The decomposition principle

Gu Wen—Xiang

Department of Computer Science, Northeast Normal University, Chang Chun, Ji Lin 130024, China

Lu Tu

Department of Mathematics, Chang Chun Teachers Training College, Chang Chun, Ji Lin, China.

Chen De—Gang

Department of Mathematics, Northeast Normal University, Chang Chun, Ji Lin, China.

Abstract: Some scholars have obtained the way how to decompose a fuzzy set A on X as an union of some subsets of X where these subsets have some qualities, however, many algebraic properties of the subsets of X can not be kept by the operation of union, which can be kept by the operation of intersection. Hence in this paper we study how to decompose a fuzzy set A of X as the intersection of some subsets of X which have some special properties. We have obtained some results.

Keywords: Decomposition principle, λ—cut.

1. Introduction

The decomposition theorem of fuzzy sets which establish the relation between the fuzzy sets and the general sets is well known, it decomposes a fuzzy set A of X as an union of some subsets of X. By this way we can transform the study of some properties of fuzzy sets into the study of some properties of related general subsets of X, and we can use the former results and experiences. But as well known, by the operation of union many algebraic properties of a subsystem of an algebraic system can not be kept which can be kept by the operation of intersection. For examples, the union of two subspaces may not be a subspace where the intersection of two subspaces is still a subspace. So do two subgroups. Hence when we study the algebraic properties of a fuzzy system, the former decomposition theore is not well used. This impels us to find a special relation betwteen the fuzzy set A of X and the intersection of some general subsets of X. If the equalit as the former decomposition theorem can be obtained, it must be well used on the reseach of the algebraic properties of fuzzy system. In this paper we summarize our initial works and give some fundamental results.

2. Preliminaries

Let X be an universe of discourse, F(X) be the fuzzy power set of X. If $A \in F(X)$, $A_\lambda = \{x \mid x \in X,\ A(x) \geqslant \lambda\}$ is called the λ—cut of A, $A_{\underset{\cdot}{\lambda}} = \{x \mid x \in X,\ A(X) > \lambda\}$ is called the strong λ—cut. A^c is the supplementary set of the fuzzy set A. P(X) is the power set of X. If $A \in P(X)$, $\lambda \in [0,1]$ then $\lambda \circ A$: $X \to [o,1]$

$$x \to \lambda \circ A(x) = \lambda \vee A(x).$$

F(X) is the fuzzy power set of X.

The former decomposition theorem is as following :Let A be a fuzzy set of X ,then we have the following equality:

$$A = \bigcup_{\lambda \in [0,1]} \lambda A_\lambda, \qquad A = \bigcup_{\lambda \in [0,1]} \lambda A_{\underset{\cdot}{\lambda}}.$$

3. The decomposition principle

Theorem 1. For any $A \in F(X)$,we have

$$A = \bigcap_{\lambda \in [0,1]} \lambda \circ A_{\underset{\cdot}{\lambda}}$$

Proof For any $x \in X$,

$$\Big(\bigcap_{\lambda \in [0,1]} \lambda \circ A_{\underset{\cdot}{\lambda}}\Big)(x) = \bigwedge_{\lambda \in [0,1]} (\lambda \circ A_{\underset{\cdot}{\lambda}})(x)$$

$$= \bigwedge_{\lambda \in [0,1]} [\lambda \vee A_{\underset{\cdot}{\lambda}}(x)]$$

$$= \Big\{\bigwedge_{\lambda \in [0,A(x))} [\lambda \vee A_{\underset{\cdot}{\lambda}}(x)]\Big\} \wedge \Big\{\bigwedge_{\lambda \in [A(x),1]} [\lambda \vee A_{\underset{\cdot}{\lambda}}(x)]\Big\}$$

$$= 1 \wedge \Big\{\bigwedge_{\lambda \in [A(x),1]} [\lambda \vee A_{\underset{\cdot}{\lambda}}(x)]\Big\} = \bigwedge_{\lambda \in [A(x),1]} \lambda = A(x).$$

Hence $\quad A = \bigcap_{\lambda \in [0,1]} \lambda \circ A_{\underset{\cdot}{\lambda}}$

