FUZZY SYSTEMS: THE CONNECTIONIST APPROACH

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The main concern is the difference between two approaches in fuzzy sets theory. Also considered briefly is the similarity between fuzzy systems and neural networks.

KEY WORDS: The theorem of representation,Neural network implementation of fuzzy systems.

1. INTRODUCTION

The history of Fuzzy Systems is marked by two different visions of what a fuzzy set is, each having its correlated research program.

For a long time one vision saw a fuzzy set as a function. The other vision saw it as a family of crisp sets. One took the logical implication as its paradigm,the other the associative memory. One spoke about fuzzy logic. The other spoke about system thinking. (Negoita 1981)

Recently,the last vision was heavily emphasized. What follows is an attempt to explain this trend.

2. THE REPRESENTATION OF FUZZY SETS

Consider a fuzzy set

$$f: X \rightarrow [0,1]$$

and let

$$L_a f = \{x \in X \mid f(x) \geq a\}$$

be the a-level set of f. It is easy to show the following properties:

(1) $L_0 f = X$

(2) $a \leq b \Rightarrow L_a f \supseteq L_b f$

Property(2) shows that the level sets $(L_a f)_{0 \leq a \leq 1}$ form a nested family of sets.

Many constructions involving fuzzy soncepts have to start with the following question: given a family of subsets of X $(A_a)_{0 \leq a \leq 1}$ does there exist a fuzzy set $f: X \rightarrow [0,1]$ such that $A_a = L_a f$ for each $a \in [0,1]$?

Unfortunately, (1) and (2) are not enough for the existence of f . The extra property for the family $(A_a)_a$ is

(3) If $a_1 \leq a_2 \leq \ldots$ and $\lim a_n \uparrow a$,then $A_a = \bigcap_{n=1}^{\infty} A_{a_n}$

Various operations with fuzzy sets can be defined first on level sets and then extended by using the representation theorem. (Negoita and Ralescu 1975)

3. NEURAL NETWORK IMPLEMENTATION OF FUZZY SYSTEMS

Any Fuzzy System can be expressed as a set of implications

$$(V_k, W_k \Rightarrow Z_k)$$

An implication (or rule) k, for example,can be writen in the form:

if V is V_k and W is W_k then Z is Z_k

where V,W, and Z are linguistic variables and V_k, W_k, Z_k are linguistic values,which are represented by fuzzy sets. The fuzzy set for a given variable is characterized by a membership function defined over its measurement space. For example a fuzzy set V_k is defined as

$$V_k = \{(v,\ f_{V_k}(v))\}$$

where $f_{V_k}(v)$ is a membership value for V_k defined over a measurement space or universe $\{v\}$. (Negoita 1985)

The very existence of the representation theorem makes possible the implementation of the implication by a neural network. The logic implication of the Fuzzy System is replaced by a single hidden layer feedforward network. The input and output fuzzy sets are expressed in terms of level sets. The network is trained using the back-propagation algorithm to establish fuzzy associations between the input and output fuzzy sets.Each input and output unit of the network corresponds to a level in the universe of the input and output variables.

4. CONCLUSIONS

It may be worth going a bit deeper into the ideas behind the neural network implementation of a fuzzy system. If used as a Rosetta Stone, one can start thinking again about a categorial interpretation of the learning process (Negoita 1992)

Objections to artificial intelligence are often heard,to the effect that work in this field is ad hoc and incapable of exhibiting a sustained and meaningful growth. This has been true of some past work but the situation seems to be changing.

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