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FUZZY W-PRE-SEMICONTINUOUS MAPPINGS

Ye Xiao-Hong

Department of Mathematics, Yanan University, Yanan, China

Abstract

In this paper we introduce and study the fuzzy w-pre-semicontinuous mapping on fuzzy topological spaces.

Key words: Fuzzy regular open sets; fuzzy pre-semiopen sets; fuzzy w-pre-semicontinuous mappings.

1. PRELIMINARIES

In this work, A°, A^{-}, $A_{\circ}$, A_{-} and A' will denote respectively the interior, closure, semi-interior, semi-closure and complement of the fuzzy set A. Let A be a fuzzy set of a fuzzy space X. Then A is called (1) a fuzzy pre-semiopen set of X iff $A < (A^{-})_{\circ}$; (2) a fuzzy pre-semiclosed set of X iff $A > (A^{\circ})_{-}$ [4]. A mapping $f: X_1 \to X_2$ is called fuzzy pre-semicontinous if $f^{-1}(B)$ is a fuzzy pre-semiopen set of X_1 for each fuzzy open set B of X_2 [4].

2. FUZZY W-PRE-SEMICONTINUOUS MAPPINGS

Definition 1. A mapping $f: X_1 \to X_2$ from a fuzzy space X_1 to another fuzzy space X_2 is called a fuzzy w-pre-semicontinuous mapping, if $f^{-1}(B)$ is a fuzzy pre-semiopen set of X_1 for each fuzzy regular open set B for X_2.

Theorem 1. Let $f: (X_1, \delta_1) \to (X_2, \delta_2)$ be a mapping. Then the following are equivalent:

(1) f is fuzzy w-pre-semicontinuous.

(2) $f^{-1}(B)$ is a fuzzy pre-semiclosed set of X_1 for each fuzzy

regular closed set B of X_2.

(3) $((f^{-1}(B^{\circ -}))^{\circ})_{-} < f^{-1}(B)$ for each $B' \in \delta_2$.

(4) $f^{-1}(B) < ((f^{-1}(B^{-\circ}))^{-})_{\circ}$ for each $B \in \delta_2$.

(5) $((f^{-1}(B))^{\circ})_{-} < f^{-1}(B^{-})$ for each fuzzy semiopen set B of X_2.

(6) $f^{-1}(B^{\circ}) < ((f^{-1}(B))^{-})_{\circ}$ for each fuzzy semiclosed set B of X_2.

Definition 2. Let $f: X_1 \to X_2$ be a mapping from a fuzzy space X_1 to another fuzzy space X_2, f is said to be fuzzy w-pre-semicontinuous at a fuzzy point e in X_1, if fuzzy regular open set B in X_2 and $f(e) < B$, there exists a fuzzy pre-semiopen set A in X_1 such that $e < A$ and $f(A) < B$.

Theorem 2. A mapping $f: X_1 \to X_2$ is fuzzy w-pre-semicontinuous iff f is fuzzy w-pre-semicontinuous for each fuzzy point e in X_1.

Theorem 3. Let $f: X_1 \to X_2$ be fuzzy w-pre-semicontinuous mapping. Then

(1) $((f^{-1}(B))^{\circ})_{-} < f^{-1}(B^{\circ -})$ for each fuzzy strongly semiopen set [3] B of X_2.

(2) $f^{-1}(B^{-\circ}) < ((f^{-1}(B))^{-})_{\circ}$ for each fuzzy strongly semiclosed set [3] B of X_2.

Theorem 4. Let X_1, X_2, Y_1 and Y_2 be fuzzy spaces such that X_1 is product related to X_2 and Y_1 is to Y_2. Then the product $f_1 \times f_2$: $X_1 \times X_2 \to Y_1 \times Y_2$ of fuzzy w-pre-semicontinuous mappings $f_1: X_1 \to Y_1$ and $f_2: X_2 \to Y_2$ is fuzzy w-pre-semicontinuous.

Remark 1. For the mapping $f: X_1 \to X_2$, the following statements are valid:

(1) f is fuzzy pre-semicontinuous==>f is fuzzy w-pre-semicontinuous.

(2) f is fuzzy almost precontinuous [2] ==> f is fuzzy w-pre-semicontinuous.

(3) f is fuzzy almost semicontinuous [2] ==> f is fuzzy w-pre-semicontinuous.

The converses of (1)-(3) need not be true.

Example 1. Let X={a, b, c} and A, B, C, D be fuzzy sets of X defined as follows:

$A(a)=0.2$ $A(b)=0.4$ $A(c)=0.5$;

$B(a)=0.8$ $B(b)=0.8$ $B(c)=0.6$;

$C(a)=0.3$ $C(b)=0.2$ $C(c)=0.4$;

$D(a)=0.1$ $D(b)=0.2$ $D(c)=0.3$.

Let $\delta_1=\{0, A, B, 1\}$, $\delta_2=\{0, D, C, 1\}$. Consider the identity mapping $f: (X, \delta_1) \to (X, \delta_2)$. Clearly f is fuzzy w-pre-semicontinuous. But f is not a fuzzy almost semicontinuous mapping, nor a fuzzy almost precontinuous mapping, nor a fuzzy pre-semicontinuous mapping.

Proposition 1. Let $f: X \to Y$ and $g: Y \to Z$ be mappings. If f is fuzzy pre-semicontinuous and g is fuzzy almost continuous, then $g \cdot f$ is fuzzy w-pre-semicontinuous.

Theorem 5. Let $f: X_1 \to X_2$ be a mapping from a fuzzy space X_1 to a fuzzy semiregular space X_2 [1]. Then f is fuzzy w-pre-semicontinuous iff f is fuzzy pre-semicontinuous.

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