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Comparison between PID and fuzzy control

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Abstract

The goal of this work is to make an analysis of the performances of a fuzzy controller and a comparative study of fuzzy control algorithms with a conventional control approach (PID) in the case of linear dynamic process control. This comparative study is made using computer simulation. The first part is devoted to presentations of a simulated system, and a simulated controllers. In the second part, the fuzzy controller is examined in details. A sensitivity of the fuzzy logic controller to design parameters, different shapes and superposition of membership functions, is tested. Moreover, the simulations are done for the different types of fuzzy reasoning and defuzzification methods.

1. Introduction

Fuzzy controllers were developed to imitate the performance of human expert operators by encoding their knowledge in the form of linguistic rules [Mam75]. They provide a complementary alternative to the conventional analytical control methodology. Some authors argue that fuzzy controllers are suitable where a precise mathematical model of the process being controlled is not available [Kic78] [Li88]. But, it is impossible to build a controller which need not assume anything about its environment.

An often remarked disadvantage of the methods based on the fuzzy logic is the lack of appropriate tools for analysing the controllers performance, such as stability, optimality, robustness, etc. The main advantage is the possibility to implement a human experience, intuition and heuristics into the controller. The goal of this work is to study the performances of a fuzzy controller and to compare it with a classical control approach.

2. General structure of fuzzy system

Every fuzzy system is composed of four principal blocks (Figure 1):

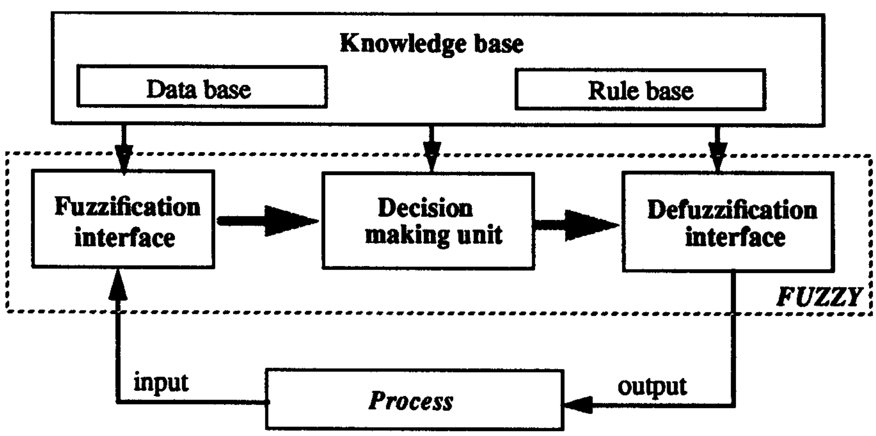

Figure 1. General structure of fuzzy inference system

1. **knowledge base** (rules and parameters for membership functions)

2. **decision unit** (inference operations on the rules)
3. **fuzzification interface** (transformation of the crisp inputs into degrees of match with linguistic variables)
4. **defuzzification interface** (transformation of the fuzzy result of the inference into a crisp output)

2. 1 Types of fuzzy reasoning

There are several types of fuzzy reasoning. The most important, in the literature, are:

- **Type 1:** Max Dot method. The final output membership function for each output is the union of the fuzzy sets assigned to that output in a conclusion after scaling their degree of membership values to peak at the degree of membership for the corresponding premise [Zim90]
- **Type 2**: Min max method. The final output membership function is the union of the fuzzy sets assigned to that output in a conclusion after cutting their degree of membership values at the degree of membership for the corresponding premise. The crisp value of output is, most usually, the center of gravity of resulting fuzzy set [Lee90].
- **Type** 3: Tsukamoto's method. The output membership function has to be monotonically non-decreasing [Tsu79]. Then, the overall output is the weighted average of each rule's crisp output induced by the rule strength and output membership functions.
- **Type 4**: Takagi and Sugeno method. Each rule's output is a linear combination of input variables. The crisp output is the weighted average of each rules's output [Jan92].

