INTUITIONISTIC FUZZY SYSTEMS

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Using and keeping all notations from [1], we shall define the object Intuitionistic Fuzzy System (IFSy). All notations related to Intuitionistic Fuzzy Sets (IFSs) and Logics (IFLs) are, e.g., from [2-8].

The IFSy $S \subset \prod_{i \in I} (V_i \times [0, 1]^2) \times [0, 1]^2$ is defined as a proper relation on the sets $\{V_i : i \in I\}$, where I is an index set, $\{V_i : i \in I\}$ represent the objects which are the constituent parts of the system and every one of the sets V_i represents a collection of alternative ways in which the corresponding object appears in the relation which defined the system (as in the case of the ordinary systems). As a difference with the standard definition, here are couples of real numbers in [0, 1] for which it is valid, that numbers μ_i and γ_i are associated to V_i $(i \in I)$, $\langle \mu_i, \gamma_i \rangle \in [0, 1]^2$ and $\mu_i + \gamma_i \leq 1$. By analogy with the IFS's and IFL's theory, the numbers μ_i and γ_i can be interpreted as degrees nof validity and non-validity (correctness and on-correctness, etc.). Moreover, two other numbers μ_S and γ_S are associated to S. They also must be interpreted as the above ones, but they are related to the system S in general. Therefore, the IFSy S can be described in the form

$$S \subset \Pi \{\langle V_i, \mu_i, \gamma_i \rangle / i \in I\} \times \langle \mu_S, \gamma_S \rangle.$$

When $\mu_i = 1$, $\gamma_i = 0$ for every $i \in I$ and $\mu_S = 1$, $\gamma_S = 0$, we obtain the ordinary system. Obviously, the new type of systems is an extension of the classical one. The sense of this extension is, that it gives a possibility to estimate the system function results (its degrees of validity and non-validity, correctness and non-correctness, etc.) in a relation to the different ways of an interpretation of the different system's μ- and γ-parameters. For example, we can define the global (final) system's validity (correctness) degrees as:

$$\langle \mu_S, \gamma_S \rangle = \langle \max_{i \in I} \mu_i, \min_{i \in I} \gamma_i \rangle,$$

or

$$\langle \mu_S, \gamma_S \rangle = \langle \min_{i \in I} \mu_i, \max_{i \in I} \gamma_i \rangle,$$

or

$$\langle \mu_S, \gamma_S \rangle = \langle \sum_{i \in I} \mu_i, \prod_{i \in I} \gamma_i \rangle,$$

etc, where for the natural number $n \geq 1$ and for real numbers a_1, $a_2, \ldots \in [0, 1]$:

$$\sum_{i=1}^{2} a_i = a_1 + a_2 - a_1 . a_2$$

and

$$\sum_{i=1}^{n+1} a_i = \sum_{i=1}^{n} a_i + a_{n+1} - a_{n+1} . \sum_{i=1}^{n} a_i,$$

and $\prod_{i=1}^{n} a_i = a_1 . a_2 . \ldots . a_n$ (standard operation "production").

Moreower, we can change the values of the global S-parameters μ_S and γ_S and the local S-parameters μ_i and γ_i for $i \in I$ by the the IF-operators $F_{\alpha, \beta}$, $G_{\alpha, \beta}$, etc. The most interesting case is obtained, when these operators (in general case - the operator $X_{a,b,c,d,e,f}$ and the operators $P_{\alpha, \beta}$ and $Q_{\alpha, \beta}$) are time-functions. In this case, we can describe the results of changing the S-parameters values at the time.

Another description of S is the following:

$$S \subset \{\langle \Pi \{\langle V_i, \mu_i, \gamma_i \rangle / i \in I\}, \mu_S, \gamma_S \rangle\}.$$

This S-form gives the possibility for a common representation of a set of systems S_j ($j \in J$; J is an index set) in the form:

$$\{\langle \Pi \{\langle V_i^j, \mu_i^j, \gamma_i^j \rangle / i \in I^j\}, \mu_S^j, \gamma_S^j \rangle / j \in J\},$$

i.e., in an IFS-form.

Here we shall illustrate the above definitions, describing the IF-representations of such named cybernetic type of systems.

Let

$$X = \Pi\{\langle V_i, \mu_i, \gamma_i \rangle / i \in I_X\}$$

and

$$Y = \Pi\{\langle V_i, \mu_i, \gamma_i \rangle / i \in I_Y\},$$

where $I_X \cap I_Y = \emptyset$ and $I_X \cup I_Y = I$. The sets X and Y are called IF-input and IF-output objects, respectively. The system

$S \subset X \times Y \times [0, 1]^2$ or $S \subset X \times Y \times \langle \mu_S, \gamma_S \rangle$ where μ_S and γ_S are the global S-parameters, as above, is called an IF-input/output or an IF-terminal system.

Following [1], we shall define the concept IF-goal-seeking system. For this aim, we shall define the functions and P.

F represents the goal-seeking component with the object M as the outcome of the goal-seeking activity, where

$$M = \{\langle m_k, \mu_k, \gamma_k \rangle / k \in K\}$$

is an "internal input" in distinction to X and Y which are IF-true (i.e., with estimations which are based on their μ_i and γ_i ($i \in I$) values) input/output objects of S and K is an index set. In the particular case, the sets I and K can coincide. M represents the domain of choices which F has. μ_k and γ_k are the degrees of validity and non-validity (correctness and non-correctness, etc.) of the k-th choice. Therefore $F \subset X \times Y \times M$. On the other hand, let $P \subset M \times X \times Y$. The relations P and F must be consistent with the system S, i.e. they must satisfy the condition

$\langle x, y, \mu_S, \gamma_S \rangle \in S$ iff $(\exists m \in M)(\langle m, x, y\rangle \in P \ \& \ \langle x, y, m\rangle \in F)$.

