X,\tau)$. Then f is fuzzy S_1-pre-semicontinuous. Clearly C is fuzzy strongly semiopen set of (X,τ). But $f^{-1}(C)$ is not fuzzy strongly semiopen set of (X,δ). Thus f is not fuzzy S-irresolute mapping.

Example 3.3. Let X={a, b, c}, and A, B, C be fuzzy sets of X defined as follows:

$$A(a)=0.3,\quad A(b)=0.2,\quad A(c)=0.4;$$
$$B(a)=0.4,\quad B(b)=0.2,\quad B(c)=0.5;$$
$$C(a)=0.2,\quad C(b)=0.6,\quad C(c)=0.4.$$

Let $\delta=\{0, A, B, 1\}$ and $\tau=\{0, C, 1\}$. Consider the identity mapping $f:(X,\delta)\to(X,\tau)$. Then f is fuzzy pre-semicontinuous.Clearly A' is fuzzy strongly semiopen set of (X,τ). But

$$A'=f^{-1}(A')\not\leq((f^{-1}(A'))^{-})_{\circ}=(A')^{-})_{\circ}=(A')_{\circ}=B'$$

i.e., A' is not fuzzy pre-semiopen set of (X,δ). Thus f is not fuzzy S_1-pre-semicontinuous.

Proposition 3.4. Let $f:X\to Y$ and $g:Y\to Z$ be mappings. Then the following statements are valid:

(1)If f is fuzzy S_1-pre-semicontinuous and g is fuzzy S-irresolute, then $g\circ f$ is fuzzy S_1-pre-semicontinuous.

(2) If f is fuzzy S_1-pre-semicontinuous and g is fuzzy strongly semicontinuous, then $g\circ f$ is fuzzy pre-semicontinuous.

REFERENCES

[1] K.K.Azad, J. Math. Anal. Appl. 82(1981) 14-32.

[2] Bai Shi-Zhong, Fuzzy Sets and Systems, 52(1992) 345-351.

[3] Bai Shi-Zhong, ICIS'92, 918-920.

[4] Bai Shi-Zhong, Fuzzy Sets and Systems, in press.

[5] C.L.Chang, J. Math. Anal. Appl. 24(1968) 182-190.

[6] Pu Pao-Ming and Liu Ying-Ming, J.Math.Anal.Appl.76(1980)571-599.

[7] T.H.Yalvac, J. Math. Anal. Appl. 132(1988) 356-364.