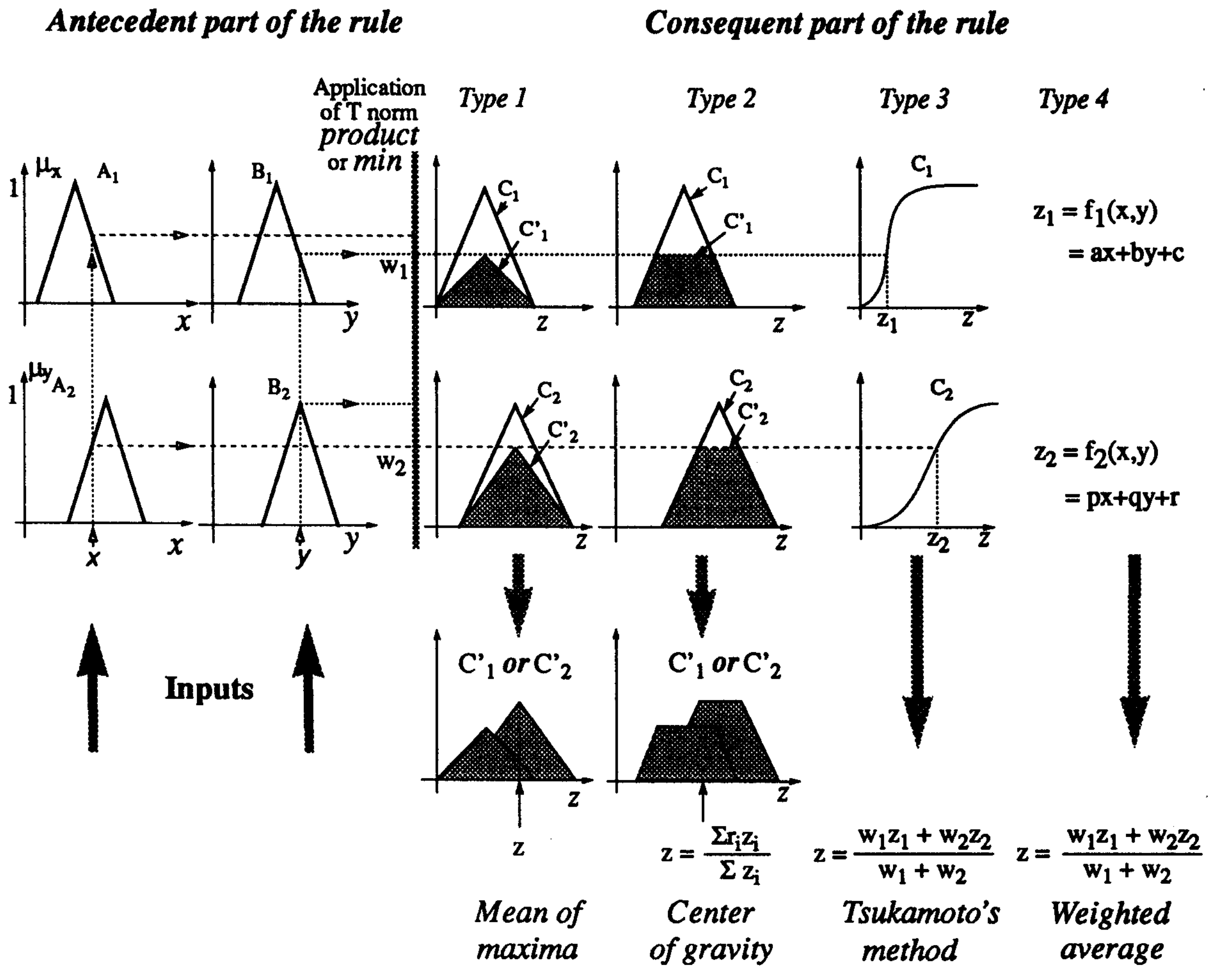

Figure 2. Four types of reasoning in fuzzy inference systems

To illustrate these four types of fuzzy reasoning, we will take the system with two inputs and one output. Suppose that two rules are activated. (A rule is activated when its firing strength is different than zero.) The rules are the following:

R_1: *If* x is A_1 and y is B_1 *then* z is C_1

R_2: *If* x is A_2 and y is B_2 *then* z is C_2

The decision procedure for the four types of fuzzy inference systems is shown on the Figure 2. Fuzzy operator ***and*** is min. One can notice that operator ***or*** is different for every type of system.

2. 2 Defuzzification strategies

Defuzzification is an operation with the aim to produce a nonfuzzy control action. It transforms an union of fuzzy sets into a crisp value. There are several methods for the defuzzification, proposed in the literature. We will describe here two of them shown in the Figure 2.

2. 2. 1 The center of gravity method

This widely used method generates a center of gravity (or center of area) of the resulting fuzzy set of a control action. If we discretize the universe it is:

$$z = \frac{\sum_{i=1}^{n} r_i z_i}{\sum_{i=1}^{n} z_i}$$

where n is the number of quantisation levels, r_i is the amount of control output at the quantisation level i and z_i represents its membership value [Ber92].

2. 2. 2 The mean of maximum method

The mean of maxima method generates a crisp control action by averaging the support values which their membership values reach the maximum. In the case of discrete universe:

$$z = \sum_{i=1}^{l} \frac{r_i}{l}$$

where l is the number of the quantized r values which reach their maximum memberships [Lee90].

