A SHORT COMMENT ON LIN's FINITE FUZZY ADEGUACY.

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In approximate environments (rough sets, fuzzy sets and neighborhood systems) Lin [2] give **adeguacy** condition to obtein **exact information**.
Here we shortly review the Lin's results and we give a different comment on their intrinsec quality: for finite fuzzy systems it is here proved that we can always to have exact information.

Keywords: point separation, fuzzy adeguacy.

1. GENERALITY ON FUNCTIONS FAMILIES.

Set-theoretical environment.

1.1 **Definition.**

A function family $\mathcal{F} \subseteq I^{\Omega}$ is said to **separate** (or **distingueshes**) **points** iff for each pair of distinct points $x,y \in \Omega$ there is an $f \in \mathcal{F}$ $(f:\Omega \to I)$ such that $f(x) \neq f(y)$.

1.2 Given $x \in \Omega$ and $f \in I^{\Omega}$ we define:

$$[x]_f = \{ y \in \Omega \mid f(y) = f(x) \} = f^{-1}(f(x)) \quad \text{f-classe of } x.$$

1.3 **Lemma.**

If the family $\mathcal{F} \subseteq I^{\Omega}$ distingueshes the points, then $\forall\, x \in \Omega$ it is:

$$\bigcap_{f \in \mathcal{F}} [x]_f = \{ x \}.$$

Topological environment.

Let (Ω, τ_{Ω}) and (I, τ_I) be topological spaces, with $I = [0,1]$.

1.4 **Definition.**

A family $\mathcal{F} \subseteq I^{\Omega}$ is said to **separate points from closed sets** iff whenever F is closed for τ_{Ω} and $x \notin F$, then there is an $f \in \mathcal{F}$ such that $f(x) \notin \overline{f(F)}$

1.5 **Rem.** The point separation and the same from closed sets are **indipendents.**

1.6 <u>Proposition</u>.

If $\mathcal{F} \subseteq I^{\Omega}$ is to separate points from closed sets and τ_{Ω} is T_1 then $\mathcal{F}$ is to separate points.

1.7 <u>Theorem</u>.

If (I,τ_I) is the euclidean real topological space, then: (Ω,τ_{Ω}) is a Tychonoff space iff there is a continous functions family $\mathcal{F} \subseteq I^{\Omega}$ such that is separate pointsnfrom closed [1].

2. <u>FUZZY ADEGUACY</u>.

2.1 For brevity, we call <u>fuzzy set</u> any $f \in I^{\Omega}$ (fuzzy sets and their membership functions are identified).

2.2 <u>Definition</u>.

A fuzzy set family $\mathcal{F}$ is called <u>adeguate to exact information</u> iff their (membership) functions distingueshes the points from closed sets.

By 1.7

2.3 <u>Corollary</u>.

There is an adeguate fuzzy sets family $\mathcal{F}$ in Ω iff Ω is a Tychonoff space [4].

2.4 <u>Remark</u>.

In 2.2 the condition "from closed sets" is essential to obtain 1.7 and hence 2.3 . In [2] Lin use the def.1.1 . By $\Omega = \{x_i\}_{i \in N}$ the function $f:\Omega \to I$ s.t. $f(x_0) = 0$, $f(x_i) = \frac{1}{i}$ $(i > 0)$ is to separate points; but, with $\tau_{\Omega} = \{\emptyset, \Omega, \{x_0\}, \{x_i\}_{i>0}\}$, is not to s.p. from closed sets ($x_0 \notin \overline{\{x_i\}}_{i>0} \Rightarrow f(x_0) = 0 \in \overline{\{\frac{1}{i}\}}_{i>0} = \overline{f(\{x_i\}_{i>0})}$).

3. FINITE ADEGUACY.

Let Ω be a finite universe.

3.1 Lemma.

If a finite topological space is T_1 then its topology is discrete [1].

3.2 Theorem.

There is an adeguate (continous function) family $\mathcal{F}$ in Ω iff the topology of Ω is discrete.

Proof.

$\Rightarrow$) Given $x \in \Omega$ and $f \in \mathcal{F}$ let $x_f = f(x) \in I$ be. Any point x_f is closed in I (that is T_1); then $[x]_f = f^{-1}(x_f) \ni x$ is closed in $\tau_{\mathcal{F}}$ (f is continous), and that is so $\forall f \in \mathcal{F}$; hence it is closed too the $\bigcap_{f \in \mathcal{F}} [x]_f = \{x\}$ (by lemma 1.3): $\tau_{\mathcal{F}}$ is T_1. By lemma 3.1 $\tau_{\mathcal{F}}$ is the discrete topology.

$\Leftarrow$) Any $f \in I^{\Omega}$ is continous. Given $x \in \Omega$, let $f = \chi_{\{x\}}(f(x) = 1;$ $f(y) = 0 \ \forall y \neq x)$ be: $\mathcal{F} = \{\chi_{\{x\}}\}_{x \in \Omega}$ is adeguate.∎

4. CONCLUSIONS.

For Lin "the theorem 1.7 gives an intrinsec characterization of adeguate fuzzy knowledge about the universe Ω : Tychonoff is an intrinsec property of Ω , while the definition of adeguacy is an external property of Ω (represented by function of Ω).

So Lin believe neighborhood systems is BETTER, because we could know that Ω is a Tychonoff from other reasons without explicit constructing the membership functions" [2] .

For us, instead, the adeguacy (def. 1.9) is not external propperty of Ω but linked at topology of Ω : "fuzzy set" is a set-theoretical concept, "Tychonoff" as "adeguacy" is topological concept (in "adeguacy" the fuzzy sets are "continous" function);

and this ideas are proved by the theorem 3.2 "finite adeguacy = discrete topology".

(In general case "fuzzy adeguacy = normality T_1").

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