NEW OPERATION, DEFINED OVER THE INTUITIONISTIC FUZZY SETS. 4

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Some operations (as "$\cup$", "$\cap$", "+", ".", "@", "\$", "#") are defined over the Intuitionstic Fuzzy Sets (IFSs) in [1-5]. Here we shall introduce a new one and we shall show its basic properties.

Let a set E be fixed. An IFS A^* in E is an object having the form:

$$A^* = \{<x, \mu_A(x), \gamma_A(x)>/x \in E\},$$

where the functions $\mu_A : E \to [0, 1]$ and $\gamma_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$ to the set A, which is a subset of E, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \gamma_A(x) \leq 1.$$

For every two IFSs A and B are valid (see [1-5]) the following definitions (let $\alpha, \beta \in [0, 1]$):

$A \subset B$ iff $(\forall x \in E)(\mu_A(x) \leq \mu_B(x)\ \&\ \gamma_A(x) \geq \gamma_B(x))$;

$A = B$ iff $(\forall x \in E)(\mu_A(x) = \mu_B(x)\ \&\ \gamma_A(x) = \gamma_B(x))$

$A \subset_{\Box} B$ iff $(\forall x \in E)(\mu_A(x) \leq \mu_B(x))$;

$A \subset_{\Diamond} B$ iff $(\forall x \in E)(\gamma_A(x) \geq \gamma_B(x))$;

$\bar{A} = \{<x, \gamma_A(x), \mu_A(x)>/x \in E\}$;

$A \cap B = \{<x, \min(\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x))>/x \in E\}$;

$A \cup B = \{<x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma_B(x))>/x \in E\}$;

$A + B = \{<x, \mu_A(x)+\mu_B(x)-\mu_A(x).\mu_B(x), \gamma_A(x).\gamma_B(x)>/x \in E\}$;

$A . B = \{<x, \mu_A(x).\mu_B(x), \gamma_A(x)+\gamma_B(x)-\gamma_A(x).\gamma_B(x)>/x \in E\}$;

$A @ B = \{<x, (\mu_A(x)+\mu_B(x))/2, (\gamma_A(x)+\gamma_B(x))/2>/x \in E\}$;

$$A \$ B = \{\langle x, \sqrt{\mu_A(x).\mu_B(x)}, \sqrt{\gamma_A(x).\gamma_B(x)}\rangle / x \in E\};$$

$$A \# B = \{\langle x, 2.\mu_A(x).\mu_B(x)/(\mu_A(x) + \mu_B(x)), 2.\gamma_A(x).\gamma_B(x)/ (\gamma_A(x) + \gamma_B(x))\rangle / x \in E\};$$

Here we shall define the new operation:

$$A * B = \{\langle x, \frac{\mu_A(x) + \mu_B(x)}{2.(\mu_A(x).\mu_B(x) + 1)}, \frac{\gamma_A(x) + \gamma_B(x)}{2.(\gamma_A(x).\gamma_B(x) + 1)}\rangle / x \in E\}.$$

THEOREM 1: For every two IFSs A and B, $A * B$ is an IFS.

Proof: For every $x \in E$:

$$\frac{\mu_A(x) + \mu_B(x)}{2.(\mu_A(x).\mu_B(x) + 1)} + \frac{\gamma_A(x) + \gamma_B(x)}{2.(\gamma_A(x).\gamma_B(x) + 1)} \leq 1,$$

because for certain real numbers $a, b, c, d \in [0, 1]$ for which $a + b \leq 1$ and $c + d \leq 1$ the following is valid:

$$2 + 2.a.c + 2.b.d + 2.a.b.c.d - a - b - c - d - a.b.c - a.b.d - a.c.d - b.c.d$$

$$\geq 2.a.c + 2.b.d - a.b.c - a.b.d - a.c.d - b.c.d \geq 0.$$

From the above definitions follows the validity of

THEOREM 2: For every three IFSs A, B and C:

(a) $A * B = B * A$;

(b) $\overline{\bar{A} * \bar{B}} = A * B$;

The different relations between the operation "*" and the other operations are not valid.

THEOREM 3: For every two IFSs A and B and for every two numbers $\alpha, \beta \in [0, 1]$:

(a) $G_{\alpha,\beta}(A * B) =_{\Box} G_{\alpha,\beta}(A) * G_{\alpha,\beta}(B)$,

(b) $G_{\alpha,\beta}(A * B) =_{\Diamond} G_{\alpha,\beta}(A) * G_{\alpha,\beta}(B)$,

Easily can be seen, that the equality:

$$(A * B) * C = A * (B * C)$$

is not valid.

THEOREM 4 (cf. [4, 5]): For every two IFSs A and B:

$$\text{(a)}\quad A \,.\, B \subset \left\{ \begin{array}{c} A \cap B \subset \left\{ \begin{array}{c} A \# B \\ A \$ B \\ A @ B \end{array} \right\} \subset A \cup B \\ A * B \end{array} \right\} \subset A + B;$$

$$\text{(b)}\quad A * B \subset_{\Box} A @ B;$$

$$\text{(c)}\quad A * B \subset_{\Diamond} A \# B.$$

The validity of these inclusions follows from the validity of the following inequalities for the arbitrary real numbers $a, b \in [0, 1]$:

$$a.b \le \min(a, b) \le \frac{2.a.b}{a + b} \le \sqrt{a.b} \le \frac{a + b}{2} \le \max(a, b) \le a + b - a.b,$$

$$a.b \le \frac{a + b}{2.(a.b + 1)} \le \frac{a + b}{2}.$$

This operation can be transferred to an operation on ordinary fuzzy sets as follows:

$$A * B = \{<x, \frac{\mu_A(x) + \mu_B(x)}{2.(\mu_A(x).\mu_B(x) + 1)}>/ x \in E\}$$

Now, Theorem 4 obtains the form:

$$A \,.\, B \subset \left\{ \begin{array}{c} A \cap B \subset A * B \subset A \$ B \subset A @ B \subset A \cup B \\ A * B \end{array} \right\} \subset A + B.$$

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