3. Structure of a controlled system

The objectives of this simulation is to control the position of a servomotor based on the use of a direct current motor with separated excitation [Bov91]. The purpose of the regulation is to keep process variables close to specified values inspite of process disturbances. In the servo problem, the task is to make the process variables respond to changes in a command signal in a given way, where the command signal must be known. One way to express how the system should respond to a command signal is to give a model of the desired response. This can be done in specifying a desired transfer function from the command signal to the process variables. Moreover, it is possible to express servo response in terms of specifications on the desired closed loop step response or frequency response. We will mention here only the time domain specifications (Figure 3). They are:

- risetime T_r
- overshoot M

- settling time T_s (time before the step response is within ±p from its steady state value)
- steady-state error e_0

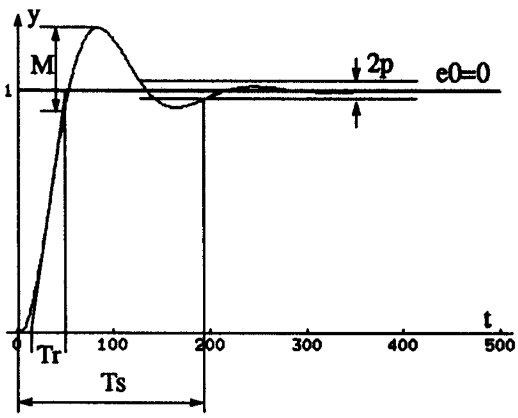

Figure 3. Expressing servo specifications in terms of requirements on the step response [Buh83]

The example system that we simulated is the same as in [Li88] and [Bov91].

That system can be represented by a diagram in Figure 4, where:

- *u* is an input voltage (control variable)
- y observed output
- τ_e - electrical time constant (0.0028 sec)
- τ_m mechanical time constant (0.28sec)
- *K* static gain of the motor (*K*=0.25)

As $\tau_e \ll \tau_m$, the equivalent transfer function will be:

$$G(s) = \frac{K}{s(1+\tau_m s)}$$

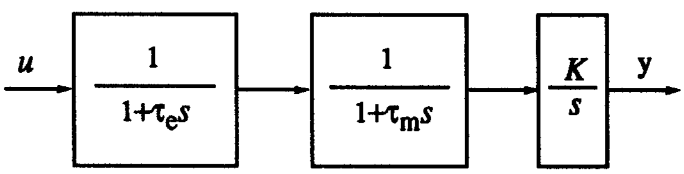

Figure 4. Structure of the simulated system

General structure of this system in a control loop is shown in the Figure 5a. Reference signal is a step function shown in the Figure 5b.

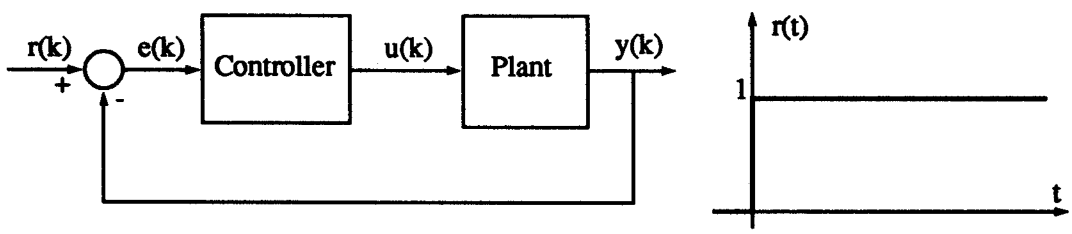

Figure 5. Structure of the controlled system and a reference signal

Sampling period taken in this simulation is 10ms according to the Shannon's theorem. Discretised transfer function [Ast89] is:

$$G(z) = K(1-z^{-1})Z\left[\frac{1}{s}\frac{1}{s(1+s\tau_m)}\right] = \frac{b_1 z + b_2}{z^2 + a_1 z + a_2} \quad 3.1$$

where:

$$b_1 = K\tau_m\left(\frac{T}{\tau_m} - 1 + e^{-\frac{T}{\tau_m}}\right) \quad , \quad b_2 = K\tau_m\left[1 - (1+\frac{T}{\tau_m})e^{-\frac{T}{\tau_m}}\right]$$

$$a_1 = -\left(1 + e^{-\frac{T}{\tau_m}}\right) \quad , \quad a_2 = e^{-\frac{T}{\tau_m}}$$

Recursive model of the system is:

$$y(kT) = -a_1 y[(k-1)T] - a_2 y[(k-2)T] + b_1 e[(k-1)T] + b_2 e[(k-2)T]$$

4. Classical control approach

The control problem presented here is simple problem and can be handled very well by PID control. PID is very well known and proved as very efficient. The textbook version of the algorithm is:

$$u(t) = K_p e(t) + K_i \int_0^t e(s)\,ds + K_d \frac{de}{dt}$$

where u is the control variable, e is the error defined as $e = r - y$ (Figure 5a) where r is the reference value (Figure 5b) and y is the process output.

