FUZZY WEAKLY S-IRRESOLOTE MAPPINGS

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ABSTRACT

In this paper,we introduce and study the fuzzy weakly S-irresolute mapping in fuzzy topological spaces which is the weaker form of fuzzy S-irresolute mapping and is the stronger form of fuzzy precontinuous mapping.

Key words: Fuzzy topological spaces; fuzzy preopen sets; fuzzy strongly semiopen sets; fuzzy weakly S-irresolute mapping.

1. PRELIMINARIES

In this paper, A°, A^- and A' will denote respectively the interior, closure, and complement of the fuzzy set A. A fuzzy set A of a fuzzy space (X,δ) is called (1) a fuzzy strongly semiopen set of X iff there is a $B\in\delta$ such that $B\leqslant A\leqslant B^{-\circ}$[1]. (2) a fuzzy strongly semiclosed set of X iff there is a fuzzy closed set B in X such that $B^{\circ-}\leqslant A\leqslant B$[1]. (3) a fuzzy preopen set of X iff $A\leqslant A^{-\circ}$[4]. (4) a fuzzy preclosed set of X iff $A\geqslant A^{\circ-}$[4]. A mapping $f:(X,\delta)\to(Y,\tau)$ from a fuzzy space X to another fuzzy space Y is called (1) a fuzzy strongly semicontinuous mapping if $f^{-1}(B)$ is a fuzzy strongly semiopen set of X for each $B\in\tau$ [1].(2) a fuzzy precontinuous mapping if $f^{-1}(B)$ is a fuzzy preopen set of X for each $B\in\tau$[4]. (3) a fuzzy S-irresolute mapping if $f^{-1}(B)$ is a fuzzy strongly semiopen set of X for each fuzzy strongly semiopen set B of Y[2].

Definition 1.1. Let A be a fuzzy set of a fuzzy space (X,δ). Then

$A^{\Delta}=\bigcup\{B: B\leqslant A, B \text{ fuzzy strongly semiopen}\}$,

$A^{\sim}=\bigcap\{B: A\leqslant B, B \text{ fuzzy strongly semiclosed}\}$,

$A_{\Delta}=\bigcup\{B: B\leqslant A, B \text{ fuzzy preopen}\}$,

$A_{\sim}=\bigcap\{B: A\leqslant B, B \text{ fuzzy preclosed}\}$

are called the strong semi-interior[1], strong semi-closure[1], pre-interior and pre-closure of A respectively.

2. FUZZY WEAKLY S-IRRESOLUTE MAPPINGS

Definition 2.1. Let $f:(X,\delta)\to(Y,\tau)$ be a mapping from a fuzzy space X to another fuzzy space Y, f is called a fuzzy weakly S-irresolute mapping if $f^{-1}(B)$ is a fuzzy preopen set of X for each fuzzy strongly semiopen set B of Y.

Definition 2.2. Let $f:(X,\delta)\to(Y,\tau)$ be a mapping from a fuzzy space X to another fuzzy space Y, f is said to be fuzzy weakly S-irresolute at a fuzzy point p in X, if fuzzy strongly semiopen set B of Y and $f(p)\leqslant B$, there exists a fuzzy preopen set A of X such that $p\leqslant A$ and $f(A)\leqslant B$.

Theorem 2.3. Let $f: (X,\delta)\to(Y,\tau)$ be a mapping from a fuzzy space X to another fuzzy space Y. Then the following are equivalent:

(1) f is fuzzy weakly S-irresolute.

(2) $f^{-1}(B)$ is a fuzzy preclosed set of X for each fuzzy strongly semiclosed set B of Y.

(3) $f(A_{\sim})\leqslant(f(A))^{\sim}$ for each fuzzy set A of X.

(4) $(f^{-1}(B))_{\sim}\leqslant f^{-1}(B^{\sim})$ for each fuzzy set B of Y.

(5) $f^{-1}(B^{\Delta})\leqslant(f^{-1}(B))_{\Delta}$ for each fuzzy set B of Y.

Theorem 2.4. A mapping $f:(X,\delta)\to(Y,\tau)$ is fuzzy weakly S-irresolute iff f is fuzzy weakly S-irresolute for each fuzzy point p in X.

Theorem 2.5. Let $f:(X,\delta)\to(Y,\tau)$ be one-to-one and onto, where X and Y are fuzzy spaces. f is a fuzzy weakly S-irresolute mapping iff $(f(A))^{\Delta}\leqslant f(A_{\Delta})$ for each fuzzy set A of X.

Theorem 2.6. Let X_1, X_2, Y_1 and Y_2 be fuzzy spaces such that X_1 is product related to X_2. Then the product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ of fuzzy weakly S-irresolute mappings $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$ is fuzzy weakly S-irrselote.

Remark 2.7. For the mapping $f : X \to Y$ the following statements are valid:

f fuzzy S-irresolute => f fuzzy weakly S-irresolute =>

f fuzzy precontinuous.

None is reversible.

Proposition 2.8. Let $f : X \to Y$ and $g : Y \to Z$ be mappings. Then the following statements are valid:

(1) If f is fuzzy weakly S-irresolute and g is fuzzy S-irresolute, then $g \circ f$ is fuzzy weakly S-irresolute.

(2) If f is fuzzy weakly S-irresolute and g is fuzzy strongly semi-continuous, then $g \circ f$ is fuzzy precontinuous.

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