ANOTHER KIND OF PATTERN RECOGNITION FOR INTERVAL-VALUED FUZZY SETS

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Abstract : A kind of problems of pattern recognition for interval-valued fuzzy sets [2] (IVFS's , for short) have been solved in [1] by the author. In this paper, another kind of problems of pattern recognition for IVFS's is studied. First, the concept of induced fuzzy sets of given IVFS is introduced, and then based on this, the concept of nearness degree for IVFS's is presented. Second, the principle of choose nearness is built. Finally, a concrete example of pattern recognition for IVFS's is given.

Keywords: Interval-valued fuzzy set ; pattern recognition ; nearness degree; the principle of choose nearness.

The problems will be solved in this paper are: given IVFS's $A_1, A_2, \ldots, A_n$, let IVFS B be the pattern will be recognized, we want to judge which pattern A_i ($i=1, 2, \ldots, n$) is the nearest nestled up against B. This kind of problems can be solved by introducing nearness degree and the principle of choose nearness of IVFS's.

This paper is a continuation of [1]. The terminology and symbol used in this paper follow those of Zhang [1].

We now introduce the concept of induced fuzzy sets for IVFS's.

Definition 1. Given an $A \in IF(X)$ (cf. Definition 4 of [1]). For each $x \in X$, suppose that $A(x) = [A^-(x), A^+(x)]$. Take arbitrarily $u(x) \in [A^-(x), A^+(x)]$, then we obtain a fuzzy set

$$\omega(A): X \rightarrow [0,1]$$
$$x \mapsto u(x)$$

and, $\omega(A)$ is called a induced fuzzy set of A . We denote by $I(A)$ the set consisting of all induced fuzzy sets of A.

Remark 1. In general, an IVFS A has infinite induced fuzzy sets. However, when A degenerate into a fuzzy set, A has only a induced fuzzy set, i.e. A itself.

Example 1. Let $X=\{x_1, x_2\}$, $A \in IF(X)$ as follows:

$$A(x_1) = [0.1, 0.2], \quad A(x_2) = [0.3, 0.4].$$

Then all induced fuzzy sets of A is

$$\{\ \omega(A):\ \omega(A)(x_1) = u(x_1) \in [0.1, 0.2],\ \omega(A)(x_2) = u(x_2) \in [0.3, 0.4]\ \}.$$

Let A, B be two fuzzy sets. We denote by $N(A, B)$ the nearness degree of A with B.

Definition 2. Let $A, B \in IF(X)$. Then

$$IN(A,B) = \{\ N(\omega(A), \omega(B)):\ \omega(A) \in I(A),\ \omega(B) \in I(B)\ \}$$

is called the nearness degree of A with B.

Since $N(\omega(A), \omega(B)) \in [0, 1]$, $IN(A, B)$ is a subset of real interval $[0, 1]$. In general, $IN(A, B)$ is an interval number which contained in $[0, 1]$.

Example 2. Let $X=\{x_1, x_2\}$, $A, B \in IF(X)$ as follws:

$$A(x_1) = [0.1, 0.2], \quad A(x_2) = [0.3, 0.4],$$

$$B(x_1) = [0, 0.1], \quad B(x_2) = [0.2, 0.3].$$

Now define

$$N(\omega(A), \omega(B)) = 0.5(|\omega(A)(x_1) - \omega(B)(x_1)| + |\omega(A)(x_2) - \omega(B)(x_2)|),$$

then from the definition of operations for interval numbers we obtain

$$IN(A, B) = 0.5(|[0.1, 0.2] - [0, 0.1]| + |[0.3, 0.4] - [0.2, 0.3]|) = [0, 0.2].$$

In the following, we only consider the case in which the nearness degree of IVFS's is interval number.

Definition 3 (the principle of cheese nearness of IVFS's). Given patterns $A_1, A_2, \ldots, A_n \in IF(X)$, and $A \in IF(X)$ is pattern awaiting recognition. If $IN(A_i, A) > IN(A_j, A)$ $(i \neq j)$, then we say that A_j is not nestle against A, where $j=1, 2, \ldots, i-1, i+1, \ldots, m$, then say that A_i is the nearest nestled up against A with respect to $A_1, A_2, \ldots, A_{i-1}, A_{i+1}, \ldots, A_m$. Otherwise, from above criterion we only know that A is not nestle against $A_{j1}, A_{j2}, \ldots, A_{jk}$ $(1 < k < m-1)$, or do not exist $i, j, i \neq j, i, j \in \{1, 2, \ldots, m\}$, such that $IN(A_i, A) > IN(A_j, A)$, in this case, the interval numbers, formed by these nearness degree of others IVFS's with A, are erlational each other. There is no harm in assuming these interval numbers satisfy:

$IN(A_{i_1}, A) \not> IN(A_{i_2}, A) \not> \ldots \not> IN(A_{i_1}, A)$, $2 < k+1 < n$, then we say that A 1-th pre-nestle against A_{11} , and A 2-th pre-nestle against A_{12}, . . . , and A 1-th pre-nestle against A_{1l}.

Remark 2. Since the nearness degree here discussed are interval numbers we can define the confidence of 1-th pre-nearness degree following the confidence of 1-th pre-belong (cf. Definition 4 of [1]).