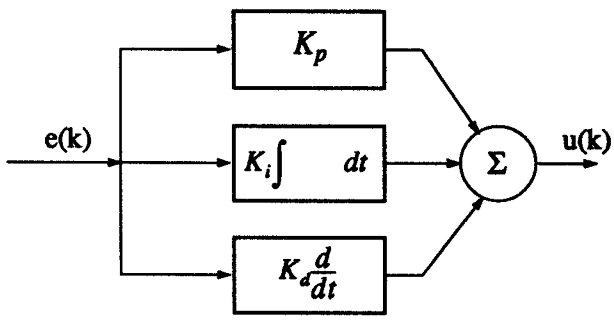

Figure 6. Structure of the PID controller

There are three parameters to adjust K_p, K_i and K_d. The structure of the controller is represented in the Figure 6. A reasonably realistic PID regulator can be described by:

$$u(kT) = K_p e(kT) + K_i \sum_{j=0}^{k} e(jT) + K_d [e(kT) - e((k-1)T)] \quad 4.1$$

One of the most used method for the adjustment of parameters for PID controller is Ziegler-Nichols method [Ast89]. This simulation is done with PD because the controlled system has an intergrator term. The response of the system with the controller in the closed loop is given in the Figure 7.

Control signal and error signal are represented in the Figure 8. One can notice that the range for the control signal is [0, 12] and the range for the error signal is [0, 1]. Changes in the error Δe are in the [0,1]. It is very important for the design of the fuzzy controller.

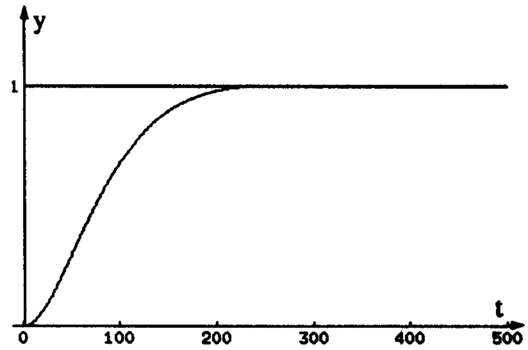

Figure 7. Step response of the system controlled with the PID in the closed loop

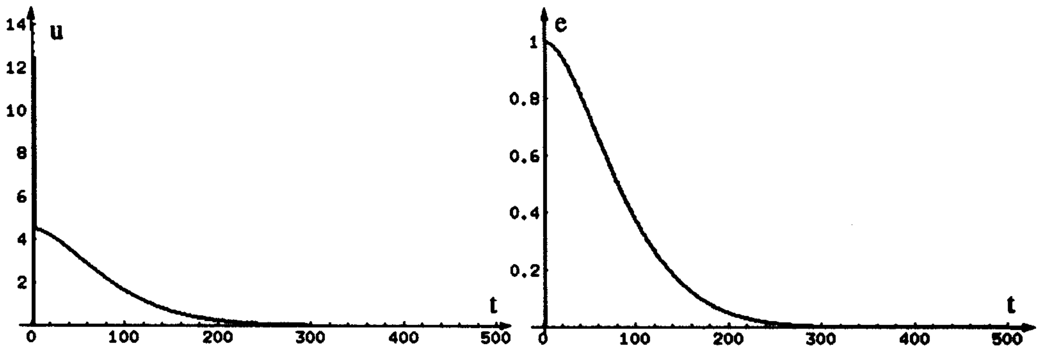

Figure 8. Control and error signal of the system

5. Fuzzy controller

There are several methods to design a fuzzy controller:

- modellization of the knowledge of the control engineer
- modellization of the human operator actions and his experience
- fuzzy modellization of the controlled plant

There is no systematic methodology to design the fuzzy controller. The most used approach is to define membership functions of the inputs and outputs, after rule data base and to test a controller. Fuzzy controller is nonlinear and it is very difficult to examine the influence of certain parameters. Because of that, the only method would be to test the controller on the system, and to adjust parameters which seem to be wrong. Our goal is to examine the influence of certain parameters and to control a linear system with known parameters and transfer function.

A basic structure of a system controlled by the fuzzy controller is presented in the Figure 5. Inputs variables, or process states in the fuzzy controller are:

- the error $e(k)$
- the change in error $\Delta e(k) = e(k) - e(k-1)$

Since the inputs in this controller are the same as for one PD controller, we can consider that the fuzzy controller simulated here corresponds to the classical PD controller. Its structure is presented in the Figure 1. The design of fuzzy controller is related with a choice of following parameters:

1. Knowledge base
   - the rule base (choice of input and control variables and control rules)
   - the universe of dicourse for every process state (choice of membership functions with their

parameters and shapes)

2. Decision making logic
   - definition of fuzzy implication
   - interpretation of the terms ***and*** and ***also*** (choice of the type of fuzzy reasoning)
3. Defuzzification mechanism

In this simulation, we partitioned a space of input and output variables into 7 fuzzy subsets. They are presented by 7 membership functions as in the Figure 3. These functions are:

