On semi–continous and irresolute fuzzy multifunctions

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Abstract: The paper deals with the study of semi–continous and irresolute multifunctions in fuzzy setting. Several characterizations of these fuzzy multifunctions are eslablished.

1. Preliminaries

Throughout the paper, by (X,T) or simply by X we will mean a topological space in classical sense, and (Y,T_1) or simply Y will stand for a fuzzy topological space (fts, for short) as defined by Chang [1]. Fuzzy sets in Y will be denoted by A、B、U、V etc. and a fuzzy set which is a fuzzy point which support y and value α $(0<\alpha\leq 1)$ at some $y\in Y$, will be designated by y_α. The interior and closure of a fuzzy set A in an fts Y will be denoted by IntA and ClA respectively. The semi–interior and semi–closure of a fuzzy set A in an fts Y will be denoted by SIntA and SClA respectively. Two fuzzy set A and B in an fts Y will be called q–coincident, denoted as AqB, iff there exists an $y\in Y$ such that $A(y)+B(y)>1$.

Definition 1.1[2] Let (X,T) be a topological space and (Y,T_1) be an fts, $F: X\rightarrow Y$ is called a fuzzy multifunction iff for each $x\in X$, F(x) is a fuzzy set in Y.

Definition 1.2 [3] For a fuzzy multifunctions $F: X\rightarrow Y$, the upper inverse $F^+(A)$ and lower inverse $F^-(A)$ of a fuuzy set A in Y are defined as follows:

$$F^+(A)=\{x\in X:\ F(x)\leq A\}$$
$$F^-(A)=\{x\in X:\ F(x)qA\}$$

Proposition 1.3 For a fuzzy multifunctions F: $X \to Y$, we have $F^{-}(1-G) = X - F^{+}(G)$ for any fuzzy set G in Y.

Definition 1.4 A fuzzy set U in (Y,T_1) is said to be a fuzzy semi-neighbourhood (f.semi-nbd, in short) of fuzzy point x_α iff there is a fuzzy semi-open set V in Y such that $x_\alpha \in V \leqslant U$.

A set U in (X,T) is said to be a semi-neghbourhood (semi-nbd, in short) of point $x \in X$ iff there is a semi-open set V in X, such that $x \in V \subset U$.

2. Lower and upper semi-continous fuzzy multifunctions

Definition 2.1 A fuzzy multifunction F: $X \to Y$ is called

(a) fuzzy lower semi-continous (f.l.s.c., in short) at a point $x_0 \in X$ iff for every fuzzy open set V in Y which $x_0 \in F^{-}(V)$, there exists a semi-open nbd U of x_0 in X such that $U \subset F^{-}(V)$, i.e. F(x)qV for each $x \in U$.

(b) fuzzy upper semi-contious (f.u.s.c., in short) at a point $x_0 \in X$ iff for every fuzzy open set V in Y wich $x_0 \in F^{+}(V)$, there exists a semi-open nbd U of x_0 in X such that $U \subset F^{+}(V)$, i.e. $F(x) \leqslant V$ for each $x \in U$.

(c) f.l.s.c.(f.u.s.c.) on X iff it is respectively so at each $x_0 \in X$.

Remark 2.2 In [2] and [3], lower semi-continuous and upper semi-continuous fuzzy multifunctions is defined in the same way as above, but our definition of f.l.s.c. and f.u.s.c. multifunctions differs from that given in [2] and [3].

Theorem 2.3 A fuzzy multifunction F: $X \to Y$ is f.l.s.c. iff for any fuzzy open set U in Y, $F^{-}(U)$is semi-open in X.

Proof. Let F be f.l.s.c. and let $x \in F^{-}(U)$, where U is any fuzzy open set in Y. By fuzzy lower semi-continity of F, there exists a semi-open nbd U of x such that $V \subset F^{-}(U)$. Then $V \subset \mathrm{SInt}F^{-}(U)$ and hence $x \in \mathrm{SInt}F^{-}(U)$. This shows that $F^{-}(U) \subset$

SIntF$^-$(U) and consequently F$^-$(U) is semi–open in X.

Conversely, let $x \in X$ and U be a fuzzy semi– open set in Y such that $x \in F^-(U)$. Then $V = F^-(U)$ is a fuzzy semi–open nbd of x such that $V \subset F^-(U)$.Hence F is f.l.s.c. at x and consequently, f.l.s.c. on X.

Theorem 2.4 A fuzzy multifunction F: X→Y is f.u.s.c. iff for any fuzzy open set U in Y $F^+(U)$ is semi–open in X.

Proof. Similar to that of Theorem 2.3 and omitted.

Theorem 2.5 For a fuzzy multifunction F: X→Y the following statements are equivalent:

(1) F is f.l.s.c.

(2) $F^+(V)$ is semi–closed in X, for every fuzzy closed set V in Y.

(3) $F^+(V) \supset \mathrm{IntCl}\ F^+(V)$, for every fuzzy closed set V in Y.

(4) $F^-(V) \subset \mathrm{ClInt}\ F^-(V)$, for every fuzzy open set V in Y.