Example 3. Glyn has three daughters: A, B, and C. Albin will judge which daughter is alikest to her mother (notic, Albin and the Glyns haven't met often and don't know each other very well. Now Albin considers this problem from the following four quarters: height, appearance , fat-thin and disposition. The principle of determining data as follows: the data of Glyn's appearance is fixed to be 0, and the data of three daughters's appearance are determined from the degree of look alike between each daughter and mother; For the data of fat-thin, neither fat nor thin is fixed to be 0.5, and the fat one is fixed to be greater than 0.5, and the thin one is fixed to be smaller than 0.5; The data of disposition are determined by the same principle as appearance because the information obtained are incomplete, Albin has only got some approximate data for the heights about Glyn and her daughters. All data are shown in the following table.

	height	appearance	fat-thin	disposition
Glyn	[1.60, 1.61]	0	0.5	0

A	[1.61, 1.62]	0.1	0.4	0.1
B	[1.59, 1.60]	0.05	0.6	[0.2, 0.3]
C	[1.60, 1.61]	0.05	0.5	0.05

For height, we handle as follows:

Glyn: $[(1.60-1.59)/10,\ (1.61-1.59)/10] = [0.1, 0.2]$;

A: $[(1.61-1.59)/10,\ (1.62-1.59)/10] = [0.2, 0.3]$;

B: $[(1.59-1.59)/10,\ (1.60-1.59)/10] = [0, 0.1]$;

C: $[(1.60-1.59)/10,\ (1.61-1.59)/10] = [0.1, 0.2]$.

Put $X=\{x_1, x_2, x_3, x_4\}$, where x_1: height; x_2: appearance; x_3: fat-thin; x_4: disposition. Then we obtain the following four IVFS's on X -- W_G(Glyn), W_A (daughter A), W_B(daughter B), W_C(daughter C) :

$$W_G(x) = \begin{cases} [0.1, 0.2] & x = x_1 \\ 0 & x = x_2 \\ 0.5 & x = x_3 \\ 0 & x = x_4 \end{cases},$$

$$W_A(x) = \begin{cases} [0.2, 0.3] & x = x_1 \\ 0.1 & x = x_2 \\ 0.4 & x = x_3 \\ 0.1 & x = x_4 \end{cases},$$

$$\begin{cases} [0, 0.1] & x = x_1 \end{cases}$$

$$W_B(x) = \begin{cases} 0.05 & x = x_2 \\ 0.6 & x = x_3 \\ [0.2, 0.3] & x = x_4 \end{cases},$$

$$W_C(x) = \begin{cases} [0.1, 0.2] & x = x_1 \\ 0.05 & x = x_2 \\ 0.5 & x = x_3 \\ 0.05 & x = x_4 \end{cases}.$$

For $A, B \in IF(X)$, we define

$$IN(A, B) = \{ N(\omega(A), \omega(B)) : N(\omega(A), \omega(B)) = 1 - 0.25\sum_{i=1}^{4} | \omega(A)(x_i) - \omega(B)(x_i) |, \ \omega(A) \in I(A), \ \omega(B) \in I(B) \}.$$

From the definition of the operations for interval numbers it can be seen that

$$IN(A, B) = 1 - 0.25\sum_{i=1}^{4} | A(x_i) - B(x_i) |.$$

Then we get

$$IN(W_G, W_A) = 1 - 0.25(| [0.1, 0.2] - [0.2, 0.3] | + | 0 - 0.1 | + | 0.5 - 0.4 | + | 0 - 0.1 |) = 0.25[3.5, 3.7],$$

$$IN(W_G, W_B) = 1 - 0.25(| [0.1, 0.2] - [0, 0.1] | + | 0 - 0.05 | + | 0.5 - 0.6 | + | 0 - [0.2, 0.3] |) = 0.25[3.35, 3.65],$$

$$IN(W_G, W_C) = 1 - 0.25(| [0.1, 0.2] - [0.1, 0.2] | + | 0 - 0.05 | + | 0.5 - 0.5 | + | 0 - 0.05 |) = 0.25[3.8, 3.9].$$

It is clear that

$IN(W_G, W_C) > IN(W_G, W_A)$, $IN(W_G, W_C) > IN(W_G, W_B)$.

Hence from Definition 3, W_C is the nearest nestled up against W_G, that is , daughter C is alikest to her mother.

Next we will judge which daughter is aliker to her mother of daughter A and B.

Since $IN(W_G, W_A) \cap IN(W_G, W_B) \neq \varphi$, this two interval numbers are relational (cf. Definition 1 of [1]) , then we should do judgment according to pre-greater relation (cf. Remark 1 of [1]).

Since the mid-point value of interval number $IN(W_G, W_A) = [3.5/4, 3.7/4]$ is 0.9, and the mid-point value of $IN(W_G, W_B) = [3.35/4, 3.65/4]$ is 0.875 , thus

$$IN(W_G, W_A) \succ IN(W_G, W_B).$$

This shows that daughter A is aliker to her mother than daughter B . Therefore we get the order of degree which daughters are alike to mother :

daughter C, daughter A, daughter B.

The confidence (cf. Definition 6 of [1]) for daughter C is the first in above order is 1, and the confidence for daughter A is the second in above order is

$$\alpha = 1 - [(3.65/4 - 3.5/4) / (3.7/4 - 3.35/4)] = 0.57.$$

References

[1] Zhang Xingfang , On pattern recognition for interval-valued fuzzy sets, BUSEFAL, 56 (1993), 72--78.

[2] B. Turksen , Interval-valued fuzzy sets based on normal forms , Fuzzy Sets and systems, 20 (1986), 191--210.