A SECOND TYPE OF INTUITIONISTIC FUZZY SETS

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Following the definition of the concept Intuitionistic Fuzzy Set (IFS) from [1], here we shall introduce the concept a Second Type of IFS (IFS2). The idea for this new object is given (unformally) in [2].

Let a set E be fixed. An IFS2 A^* in E is an object with the following form:

$$A^* = \{<x, \mu_A(x), \nu_A(x)>/x \in E\},$$

where the functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \le \mu_A(x)^2 + \nu_A(x)^2 \le 1.$$

Obviously, every ordinary fuzzy set has the form:

$$\{<x, \mu_A(x), \sqrt{1 - \mu_A(x)^2}>/x \in E\}.$$

If

$$\pi_A(x) = \sqrt{1 - \mu_A(x)^2 - \nu_A(x)^2},$$

then $\pi_A(x)$ is the degree of indeterminacy of the element $x \in E$ to the set A. In the ordinary fuzzy sets, $\pi_A(x) = 0$ for every $x \in E$.

For simplicity below we shall write A instead of A^*.

In a difference of the geometrical interpretation of the ordinary IFSs (see Fig. 1), the geometrical interpretation of the IFS2s has the form from Fig. 2.

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Following the definitions of the relations and operations over IFSs (see [1, 3, 4, 6]), we shall define over IFS2s only these of the IFS-operations and relations and we shall show only these of their properties which have a sense here. The fuzzy set relations and operations directly follow from the given below.

For every two IFS2s A and B the following relations and operations can be defined:

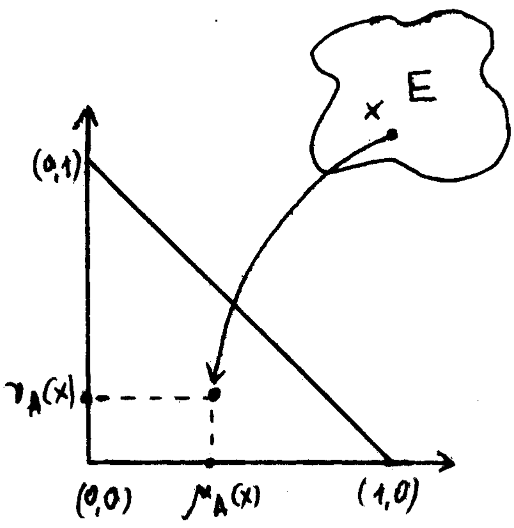

Fig. 1

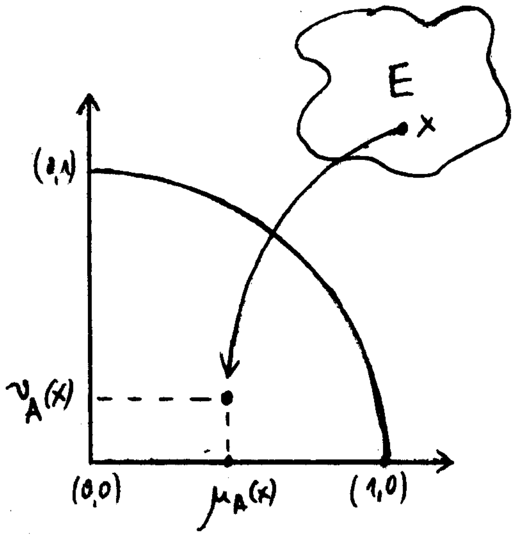

Fig. 2

$A \subset B$ iff $(\forall x \in E)(\mu_A(x) \le \mu_B(x)\ \&\ \nu_A(x) \ge \nu_B(x))$;

$A \supset B$ iff $B \subset A$;

$A = B$ iff $(\forall x \in E)(\mu_A(x) = \mu_B(x)\ \&\ \nu_A(x) = \nu_B(x))$

$\bar{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in E\}$;

$A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle / x \in E\}$;

$A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))\rangle / x \in E\}$;

$A\ @\ B = \{\langle x, (\mu_A(x)+\mu_B(x))/2, (\nu_A(x)+\nu_B(x))/2\rangle / x \in E\}$;

$A\ \$\ B = \{\langle x, \sqrt{\mu_A(x).\mu_B(x)}, \sqrt{\nu_A(x).\nu_B(x)}\rangle / x \in E\}$.

It is easy to convince oneself of the correctness of the defined operations and relations.

THEOREM 1: For every three IFS2s A, B and C:

(a) $A \cup B = B \cup A$;

(b) $A \cap B = B \cap A$;

(c) $A\ @\ B = B\ @\ A$;

(d) $A\ \$\ B = B\ \$\ A$;

(e) $(A \cup B) \cup C = A \cup (B \cup C)$;

(f) $(A \cap B) \cap C = A \cap (B \cap C)$;

(g) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$;

(h) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$;

(i) $(A \cap B)\ @\ C = (A\ @\ C) \cap (B\ @\ C)$;

(j) $(A \cup B) \bullet C = (A \bullet C) \cup (B \bullet C)$;

(k) $\overline{\bar{A} \cup \bar{B}} = A \cap B$;

(l) $\overline{\bar{A} \cap \bar{B}} = A \cup B$;

(m) $\overline{\bar{A} \bullet \bar{B}} = A \bullet B$;

(n) $\overline{\bar{A} \$ \bar{B}} = A \$ B$.

