SOME MODAL TYPE OF OPERATORS IN INTUITIONISTIC FUZZY MODAL LOGIC

Part. II

Krassimir T. Atanassov

Math. Research Lab. - IPACT, P.O.Box 12, Sofia-1113, BULGARIA

Following the ideas and using the notation from [1-3], we shall define some new operators over the Intuitionistic Fuzzy Modal Logic (IFML) (cf. [4-7]).

Let everywhere A be a given propositional form (c.f. [8]: each proposition is a propositional form; if A is a propositional form then $\neg A$ is a propositional form; if A and B are propositional forms, then $A \& B$, $A \vee B$, $A \supset B$ are propositional forms) and let $V(A) = \langle a, b \rangle$, where $a, b \in [0, 1]$ and $a + b \leq 1$.

Firstly, we shall define the following two operators:

$$V(P_{\alpha,\beta}(A)) = \langle \max(\alpha, a), \min(\beta, b) \rangle,$$

$$V(Q_{\alpha,\beta}(A)) = \langle \min(\alpha, a), \max(\beta, b) \rangle.$$

Obviously, for operators "!" and "?" from [1], for which

$$V(!(A)) = \langle \max(1/2, a), \min(1/2, a \rangle,$$

$$V(?(A)) = \langle \min(1/2, a), \max(1/2, a \rangle,$$

it is valid that:

$$!(A) = P_{1/2, 1/2}(A),$$

$$?(A) = Q_{1/2, 1/2}(A).$$

Let

$$P_{\alpha,\beta}(V(A)) = V(P_{\alpha,\beta}(A)),$$

$$Q_{\alpha,\beta}(V(A)) = V(Q_{\alpha,\beta}(A)).$$

We must note, that for every propositional form A

$$V(P_{\alpha,\beta}(A)) = V(A) \vee \langle \alpha, \beta \rangle$$

and

$$V(Q_{\alpha,\beta}(A)) = V(A) \& \langle \alpha, \beta \rangle.$$

THEOREM 1: For every propositional form A and for every α, β, Γ, $\partial \in [0, 1]$, such that $\alpha + \beta \leq 1$, $\Gamma + \partial \leq 1$:

(a) $V(\overline{P_{\alpha,\beta}(\bar{A})}) = V(Q_{\beta,\alpha}(A))$;

(b) $V(P_{\alpha,\beta}(Q_{\Gamma,\partial}(A))) = V(Q_{\max(\alpha,\Gamma),\min(\beta,\partial)}(P_{\alpha,\beta}(A)))$;

(c) $V(Q_{\alpha,\beta}(P_{\Gamma,\partial}(A))) = V(P_{\min(\alpha,\Gamma),\max(\beta,\partial)}(Q_{\alpha,\beta}(A)))$;

(d) $V(P_{\alpha,\beta}(P_{\Gamma,\partial}(A))) = V(P_{\max(\alpha,\Gamma),\min(\beta,\partial)}(A))$;

(e) $V(Q_{\alpha,\beta}(Q_{\Gamma,\partial}(A))) = V(Q_{\min(\alpha,\Gamma),\max(\beta,\partial)}(A))$.

Proof: (b) $V(P_{\alpha,\beta}(Q_{\Gamma,\partial}(A)))$

$= V(P_{\alpha,\beta}(\langle\min(\Gamma, a), \max(\partial, b)\rangle)$

$= \langle\max(\alpha, \min(\Gamma, a)), \min(\beta, \max(\partial, b))\rangle$

$= \langle x, \min(\max(\alpha, \Gamma), \max(\alpha, a)), \max(\min(\beta, \partial), \max(\beta, a))\rangle$

$= Q_{\max(\alpha,\Gamma),\ \min(\beta,\partial)}(\langle\max(\alpha, a), \max(\beta, b)\rangle)$

$= Q_{\max(\alpha,\Gamma),\ \min(\beta,\partial)}(P_{\alpha,\beta}(A))$.

THEOREM 2: For every two propositional forms A and and B and for every α, $\beta \in [0, 1]$, such that $\alpha + \beta \leq 1$:

(a) $P_{\alpha,\beta}(A \mathbin{\&} B) = P_{\alpha,\beta}(A) \mathbin{\&} P_{\alpha,\beta}(B)$,

(b) $P_{\alpha,\beta}(A \veebar B) = P_{\alpha,\beta}(A) \veebar P_{\alpha,\beta}(B)$,

(c) $Q_{\alpha,\beta}(A \mathbin{\&} B) = Q_{\alpha,\beta}(A) \mathbin{\&} Q_{\alpha,\beta}(B)$,

(d) $Q_{\alpha,\beta}(A \veebar B) = Q_{\alpha,\beta}(A) \veebar Q_{\alpha,\beta}(B)$.