Let $G \subset M \times X \times Y \times [0, 1]^2$, i.e., $G \subset M \times X \times Y \times \langle \mu_G, \gamma_G \rangle$, where μ_G and γ_G determine the degree of validity and non-validity (correctness and non-correctness, etc.) of the evaluation of G. In [1] there is only the value of μ_G. For example, for given m, x, y, i.e. for given μ_k, γ_k (for m_k, $k \in K$), $\mu_{i'}$, $\gamma_{i'}$ (for $x_{i'}$, $i' \in I_X$), $\mu_{i''}$, $\gamma_{i''}$ (for $x_{i''}$, $i'' \in I_Y$), G can be defined as:

$$\langle \mu_G, \gamma_G \rangle = G(\mu_k, \gamma_k, \mu_{i'}, \gamma_{i'}, \mu_{i''}, \gamma_{i''})$$
$$= \langle \min(\mu_k, \max(\mu_{i'}, \mu_{i''})), \max(\gamma_k, \min(\gamma_{i'}, \gamma_{i''}))\rangle.$$

More complete form of , when G determines a relation among sets $\{m_k / k \in K'\}$, $\{x_{i'} / i' \in I'_X\}$ and $\{y_{i''} / i'' \in I''_Y\}$ for $K' \subset K$, $I'_X \subset I_X$ and $I''_Y \subset I_Y$, are, e.g., the following two:

$$\langle \mu_G, \gamma_G \rangle = G(\{\langle \mu_k, \gamma_k \rangle / k \in K'\}, \{\langle \mu_{i'}, \gamma_{i'} \rangle / i' \in I'_X\},$$
$$\{\langle \mu_{i''}, \gamma_{i''} \rangle / i'' \in I''_Y\})$$

$$= \langle \min_{k \in K}(\mu_k, \max_{i' \in I'_X,\ i'' \in I''_Y}(\mu_{i'}, \mu_{i''}) - \mu_k . \max_{i' \in I',\ i'' \in I''}(\mu_{i'}, \mu_{i''}),$$

$$\max_{k \in K}(\gamma_k . \min_{i' \in I'_X,\ i'' \in I''_Y}(\gamma_{i'}, \gamma_{i''})\rangle$$

and

$$\langle \mu_G, \gamma_G \rangle = G(\{\langle \mu_k, \gamma_k \rangle / k \in K'\}, \{\langle \mu_{i'}, \gamma_{i'} \rangle / i' \in I'_X\},$$

$$\{\langle \mu_{i''}, \gamma_{i''} \rangle / i'' \in I''_Y\})$$

$$= P_{\max_{k \in K} \mu_k,\ \min_{k \in K} \gamma_k}(\langle \max_{i' \in I'_X,\ i'' \in I''_Y}(\mu_{i'} + \mu_{i''} - \mu_{i'} . \mu_{i''}),$$

$$\min_{i' \in I'_X,\ i'' \in I''_Y}(\gamma_{i'} . \gamma_{i''})\rangle).$$

A selection, or search, relation $E \subset X \times Y \times [0, 1]^2 \times M$ is used by F to select the internal value m, on the basis of the evaluation function G and parametrized system representation, if

$$(\forall x \in X)(\forall y \in Y)(\forall m \in M)(\langle x, y, G(m, x, y), m\rangle \in E$$

$$\text{iff } \langle x, y, m\rangle \in F).$$

Finally, we can define a condition for a correctness of S, e.g. in the form:

$$(\min\{\mu_G / (\forall i' \in I'_X)(\forall i'' \in I''_Y)(\forall k \in I_M)(G = G(m_k, x_{i'}, y_{i''}))\} \geq \mu)$$

$$\& (\max\{\gamma_G / (\forall i' \in I'_X)(\forall i'' \in I''_Y)(\forall k \in I_M)(G = G(m_k, x_{i'}, y_{i''}))\}$$

$$\leq \gamma).$$

The IF-representation of the concrete system gives a possibility to describe simultaneously as the results of the system functioning, as well as the degrees of validity and non-validity (correctness and non-correctness) ot these results. Moreover, we can make expert estimations of the system functioning and we can correct them.

For example, we cam change the x, y and m $\langle \mu, \gamma \rangle$-components with components $F_{\alpha,\beta}(\langle \mu, \gamma \rangle)$, $G_{\alpha,\beta}(\langle \mu, \gamma \rangle)$, etc., $P_{\alpha,\beta}(\langle \mu, \gamma \rangle)$ or $Q_{\alpha,\beta}(\langle \mu, \gamma \rangle)$. They can be related to a fixed time-scale, too and they can modify the $\langle \mu, \gamma \rangle$-degrees of x, y and m before, at the time or after every system action.

The same is valid for the global S-components, too.

On the other hand (in some IF-interpretations of systems), the last two parameters can be related to the interstitial values of the other $\langle \mu, \gamma \rangle$-parameters. After the functioning of the system, the values of μ and γ will correspond to the final system state and they can be used in the first sense in a next functioning of the system.

The above approach for IF-interpretation of the concept system can be transformed on the more concrete types of systems, too. For example, it can be used for IF-interpretation of the dynamical fuzzy systems from [9].

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