(5) $\mathrm{SCl}\ F^+(G) \subset F^+(\mathrm{Cl}G)$, for every fuzzy set G in Y.

(6) $F^-(\mathrm{Int}G) \subset \mathrm{SInt}\ F^-(G)$, for every fuzzy set G in Y.

Theorem 2.6 For a fuzzy multifunction F: X→Y the following statements are equivalent:

(1) F is f.u.s.c.

(2) $F^-(V)$ is semi–closed set in X, for every fuzzy closed set V in Y.

(3) $F^-(V) \supset \mathrm{IntCl}F^-(V)$,for every fuzzy closed set V in Y.

(4) $F^+(U) \subset \mathrm{ClInt}\ F^+(U)$,for every fuzzy open set U in Y.

(5) $\mathrm{SCl}\ F^-(G) \subset F^-(\mathrm{Cl}G)$, for every fuzzy set G in Y.

(6) $F^+(\mathrm{Int}G) \subset \mathrm{SInt}F^+(G)$, for every fuzzy set G in Y.

3. Upper and lower irresolute fuzzy multifunctions.

Definition 3.1 A fuzzy mulitifunction F: X→Y is called

(a) fuzzy lower irresolute (f.l.i.,in short) at a point $x_o \in X$ iff for every fuzzy semi–open set V in Y with $x_o \in F^-(V)$, there exists a semi–open nbd V of x_o in X, such that $U \subset F^-(V)$, i.e. $F(x)qV$ for each $x \in U$.

(b) fuzzy upper irresolute (f.u.i.,in short) at a point $x_o \in X$, iff for every fuzzy semi-open set V in Y with $x_o \in F^+(V)$, there exists on semi-open nbd U of x_o in X, such that $U \subset F^+(V)$, i.e. $F(x) \leqslant V$ for each $x \in U$.

(c) f.l.i.(f.u.i.) on X iff it is respeetively so at eaach $x_o \in X$.

Theorem 3.2 A fuzzy multifunction $F: X \to Y$ is f.l.i. iff for any fuzzy semi-open set U in Y, $F^-(U)$ is an semi-open set in X.

Proof. Let F be f.l.i. and let $x \in F^-(U)$, where U is any fuzzy semi-open set in Y. By fuzzy lower irresolute of F, there exists an semi-open nbd V of x such that $V \subset F^-(U)$. Then $V \subset$ SInt$F^-(U)$ and hence $x \in$ SInt $F^-(U)$ and consequently $F^-(U)$ is an semi-open set in X.

Conversely, let $x \in X$ and U be a fuzzy semi-open set in such that $x \in F^-(U)$. Then $V = F^-(U)$ is an semi-open nbd of x such that $V \subset F^-(U)$. Hence F is f.l.i. at x and consequently f.l.i. on X.

Theorem 3.3 A fuzzy multifunction $F: X \to Y$ is f.u.i. iff for any fuzzy semi-open set U in Y, $F^+(U)$ is an semi-open set in X.

Proof. Similar to that of Theorem 3.2 and omitted.

Theorem 3.4 For a fuzzy multifunction $F: X \to Y$ the following statements are equivalent:

(1) F is f.l.i.

(2) $F^+(V)$ is semi-closed in X, for every fuzzy semi-closed set V in Y.

(3) $F^+(V) \supset$ SIntSCl $F^+(V)$, for every fuzzy semi-closed set V in Y.

(4) $F^-(U) \subset$ SClSInt$F^-(U)$, for every fuzzy semi-open set V in Y.

(5) SCl $F^+(G) \subset F^+($SClG$)$, for every fuzzy set G in Y.

(6) $F^-($SIntG$) \subset$ SInt$F^-(G)$, for every fuzzy set G in Y.

Theorem 3.5 For a fuzzy multifunction $F: X \to Y$ the following statements are equivalent:

(1) F is f.u.i.

(2) $F^{-}(V)$ is semi-closed in X, for every fuzzy semi-closed set V in Y.

(3) $F^{-}(V) \supset SIntSClF^{-}(V)$, for every fuzzy semi-closed set V in Y.

(4) $F^{+}(U) \subset SClSIntF^{+}(U)$, for every fuzzy semi-open set U in Y.

(5) $SClF^{-}(G) \subset F^{-}(SClG)$, for every fuzzy set G in Y.

(6) $F^{+}(SIntG) \subset SIntF^{+}(G)$, for every fuzzy set G in Y.

Remark 3.6 It is obvious that:

For a fuzzy mulitifunction $F:X \to Y$

(a) F is f.l.s.c. (in the sense of Mukherjee and Malakar [3]) => F is f.l.s.c.

F is f.u.i. => F is f.l.s.c.

(b) F is f.u.s.c. (in the sense of Mukherjee and Malakar [3]) => F is f.u.s.c.

F is f.u.i. => F is f.u.s.c.

In the next paper we shall show that under certain additionall conditions, The implications in Remark 3.6 can be reversed. Also example can easily be framed to show that fuzzy irresolute and fuzzy semi-continuous (in the sense of Mukherjee and Malakar[3]) multifunctions are in dependent notions.

References

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