NEW OPERATION, DEFINED OVER THE INTUITIONISTIC FUZZY SETS.2

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Some operations (as "$\cup$", "$\cap$", "+", ".", "$\bullet$") are defined over the Intuitionstic Fuzzy Sets (IFSs) in [1-3]. Here we shall introduce a new one, and we shall show its basic properties.

Let a set E be fixed. An IFS A^* in E is an object having the form:

$$A^* = \{<x, \mu_A(x), \gamma_A(x)>/x \in E\},$$

where the functions $\mu_A : E \to [0, 1]$ and $\gamma_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$ to the set A, which is a subset of E, respectively, and for every $x \in E$:

$$0 \le \mu_A(x) + \gamma_A(x) \le 1.$$

Obviously, every ordinary fuzzy set has the form:

$$\{<x, \mu_A(x), 1-\mu_A(x)>/x \in E\}.$$

For every two IFSs A and B are valid (see [1-3]) the following definitions (let $\alpha, \beta \in [0, 1]$):

$A \subset B$ iff $(\forall x \in E)(\mu_A(x) \le \mu_B(x) \ \& \ \gamma_A(x) \ge \gamma_B(x))$;

$A \supset B$ iff $B \subset A$;

$A = B$ iff $(\forall x \in E)(\mu_A(x) = \mu_B(x) \ \& \ \gamma_A(x) = \gamma_B(x))$;

$A \subset_\Box B$ iff $(\forall x \in E)(\mu_A(x) \le \mu_B(x))$;

$A \subset_\Diamond B$ iff $(\forall x \in E)(\gamma_A(x) \ge \gamma_B(x))$;

$\bar{A} = \{<x, \gamma_A(x), \mu_A(x)>/x \in E\}$;

$A \cap B = \{<x, \min(\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x))>/x \in E\}$;

$A \cup B = \{<x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma_B(x))>/x \in E\}$;

$$A + B = \{\langle x, \mu_A(x)+\mu_B(x)-\mu_A(x).\mu_B(x), \gamma_A(x).\gamma_B(x)\rangle / x \in E\};$$

$$A . B = \{\langle x, \mu_A(x).\mu_B(x), \gamma_A(x)+\gamma_B(x)-\gamma_A(x).\gamma_B(x)\rangle / x \in E\};$$

$$A @ B = \{\langle x, (\mu_A(x)+\mu_B(x))/2, (\gamma_A(x)+\gamma_B(x))/2\rangle / x \in E\};$$

$$\Box A = \{\langle x, \mu_A(x), 1-\mu_A(x)\rangle / x \in E\};$$

$$\Diamond A = \{\langle x, 1-\gamma_A(x), \gamma_A(x)\rangle / x \in E\};$$

$$C(A) = \{\langle x, K, L\rangle / x \in E\},$$

where $K = \max_{x \in E} \mu_A(x)$, $L = \min_{x \in E} \gamma_A(x)$;

$$I(A) = \{\langle x, k, l\rangle / x \in E\},$$

where $k = \min_{x \in E} \mu_A(x)$, $l = \max_{x \in E} \gamma_A(x)$;

Here we shall define the operation:

$$A \$ B = \{\langle x, \sqrt{\mu_A(x).\mu_B(x)}, \sqrt{\gamma_A(x).\gamma_B(x)}\rangle / x \in E\};$$

THEOREM 1: For every two IFSs A and B, $A \$ B$ is an IFS.

Proof: For every $x \in E$:

$$\sqrt{\mu_A(x).\mu_B(x)} + \sqrt{\gamma_A(x).\gamma_B(x)}$$

$$\leq (\mu_A(x) + \mu_B(x))/2 + (\gamma_A(x) + \gamma_B(x))/2$$

$$= (\mu_A(x) + \gamma_A(x))/2 + (\mu_B(x) + \gamma_B(x))/2 \leq 1.$$

THEOREM 2: For every two IFSs A and B:

(a) $A \$ B = B \$ A$;

(b) $\overline{\bar{A} \$ \bar{B}} = A \$ B$.

The relations between the operation "\$" and the different operations (see above) are not valid.

THEOREM 3: For every two IFSs A and B and for every two numbers $\alpha, \beta \in [0, 1]$:

(a) $\Box(A \$ B) \supset \Box A \$ \Box B$,

(b) $\Diamond(A \$ B) \subset \Diamond A \$ \Diamond B$,

(c) $G_{\alpha,\beta}(A \$ B) = G_{\alpha,\beta}(A) \$ G_{\alpha,\beta}(B)$,

(d) $C(A \$ B) \subset C(A) \$ C(B)$,

(e) $I(A \$ B) \supset I(A) \$ I(B)$,

The other relations between the operation "#" and the other operators (see [1-3]) are not valid.

Easily can be seen, that

$$(A \$ B) \$ C = A \$ (B \$ C)$$

is not valid.

THEOREM 4: For every two IFSs A and B:

(a) $A . B \subset A \cap B \subset \left\{ \begin{array}{c} A \$ B \\ A @ B \end{array} \right\} \subset A \cup B \subset A + B$;

(b) $A \$ B \subset_{\Box} A @ B$;

(c) $A @ B \subset_{\Diamond} A \$ B$.

The validities of these inclusions follow from the validity of the following inequalities for the arbitrary real numbers a, b ∈ [0, 1]:

$$a.b \leq \min(a, b) \leq \sqrt{a.b} \leq \frac{a + b}{2} \leq \max(a, b) \leq a + b - a.b.$$

This operation can be transferred to an operation on ordinary fuzzy sets as follows:

$$A \$ B = \{<x, \sqrt{\mu_A(x).\mu_B(x)}>/x \in E\}.$$

Now, Theorem obtains the form:

$$A . B \subset A \cap B \subset A \$ B \subset A @ B \subset A \cup B \subset A + B.$$

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