- Negative Big (NB)
- Negative Medium (NM)
- Negative Small (NS)
- Close to Zero (ZR)
- Positive Small (PS)
- Positive Medium (PM)
- Positive Big (PB)

The rule base that we have taken the rule base proposed by Mamdani [Mam75] for the simulation of PD controller. These rules are shown in the Table 1. The table is read in the following way:

If the error is negative small (NS) ***and*** the change of error positive big (PB),
than the control action is positive medium (PM).

e Δe	NB	NM	NS	ZR	PS	PM	PB
NB	NB	NB	NB	NB	NS	ZR	PS
NM	NB	NB	NB	NM	NS	ZR	PS
NS	NB	NB	NM	NS	ZR	PS	PM
ZR	NB	NM	NS	ZR	PS	PM	PB
PS	NM	NS	ZR	PS	PM	PB	PB
PM	NS	ZR	PS	PM	PB	PB	PB
PB	ZR	PS	PM	PB	PB	PB	PB

Fuzzy reasoning methods that were simulated are the methods of type 1 and 2 presented in the Figure 2, and the defuzzification strategies are the center of gravity and mean of maximum method.

6. Results of the simulation

6. 1 Influence of the parameters of the membership functions

The first experiment is a test of the influence of the parameters of membership functions. Once they are adjusted, we can proceed to test the influences of other factors to the quality of the system response.

Membership functions for inputs ant output are symmetrical, triangular or bell shaped and uniformly distributed as in the Figure 9. We will test the sensibility of the controlled system to three parameters: limits for the universe of discourse for e, for Δe and for u and we will denote them with e_{max}, Δe_{max} and u_{max} respectively. Fuzzy inference is of the type 2 with the defuzzification method center of gravity. The reason is that these methods are most frequently used in the literature. The choice of the limits for membership functions were done with a knowledge of the range for the error and control signal of the system controlled by the PID controller (Figure 8). It was impossible to make this choice without any knowledge on the controlled system. We can consider these informations as something that we could receive from the operator.

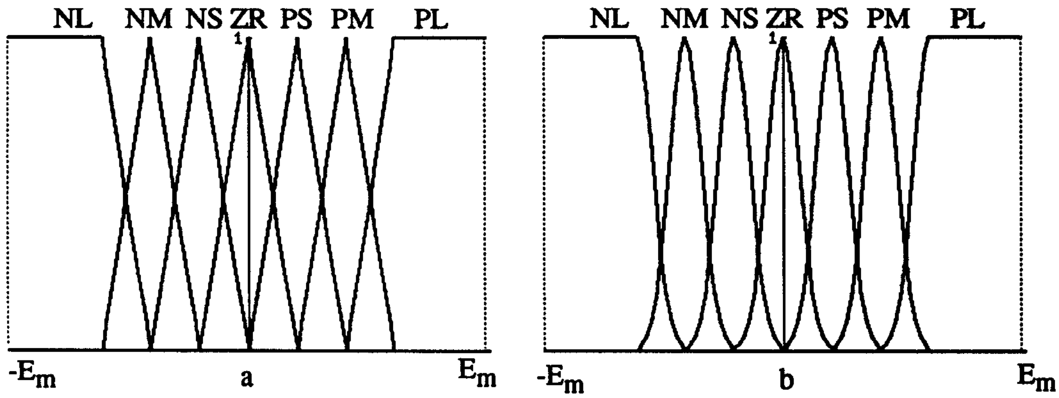

Figure 9. Uniform distribution of membership functions for e, Δe and u

a) triangular functions, b) bell-shaped functions

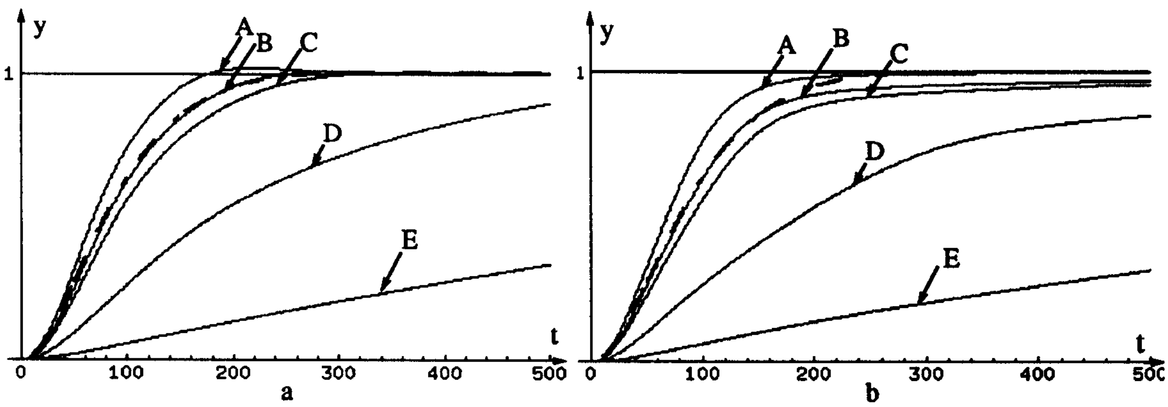

Figure 10. Responses of the system with fuzzy controller, $e_{max} = 3$, $\Delta e_{max} = 1$

