Anti extension theorem

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Abstract: The concept of anti fuzzy subgroup was initiated by R. Biswas [1]. In this paper , first we obtain some qualities of the anti fuzzy subgroups, then we introduce the anti extension theorem and by it we prove that the homomorphic image and the homomorphic preimage of the anti fuzzy subgroup are anti fuzzy subgroups.

Keywords: group; fuzzy subgroup; anti extension theorem; anti characteristic function.

1. Introduction

Since Rosenfeld gave the concept of fuzzy group in his pioneering paper [2], a lot of results on fuzzy groups appeared . In [1] R. Biswas opened a new road for himself by giving the concept of the anti fuzzy subgroup. In this paper we try to transfer some results of fuzzy subgroups on the anti fuzzy subgroup. Then in case of the extension theorem playing an important role in the fuzzy group theory, we define the anti extension theorem and prove by this theorem the homomorphic image and the homomorphic preimage of an anti fuzzy sbugroup are still anti fuzzy groups.

2. Preliminaries

Definition 2. 1 [1] Let G be a gruop. A fuzzy subset A of G is called an anti fuzzy subgroup of G if for x,y in G: (i) $A(xy)\leqslant\max(A(x),A(y))$; (ii) $A(x^{-1})\leqslant A(x)$.

Proposition 2. 2 [1] If A is an anti fuzzy subgroup of a group G, then for any x in G:
(i) $A(e)\leqslant A(x)$; (ii) $A(x^{-1})=A(x)$.

Proposition 2. 3 [1] A is an anti fuzzy subgroup of a guoup G iff for any x,y in G, $A(xy^{-1})\leqslant\max(A(x)),A(y))$ holds.

Definition 2. 4 [1] Let A be a fuzzy set of G. For t in [0,1], the lower level subset of A is the set

$$A(t)=\{x \text{ in } G: A(x)\leqslant t\}.$$

Propsition 2. 5 [1] Let A be an anti fuzzy subgroup of a group G. Then for t in [0,1] suth that $t\geqslant A(e)$, A(t) is a subgroup of G.

3. Some qualities of the anti fuzzy subgroups

Definition 3. 1 Let T be a subset of X. We define a function f[T] on X: f[T](x) =0, x is in T; f[T] (x)=1,x is not in T. Then we call it the anti characteristic funtion of T.

Proposition 3. 2 $f[T]$ is an anti fuzzy subgroup iff T is a subgroup of G.

Proposition 3. 3 Let A_t be anti fuzzy subgroups of G, $t \in I$, then $\bigcup A_t, t \in I$, is an anti fuzzy subgroup of G.

Proof. $(\bigcup_{t\in I} A_t)(xy^{-1}) = \bigvee_{t\in I} A_t(xy^{-1}) \leqslant \bigvee_{t\in I}(\vee(A_t(x), A_t(y))) = \vee(\bigvee_{t\in I} A_t(x), \bigvee_{t\in I} A_t(y)) = \max\{(\bigcup A_t(x), (\bigcup A_t)(y)\}$

Hence $\bigcup_{t\in I} A_t$ is an anti fuzzy subgroup.

Proposition 3. 4 A is an anti fuzzy subgroup of G iff $A(x) > A(y)$ following $A(xy) = A(x)$.

Proof. Let A be an anti fuzzy subgroup. Since $A(x) > A(y)$ it is clear $A(xy) \leqslant A(x)$. If $A(xy) < A(x)$, then $A(x) = A(xyy^{-1}) \leqslant \max\{A(xy), A(y^{-1})\} = \max\{A(xy), A(y)\}$, but $A(x) > A(y)$, $A(x) > A(xy)$, so there is a construction. Hence $A(xy) = A(x)$.

If $A(x) > A(y)$ following $A(xy) = A(x)$, we come to prove that A is an anti fuzzy subgroup.

For any x,y in G, if $A(x) > A(y)$ or $A(x) < A(y)$, then

$A(xy) \leqslant \max\{A(x), A(y)\}$ holds.

Suppose $A(x) = A(y)$. If $A(xy) > A(x) = A(y)$, then

$A(x) = A(xyy^{-1}) = A(xy)$.

But it is impossible. Hence $A(xy) \leqslant A(x) = A(y) = \max\{A(x), A(y)\}$.

It is easy to prove $A(x^{-1}) \leqslant A(x)$.

Hence A is an anti fuzzy subgroup.

