THE DECISION METHOD OF AN ORTHOGONAL F-MATRIX

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ABSTRACT

In this paper a principal result is that we proved the theorem: Any orthogonal F-matrix is equivalent to a diagonal absolute dominant F-matrix. And we gave practical decision method of the orthogonal F-matrix.

Keywords: Orthogonal F-matrix, Diagonal absolute dominant F-matrix, charactersistic interval I_A of an orthogonal F-matrix A.

1. PRELIMINARIES

We shall study and discuss on the lattice L = [0,1]. in this paper. The $L^{n\times m}$ is a set of all $n\times m$ matrices on the lattice L, its every element is called a fuzzy matrix or F-matrix.

Definition 1.1 [1] Let $A = (a_{ij}) \in L^{n\times n}$, $n \geqslant 2$. If there is $\lambda \in [0,1]$ such that

$$A_\lambda (A^T)_\lambda = I_n, \qquad (1.1)$$

where I_n is the n-order unit matrix, and $A_\lambda = (a_{ij}(\lambda))_{n\times n}$ is the λ-cut matrix of the A.(Its definition is

$$a_{ij}(\lambda) = \begin{cases} 1, & \text{if } a_{ij} \geqslant \lambda , \\ 0, & \text{if } a_{ij} < \lambda . \end{cases}$$

Therefore A_λ is a permutation matrix). Then the A is called the orthogonal F-matrix. And λ is called the charateristic number of the A, denoted by λ_A. And the set of all λ_A is called the characteristic interval of the A,denoted by I_A.

If a F-matrix B is not an orthogonal, we define its characteristic number is zero. i.e. $\lambda_B = 0$. And its characteristic interval is $I_B = \{0\}$.

Definition 1.2 [1] Let $A \in L^{n \times n}$, $n \geqslant 2$. The A is called the orthogonal F-matrix, if there is $\lambda \in [0,1]$ such that the λ-cut matrix of the A, A_λ, is a permutation matrix.

X.P Wang and W.J.Liu proved that the definition 1.1 is equivalent to the definition 1.2 in the document [1]. A_λ is called the permutation matrix of the A in (1.1).

2.THE DECISION METHOD OF THE ORTHOGONAL F-MATRIX

Theorem 2.1 Any permutation matrix is the orthogonal.

Definition 2.1 Let $A = (a_{ij}) \in L^{n \times n}$. The A is called the diagonal absolute dominant F-matrix, if

$$\forall i, \quad a_{ii} > a_{kl}, \text{ if } k \neq l.$$

Theorem 2.2 A diagonal absolute dominant F-matrix is the orthogonal.

Theorem 2.3 Let $A = (a_{ij}) \in L^{n \times n}$ is a diagonal absolute dominant F-matrix. Then $I_A = (\mu, \lambda]$, where

$$\lambda = \min_i \{a_{ii}\} \quad \text{and} \quad \mu = \max_{i \neq j} \{a_{ij}\}.$$

Defination 2.2 Let $A, B \in L^{m \times n}$. If there are permutation matrices $P \in L^{m \times m}$ and $Q \in L^{n \times n}$ such that

$$B = P\,A\,Q.$$

Then we say that the A is equivalent to B, noted by $B \cong A$.

Theorem 2.4 (The decision theorem of the orthogonal F-matrix) Any orthogonal F-matrix is equivalent to a diagonal absolute dominant F-matrix.

By this theorem we can obtain a method decideing the orthogonal F-matrix is as fillows:

To exchangeing the rows and columns of the A, if we can obtain a diagonal absolute dominant F-matrix, then the A is an orthogonal, otherwise it is not an orthogonal

Example 2.1 By

$$A = \begin{bmatrix} .1 & .1 & .2 & .4 & .2 \\ .7 & .2 & .1 & .3 & .2 \\ .2 & .1 & .4 & .2 & .3 \\ .3 & .2 & .1 & .3 & .6 \\ .1 & .8 & .3 & .3 & .1 \end{bmatrix}$$

$$\xrightarrow{r_1 \leftrightarrow r_2} \begin{bmatrix} .7 & .2 & .1 & .3 & .2 \\ .1 & .1 & .2 & .4 & .2 \\ .2 & .1 & .4 & .2 & .3 \\ .3 & .2 & .1 & .3 & .6 \\ .1 & .8 & .3 & .3 & .1 \end{bmatrix} = A_1$$

$$\xrightarrow{r_2 \leftrightarrow r_5} A_2 \xrightarrow{c_4 \leftrightarrow c_5} A_3 = \begin{bmatrix} .7 & .2 & .1 & .2 & .3 \\ .1 & .8 & .3 & .1 & .3 \\ .2 & .1 & .4 & .3 & .2 \\ .1 & .1 & .2 & .6 & .3 \\ .3 & .2 & .1 & .2 & .4 \end{bmatrix},$$

where $r_i \leftrightarrow r_j$ show exchange the row r_i for the row r_j, and $c_i \leftrightarrow c_j$ show exchange the column c_i for the column c_j. Because A_3 is a diagonal absolute dominant F-matrix, then the A is orthogonal. And $I_A=(.3,.4]$.

Example 2.2 For

$$B= \begin{bmatrix} .2 & .5 & .3 & .4 & .1 \\ .8 & .3 & .5 & .3 & .4 \\ .4 & .3 & .5 & .3 & .1 \\ .4 & .4 & .3 & .3 & .1 \\ .4 & .4 & .9 & .2 & .1 \end{bmatrix},$$

we can not exchange the B for a diagonal absolute dominant F-matrix, therefore B is not orthogonal.

REFERENCES

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