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A note on the quotient operator in fuzzy relational databases

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Recently Yager (1991) has proposed an approach based on Ordered Weighted Averages (OWA) to extend the quotient operator to a fuzzy database where each tuple of a relation is weighted by a number between 0 and 1. This number is supposed to be an estimate of the degree of association between the elements of the tuple. Quotient operations aim to find out the sub-tuples of a relation R which are associated with each element of a set S of complementary sub-tuples, as in the example below.

R :

Name	Skill-type
Jean	I
Jean	II
Jean	III
Barbara	I
Barbara	II
Debbie	I
Debbie	II
Debbie	III
Debbie	IV
Tina	II

÷ S :

Skill-type
I
II
III

=

Name
Jean
Debbie

When tuples in R and S are weighted, i.e. when R and S become fuzzy, the natural extension of the quotient operation is given by

$$\mu_{R \div S}(r) = \inf_s \mu_S(s) \rightarrow \mu_R(r,s) \tag{1}$$

where $\rightarrow$ is a multiple-valued implication connective. If $\rightarrow$ stands for Gödel implication ($a \rightarrow b = 1$ if $a \leq b$; $a \rightarrow b = b$ if $a > b$), it expresses that r should be associated with s at least as much as the extent to which s belongs to S, in order that r be regarded as an element of $R \div S$ with full membership. In the example, it expresses that all the skill-types required by S are included in the set of skill-types with which the name r is associated. (1) is basically the lower image of S via R (Dubois and Prade, 1992).

Clearly (1) extends to the case where S is allowed to be a compound fuzzy set defined on a sub-domain of R. (1) can be also extended in order to only require that *most* of the elements in S are included in R (instead of 'all'); see (Dubois and Prade, 1992). We are then more in the spirit of Yager's OWA-based proposal.

It is also worth noticing that the quotient operation (1) plays a basic role in abductive reasoning (e.g., Dubois and Prade, 1993) where $R \div S$ is then the set of elements that alone "cause" S according to the association R, interpreted in a causal way.

References

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