A. $u_{max} = 18$, B. $u_{max} = 14$, C. $u_{max} = 12$, D. $u_{max} = 5$, E. $u_{max} = 1$

a) triangular membership functions, b) bell-shaped membership functions

(Dashed lines represent the step response with the PID controller)

From the results shown in the Figure 11 and Figure 12, we can conclude that the choice of e_{max} and Δe_{max} corresponds to the choice of constants K_p and K_d (eq. 4.1). The changes of the Δe_{max} have the same effect to the quality of the response as the changes of the constant K_d and the changes of the e_{max} have the same effect to the quality of the response as the changes of the constant K_p which prove that this controller corresponds to the classical PD controller.

As we can see in the Figure 12 the response is not very sensitive to the changes of Δe_{max} if Δe_{max} is bigger that 0.5. It is due to the fact that the most of the time the changes of the error signal are very close to zero and that the most activated function is ZR.

The shape of the membership functions is not an important parameter, but we can observe that the triangular functions give the slightly better result. Specially, the risetime and setting time are shorter when triangular functions are used.

For the limits of u, the best choice is to take the range of control signal same as for the PID controller, since the response of the fuzzy controlled system is very sensitive to this parameter (Figure 10).

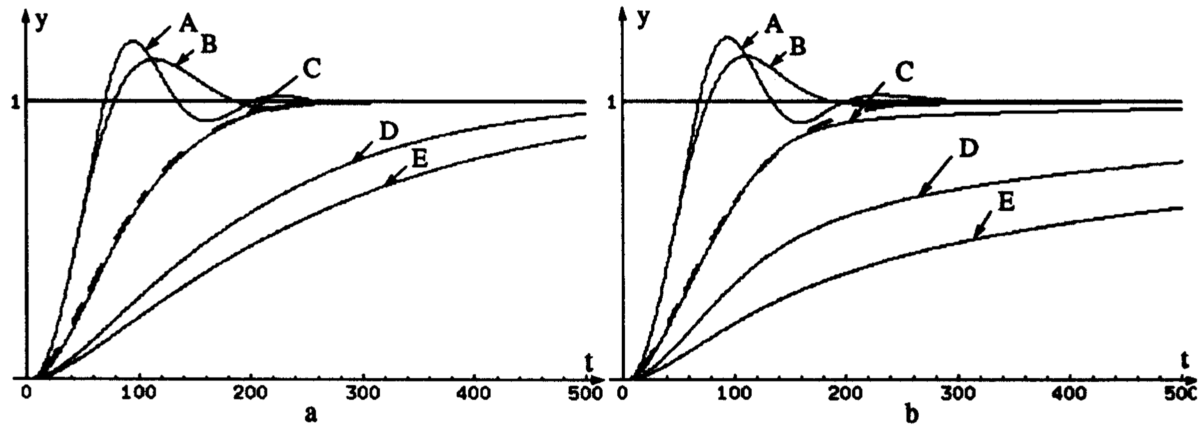

Figure 11. Responses of the system with fuzzy controller, $u_{max} = 14$, $\Delta e_{max} = 1$
A. $e_{max} = 0.5$, B. $e_{max} = 1.1$, C. $e_{max} = 3$, D. $e_{max} = 7$, E. $e_{max} = 10$
a) triangular membership functions, b) bell-shaped membership functions