*
* *

The operators Z_α, Y_α, Y^β, $Y_{\alpha,\beta}$ from [3] can be transformed over IFL. They have the following forms:

$$Z_\alpha(A) = \begin{cases} A, & \text{if } a = \alpha.b \\ F, & \text{otherwise} \end{cases},$$

$$Y_\alpha(A) = \begin{cases} A, & \text{if } a \geq \alpha.b \\ F, & \text{otherwise} \end{cases},$$

$$Y^{\beta}(A) = \begin{cases} A, & \text{if } a \le \beta.b \\ F, & \text{otherwise} \end{cases},$$

$$Y_{\alpha,\beta}(A) = \begin{cases} A, & \text{if } \alpha.b \le a \le \beta.b \\ F, & \text{otherwise} \end{cases},$$

where F is the logical false.

We can define analogical operators (with the same notations) over a set S of propositional forms by:

$$Z_{\alpha}(S) = \{A / Z_{\alpha}(A) = A\},$$

$$Y_{\alpha}(S) = \{A / Y_{\alpha}(A) = A\},$$

$$Y^{\beta}(S) = \{A / Y^{\beta}(A) = A\},$$

$$Y_{\alpha,\beta}(S) = \{A / Y_{\alpha,\beta}(A) = A\}.$$

Following [1] we can define for an IFS A over E the set

$$\Box A = \{<x, a, 1 - a> / x \in E\}.$$

For the needs of the discussion below we shall define the notion of intuitionistic fuzzy tautology (IFT) and Intuitionistic fuzzy Safety (IS) by:

"A is an IFT" iff "if $V(A) = <a, b>$, then $a \ge b$".

"A is an IS" iff "if $V(A) = <a, b>$, then $a \ge 1/2$".

Obviously, if A is an IS, then A is an IFT.

THEOREM 3: (a) $Z_{\alpha}(S)$ is a set of IFTs iff $\alpha \ge 1$.

(b) $Z_{\alpha}(\Box S)$ is a set of ISs iff $\alpha \ge 1$.

Proof: Let $Z_{\alpha}(S)$ be a set of ITSs, i.e., for every $A \in Z_{\alpha}(S)$, if $V(A) = <a, b>$, then $a \ge b$. On the other hand, $a = \alpha.b$. Therefore $\alpha \ge 1$. The opposite one can see directly.

(b) is proved analogically.

THEOREM 4: If $\alpha \ge 1$, then

(a) $Y_{\alpha}(S)$ is a set of IFTs.

(b) $Y_{\alpha}(\Box S)$ is a set of ISs.

THEOREM 5: If $\beta \le 1$, then

(a) $Y^{\beta}(S)$ is a set of IFTs.

(b) $Y^{\beta}(\Box S)$ is a set of ISs.

THEOREM 6: If $1 \leq \alpha \leq \beta$, then

(a) $Y_{\alpha,\beta}(S)$ is a set of IFTs.

(b) $Y_{\alpha,\beta}(\Box S)$ is a set of ISs.

REFERENCES:

[1] Atanassov K., Level operators on intuitionistic fuzzy sets, BUSEFAL Vol. 54, 1993, 4-8.

[2] Atanassov K., Some modal type of operators in intuitionistic fuzzy modal logic. Part I, submitted to BUSEFAL.

[3] Atanassov K., New operators over the intuitionistic fuzzy sets Compt. rend. Acad. bulg. Sci., Tome 46, N. 11, 1993 (in press).

[4] Atanassov K., Two variants of intuitonistic fuzzy propositional calculus. Preprint IM-MFAIS-5-88, Sofia, 1988.

[5] Atanassov K., Two variants of intuitionistic fuzzy modal logic. Preprint IM-MFAIS-3-89, Sofia, 1989.

[6] Atanassov K., Gargov G., Intuitionistic fuzzy logic. Compt. rend. Acad. bulg. Sci., Tome 43, N. 3, 1990, 9-12.

[7] Gargov G., Atanassov K., Two results in intuitionistic fuzzy logic. Compt. rend. Acad. bulg. Sci., Tome 45, N. 12, 1992 (in press).

[8] Mendelson E., Introduction to mathematical logic, Princeton, NJ: D. Van Nostrand, 1964.