Similarly we can prove

Theorem 2. For any $A \in F(X)$,we have

$$A = \bigcap_{\lambda \in [0,1]} \lambda \circ A_\lambda$$

Proof For any $x \in X$,

$$\Big(\bigcap_{\lambda \in [0,1]} \lambda \circ A_\lambda\Big)(x) = \bigwedge_{\lambda \in [0,1]} (\lambda \circ A_\lambda)(x)$$

$$= \bigwedge_{\lambda \in [0,1]} [\lambda \vee A_\lambda(x)]$$

$$= \Big\{\bigwedge_{\lambda \in [1,A(x)]} [\lambda \vee A_\lambda(x)]\Big\} \wedge \Big\{\bigwedge_{\lambda \in (A(x),1]} [\lambda \vee A_\lambda(x)]$$

$$= \Big\{\bigwedge_{\lambda \in [0,A(x)]} [\lambda \vee 1]\Big\} \wedge \Big\{\bigwedge_{\lambda \in (A(x),1]} [\lambda \vee 0]\Big\}$$

$$= 1 \wedge \Big\{\bigwedge_{\lambda \in (A(x),1]} \lambda\Big\}$$

$$= \bigwedge_{\lambda \in (A(x),1]} \lambda = \inf_{\lambda \in (A(x),1]} \lambda = A(x)$$

Theorem 3. Let $A \in F(X)$,

$$H : [0,1] \rightarrow P(X)$$
$$\lambda \rightarrow H(\lambda)$$

satisfying $A_\lambda \supseteq H(\lambda) \supseteq A_{\dot\lambda}$ $(\forall \lambda \in [0,1])$

then $A = \bigcap_{\lambda \in [0,1]} \lambda \circ H(\lambda)$

Proof Since $A_\lambda \supseteq H(\lambda) \supseteq A_{\dot\lambda}$

then

$$\lambda \circ A_\lambda \supseteq \lambda \circ H(\lambda) \supseteq \lambda \circ A_{\dot\lambda}$$

$$\bigcap_{\lambda \in [0,1]} \lambda \circ A_\lambda \supseteq \bigcap_{\lambda \in [0,1]} \lambda \circ H(\lambda) \supseteq \bigcap_{\lambda \in [0,1]} \lambda \circ A_{\dot\lambda}$$

But

$$\bigcap_{\lambda \in [0,1]} \lambda \circ A_\lambda = A = \bigcap_{\lambda \in [0,1]} \lambda \circ A_{\dot\lambda}$$

Hence

$$A = \bigwedge_{\lambda \in [0,1]} \lambda \circ H(\lambda)$$

$H(\lambda)$ may be not a λ—cut ,hence it is more important.

Theorem 4. Let $A \in F(X)$, $A(X) \subseteq S \subseteq [0,1]$,

$$A(X) = \{A(x) \mid x \in X\}$$

then

$$A = \bigcap_{\lambda \in S} (\lambda \circ A_{\dot\lambda})$$

Proot For any $x \in X$,

$$(\bigcap_{\lambda \in S} (\lambda \circ A_{\dot\lambda}))(x) = ([\bigcap_{\lambda < A(x)} (\lambda \circ A_{\dot\lambda})] \cap [\bigcap_{\lambda \geq A(x)} (\lambda \circ A_{\dot\lambda})](x)$$
$$= (\bigcap_{\lambda < A(x)} (\lambda \circ A_{\dot\lambda}))(x) \wedge (\bigcap_{\lambda \geq A(x)} (\lambda \circ A_{\dot\lambda}))(x)$$
$$= \{\bigwedge_{\lambda < A(x)} [\lambda \vee A_{\dot\lambda}(x)]\} \wedge \{\bigwedge_{\lambda \geq A(x)} [\lambda \vee A_{\dot\lambda}(x)]\}$$
$$= \{\bigwedge_{\lambda < A(x)} (\lambda \vee 1)\} \wedge \{\bigwedge_{\lambda \geq A(x)} [\lambda \vee 0]\}$$
$$= 1 \wedge \{\bigwedge_{\lambda \geq A(x)} \lambda\} = \bigwedge_{\lambda \geq A(x)} \lambda = A(x)$$

References

[1]L. A. Zadeh ,Fuzzy Sets ,Inform. and Control . 8(1965) 338—353
[2]Jin Chang—Ze ,The lecture of fuzzy mathematics, Chang Chun (1988).