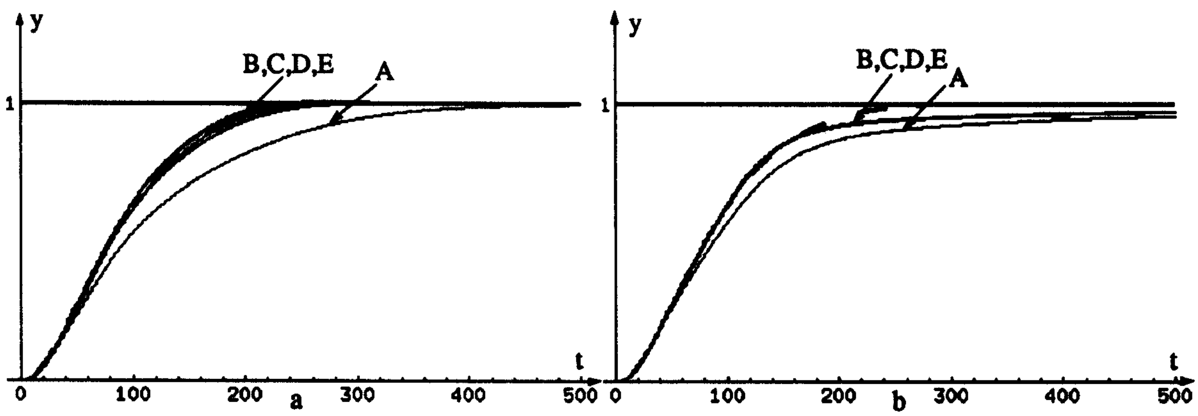

Figure 12. Responses of the system with fuzzy controller, $u_{max} = 14$, $e_{max} = 3$
A. $\Delta e_{max} = 0.1$, B. $\Delta e_{max} = 0.5$, C. $\Delta e_{max} = 1$, D. $\Delta e_{max} = 2$, E. $\Delta e_{max} = 5$
a) triangular membership functions, b) bell-shaped membership functions

6. 2 Distribution of the membership functions

From the first experiment, we noticed that the best results were received with the following parameters for membership functions:

- $e_{max} = 3$
- $\Delta e_{max} = 1$
- $u = 14$

In this experiment the sensitivity of the system response to the distribution and the overlap of the membership functions is tested. We have taken the same fuzzy inference and defuzzification method as before and we have tested only the triangular functions.

The result of the simulation is shown in the Figure 15 and different distributions simulated are presented in the Figure 13 and Figure 14.

We can notice that the system is very sensible to the distribution of the membership functions, and that the parameters that we adjusted for the regular distribution are not the adequate. It means that the controller has to be readjusted.

