RELATIONS BETWEEN BOTH TYPES OF INTUITIONISTIC FUZZY SETS

Krassimir T. Atanassov
Math. Research Lab. - IPACT, P.O.Box 12, Sofia-1113, BULGARIA

In [1] and [2] two types of Intuitionistic Fuzzy Sets (IFSs) are introduced - ordinary IFSs and IFSs from second type (IFS2).

Let a set E be fixed. An IFS (IFS2) A^* in E is an object with the form:

$$A^* = \{<x, \mu_A(x), \gamma_A(x)>/x\in E\},$$

where the functions $\mu_A : E \to [0, 1]$ and $\gamma_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \le \mu_A(x) + \gamma_A(x) \le 1 \quad (1)$$

$$(0 \le \mu_A(x)^2 + \gamma_A(x)^2 \le 1). \quad (2)$$

For simplicity below we shall write A instead of A^*.

In [2] it is noted that if A is an ordinary IFS, then A is an IFS2 too, but the opposite is not always valid.

Let E be a given universe. In the case of IFSs, the set M' with the form:

$$M' = \{<x, y>/ 0 \le x, y \le 1 \ \&\ x + y \le 1\}$$

and function F': E -> M' are given. In the case of IFS2s, the set M" with the form:

$$M'' = \{<x, y>/ 0 \le x, y \le 1 \ \&\ x^2 + y^2 \le 1\}$$

and function F": E -> M" are given.

In both cases, the sets (M' and M") render the role of model sets for E and the functions (F' and F") - of valuations for elements of E(in the sense, e.g., of [3]). For every A (subset of E) and for every $x \in E$: $F(x) = <\mu_A(x), \gamma_A(x)> \in M$, where F is one of the functions F' and F", M is one of the sets M' and M", μ_A and γ_A satisfy (1) or (2) and the three types of objects are co-ordinated. For these objects we call the ordered tuple <M, F> by "an E-model". Therefore, at the moment every universe E has two different types of E-models.

Here we shall give the transformation formulas between both types of E-models.

Let A be a fixed subset of the universe E. Let x be a fixed element of E. Then we shall show e.g. by means of the analytical geometry that for x if F'(x) = <a, b>, i.e. $\mu_A(x) = a$ and $\gamma_A(x) = b$, then

$$F''(x) = \langle \frac{a.(a + b)}{\sqrt{a^2 + b^2}}, \frac{b.(a + b)}{\sqrt{a^2 + b^2}} \rangle,$$

and if F"(x) = <c, d>, i.e. $\mu_A(x) = c$ and $\gamma_A(x) = d$ (in both places μ_A and γ_A are co-ordinated with the type of F-finction), then

$$F'(x) = \langle \frac{c.\sqrt{c^2 + d^2}}{c + d}, \frac{d.\sqrt{c^2 + d^2}}{c + d} \rangle.$$

Therefore, it can easily be seen, that there exists a bijection G: M' -> M". For it:

$$G(a,b) = \langle \frac{a.(a + b)}{\sqrt{a^2 + b^2}}, \frac{b.(a + b)}{\sqrt{a^2 + b^2}} \rangle \in M'' \text{ for } \langle a, b \rangle \in M'$$

and

$$G^{-1}(c, d) = \langle \frac{c.\sqrt{c^2 + d^2}}{c + d}, \frac{d.\sqrt{c^2 + d^2}}{c + d} \rangle \in M' \text{ for } \langle c, d \rangle \in M''.$$

In the particular case, when A is an ordinary fuzzy set if F'(x) = <a, 1 - a>, then

$$F''(x) = \langle \frac{a}{\sqrt{2.a^2 + 2.a + 1}}, \frac{1 - a}{\sqrt{2.a^2 + 2.a + 1}} \rangle,$$

and if $F''(x) = \langle c, \sqrt{1 - c^2} \rangle$, then

$$F'(x) = \langle \frac{c}{c + \sqrt{1 - c^2}}, \frac{\sqrt{1 - c^2}}{c + \sqrt{1 - c^2}} \rangle.$$

REFERENCES:

[1] Atanassov K. Intuitionistic fuzzy sets, Fuzzy sets and Systems Vol. 20 (1986), No. 1, 87-96.

[2] Atanassov K. A second tyoe of intuitionistic fuzzy sets, BUSEFAL, Vol. 56 (1993) (in press).

[3] Rasiova H., Sikorski R. The mathematics of metamathematics, Warszawa, Pol. Acad. of Sci., 1963.