Proposition 3. 5 Let A be an anti fuzzy subgroup. If $A(xy^{-1}) = A(e)$, then $A(x) = A(y)$.

Proof. Since $A(xy^{-1}) = A(e)$, then $\max\{A(xy^{-1}), A(y)\} = A(y)$.

$A(x) = A(xy^{-1}y) \leqslant \max\{A(xy^{-1}), A(y)\} = A(y)$.

Similarly we can get $A(y) \leqslant A(x)$. Hence $A(x) = A(y)$ holds.

It is clear the lower level subgroup $A(A(e))$ is a subgroup of G.

Proposition 3. 6 If A is an anti fuzzy subgroup, then A is constant on every coset $xG(A)$ $(G(A) = A(A(e)))$.

Proof. For any y in $xG(A)$, there exists a y' in $G(A)$ such that $y = xy'$, $y' = x^{-1}y$. So

$A(x^{-1}y) = A(y') = A(e)$.

Hence $A(x) = A(y)$.

Definition 3. 7 Let A be a fuzzy subset of X. If for any T a subset of X, there has a t' in T such that $A(t') = \inf_{t\in T} A(t)$, We call A has the inf property.

Gorollary 3. 8 If $G(A)$ has the finite index, then A has the inf property.

Proposition 3. 9 Let f be the homomorphism from G onto G'. If A is an anti fuzzy subgroup of G, then $f(A)$ is an anti fuzzy sutbgroup of G'.

Proof. For any y', y'' in G', if $f(x) = y'y''$, then it is clear that $x = x'x''$.

$f(x') = y'$, $f(x'') = y''$. So

$$f(A)(y'y'')=\sup_{f(x)=y'y''}A(x)=\sup_{\substack{f(x')=y'\\ f(x'')=y''}}A(x'x'')\leqslant\sup_{f(x')=y',f(x'')=y''}\max\{A(x')\ A\ (x'')\}$$

$$=\max\{\sup_{f(x')=y'}A(x'),\sup_{f(x'')=y''}A(x'')\}=\max\{f(A)(y'),f(A)(y'')\}$$

$f(A)(y^{-1})\leqslant f(A)(y)$ is clear

Hence $f(A)$ is an anti fuzzy subgroup of G'

Propossition 3. 10 Let f be as above . If B is an anti fuzzy subgroup of G', then $f^{-1}(B)$ is an anti fuzzy subgroup of G.

the proof is clear.

4. The anti extension theorem of fuzzy sets

Definiton 4. 1 Let f be a mapping from U to V. Then we define a—f a mapping from F(U) to F (V) such that:

(i) $a-f(A)(y)=\inf_{f(x)=y}A(x), A\in F(U), y\in V$;

(ii) $a-f^{-1}(B)(x)=B(f(x)), B\in F(V), x\in U$.

We call the above definition the anti extension extension theorem.

Proposittion 4. 2 The following statements hold:

(1) A is empty follows a—f(A) is empty , $A\in F(U)$;

(2) $B\subset A$ follows $a-f(B)\subset a-f(A)$, $A,B\in F(U)$;

(3) $a-f(\bigcup A_i)\subset\bigcup a-f(A_i)$, A_i in F(U) ,i in I;

(4) $a-f(\bigcap A_i)=\bigcap a-f(A_i)$, A_i in F(U), i in I;

(5) $a-f(A)=V$ iff $A=U$, A in F(U);

(6) $a-f(a-f^{-1}(a-f(B))\subset B$, B in F(U);

(7) $a-f^{-1}(a-f(A))\subset A$, A in F(U).

Proposition 4. 3 Let T be the subset of U ,X the subset of V, Then

$a-f(f[T])=f[a-f(T)]$, $a-f^{-1}(f[X])=f[a-f^{-1}(X)]$ hold.

Proposition 4. 4 Let $f:G\to G'$ be a homomorphism and A an anti fuzzy sugbroup of G. If A has the inf property, then a—f(A) is the anti fuzzy subgroup of G'.

Proof, For any y', y'' in G', we have

$$\max\{a-f(A)(y'),a-f(A)(y'')\}=\max\{\inf_{f(x)=y'}A(x),\inf_{f(x)=y''}A(x)\}=\max\{A(x'),A(x'')\}\geqslant A(x'x'')\geqslant\inf_{f(x)=y'y}A(x)=a-f(A)(y'y'')$$

It is clear that $a-f(A)(y^{-1})=a-f(A)(y)$. Hence a—f(A) is an anti fuzzy subgroup of G'.

References

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