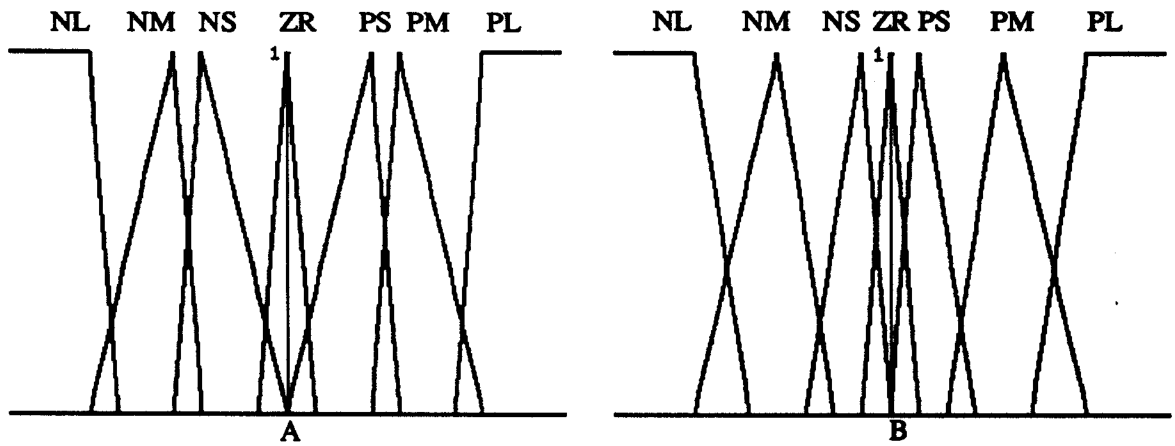

Figure 13. Nonuniform distribution of the membership functions

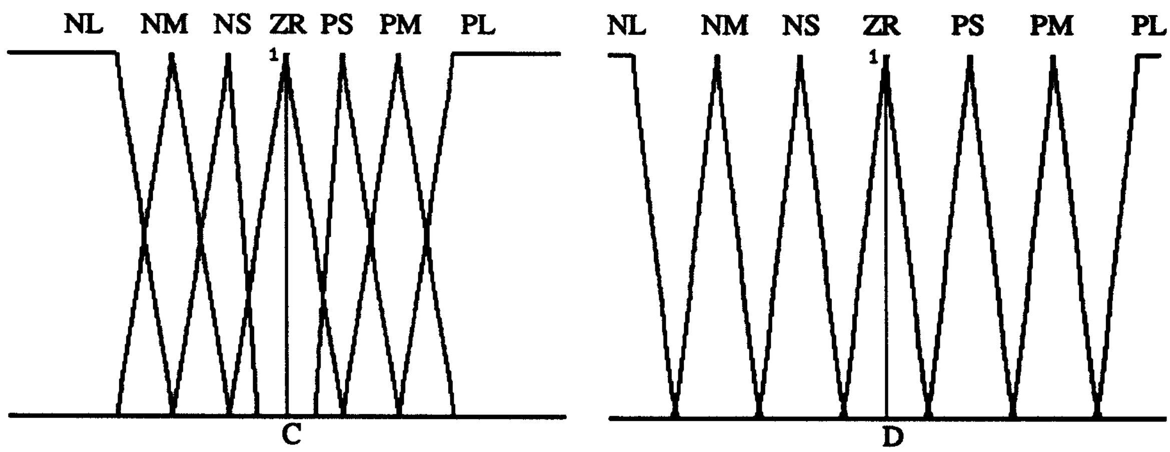

Figure 14. Nonuniform distribution of the membership functions

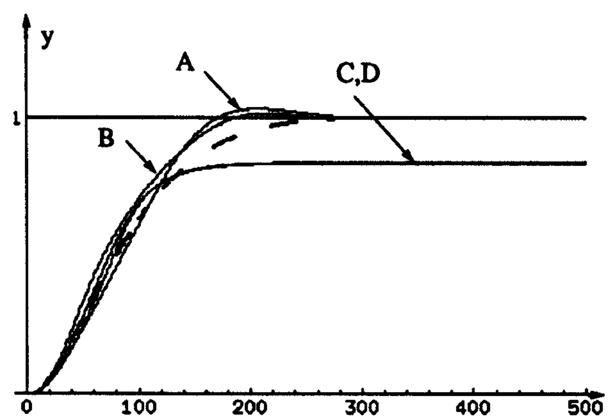

Figure 15. Response of the system for different distributions of membership functions (Figure 13 and Figure 14)

It is interesting to notice that the overlap of the functions is very important. If there is no overlap as in the Figure 14D, the system can not reach the set point. It is due to the fact that two rules can't be activated in the same time. Moreover, if the distribution is quite uniform and the membership functions NS and PS don't touch as in the Figure 14C, it is impossible to reach the set point. The reason is that for the very small values of the error and the change of the error, only one rule is activated (If the e is

ZR and Δe is ZR then u is ZR) and the control signal has always the same value.

6. 3 Different types of fuzzy reasoning and defuzzification method

The last experience is devoted to the different types of fuzzy reasoning and defuzzification method. As before, the simulation is done for the parameters that were the best in the first experiment and for the uniform distribution of the membership functions. The fuzzy reasoning methods are the type 1 and type 2 presented in the Figure 2. There is no a sensitivity of the response to the types of fuzzy reasoning A, B and C. The response of the system has a risetime and settling time the same as for the PID controlled system. The problem is the Max Dot fuzzy reasoning combined with mean of maximum defuzzification method. The reason is that the crisp values for the control signal are the centres of membership functions defined for u.

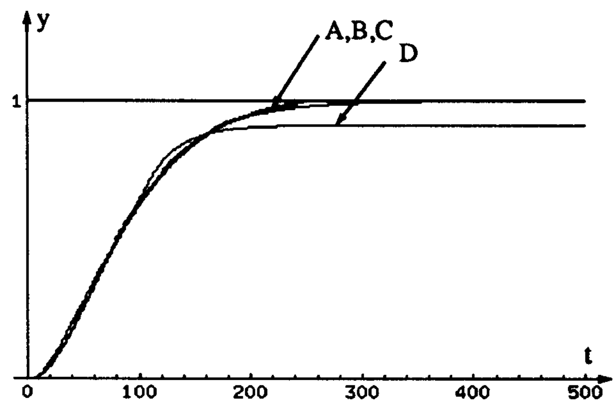

Figure 16. Response of the system for different fuzzy reasoning methods and methods for defuzzification

A. Min max and center of gravity, B. Max Dot and center of gravity
C. Min max and mean of maximum, D. Max Dot and mean of maximum

7. Conclusions

Fuzzy controllers have the advantage that can deal with nonlinear systems and use the human operator knowledge. Here we tested it with a linear system of second order with known parameters. In order to compare it with one classical controller we simulated the same system controlled by PID.

PID controller has only three parameters to adjust. Controlled system shows good results in terms of response time and precision when these parameters are well adjusted.

Fuzzy controller has a lot of parameters. The most important is to make a good choice of rule base and parameters of membership functions. Once a fuzzy controller is given, the whole system can actually be considered as a deterministic system. When the parameters are well chosen, the response of the system has very good time domain characteristics. The fuzzy controlled system is very sensitive to the distribution of membership functions but not to the shape of membership functions.

Fuzzy controlled system doesn't have much better characteristics in time domain that PID controlled system, but its advantage is that it can deal with nonlinear systems.

One of the most important problems with fuzzy controller is that the computiong time is much more long that for PID, because of the complex operations as fuzzification and particularly defuzzification. Some optimization can be done if the defuzzification method is simplified. It means that it is recommended to avoid center of gravity method.

PID controller can not be applied with the systems which have a fast change of parameters, because it would require the change of PID constants in the time. It is necessary to further study the possible combination of PID and fuzzy controller. It means that the system can be well controlled by PID which is supervised by a fuzzy system.

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