By analogy with [1], we shall define over IFS2s different operators which have no analogues in the fuzzy set theory:

$$\Box A = \{\langle x, \mu_A(x), \sqrt{1-\mu_A(x)^2}\rangle / x \in E\};$$

$$\Diamond A = \{\langle x, \sqrt{1-\gamma_A(x)^2}, \gamma_A(x)\rangle / x \in E\}.$$

THEOREM 2: For every IFS2 A: $\Box A \subset A \subset \Diamond A$.

Let $\alpha, \beta \in [0, 1]$ be a fixed number. For the IFS2 A we shall define the following operators (cf. [1,5]):

$$D_\alpha(A) = \{\langle x, \sqrt{\mu_A(x)^2 + \alpha.\pi_A(x)^2}, \sqrt{\gamma_A(x)^2 + (1-\alpha).\pi_A(x)^2}\rangle / x \in E\};$$

$$F_{\alpha,\beta}(A) = \{\langle x, \sqrt{\mu_A(x)^2 + \alpha.\pi_A(x)^2}, \sqrt{\gamma_A(x)^2 + \beta.\pi_A(x)^2}\rangle / x \in E\},$$

where $\alpha^2 + \beta^2 \le 1$;

$$G_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \beta.\gamma_A(x)\rangle / x \in E\}.$$

Obviously,

$$\Box(A) = D_0(A),$$

$$\Diamond(A) = D_1(A),$$

$$D_\alpha(A) = F_{\alpha, 1-\alpha}(A).$$

for every IFSs A as in the case of the ordinary IFSs.

THEOREM 3: For every two IFS2s A and B, and for every $\alpha, \beta \in [0, 1]$, such that $0 \le \alpha + \beta \le 1$:

(a) $F_{\alpha,\beta}(A \cap B) \subset F_{\alpha,\beta}(A) \cap F_{\alpha,\beta}(B)$;

(b) $F_{\alpha,\beta}(A \cup B) \supset F_{\alpha,\beta}(A) \cup F_{\alpha,\beta}(B)$;

(c) $F_{\alpha,\beta}(A \bullet B) = F_{\alpha,\beta}(A) \bullet F_{\alpha,\beta}(B)$;

(d) $\Box(A \$ B) \supset \Box A \$ \Box B$;

(e) $\Diamond(A \$ B) \subset \Diamond A \$ \Diamond B$;

(f) $\overline{F_{\alpha,\beta}(\bar{A})} = F_{\beta,\alpha}(A)$.

THEOREM 4: For every two IFS2s A and B and for every $\alpha, \beta \in [0, 1]$:

(a) $G_{\alpha,\beta}(A \cap B) = G_{\alpha,\beta}(A) \cap G_{\alpha,\beta}(B)$;

(b) $G_{\alpha,\beta}(A \cup B) = G_{\alpha,\beta}(A) \cup G_{\alpha,\beta}(B)$;

(c) $G_{\alpha,\beta}(A \ @ \ B) = G_{\alpha,\beta}(A) \ @ \ G_{\alpha,\beta}(B)$;

(d) $\overline{G_{\alpha,\beta}(\bar{A})} = G_{\beta,\alpha}(A)$.

THEOREM 5: For every IFS2 A and for every $\alpha, \beta, \gamma, \delta \in [0, 1]$:

(a) if $\alpha + \beta \leq 1$ and $\gamma + \delta \leq 1$, then:

$F_{\alpha,\beta}(F_{\gamma,\delta}(A)) = F_{\alpha+\gamma-\alpha.\gamma-\alpha.\delta,\ \beta+\delta-\beta.\gamma-\beta.\delta}(A)$;

(b) $G_{\alpha,\beta}(G_{\gamma,\delta}(A)) = G_{\alpha.\gamma,\ \beta.\delta}(A) = G_{\gamma,\delta}(G_{\alpha,\beta}(A))$;

(c) if $\gamma, \delta \in [0, 1]$ and $\gamma + \delta \leq 1$, then

$G_{\alpha,\beta}(F_{\gamma,\delta}(A)) \subset_{\Box} F_{\gamma,\delta}(G_{\alpha,\beta}(A))$;

(d) if $\gamma, \delta \in [0, 1]$ and $\gamma + \delta \leq 1$, then

$F_{\gamma,\delta}(G_{\alpha,\beta}(A)) \subset_{\Diamond} G_{\alpha,\beta}(F_{\gamma,\delta}(A))$.

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Let for every IFS2 A:

$C(A) = \{\langle x, K, L\rangle / x \in E\}$,

where $K = \max_{x \in E} \mu_A(x)$, $L = \min_{x \in E} \nu_A(x)$;

and

$I(A) = \{\langle x, k, l\rangle / x \in E\}$,

where $k = \min_{x \in E} \mu_A(x)$, $l = \max_{x \in E} \nu_A(x)$.

We shall call these operators "closure" and "interior". They are the same as the ones which are defined over the ordinary IFSs.

THEOREM 6: For every two IFSs A and B:

(a) $I(A) \subset A \subset C(A)$;

(b) $C(C(A)) = C(A)$;

(c) $C(I(A)) = I(A)$;

(d) $I(C(A)) = C(A)$;

(e) $I(I(A)) = I(A)$;

(f) $C(A \cup B) = C(A) \cup C(B)$;

(g) $C(A \cap B) \subset C(A) \cup C(B)$;

(h) $C(A \circledast B) \subset C(A) \circledast C(B)$;

(i) $I(A \cup B) \supset I(A) \cup I(B)$;

(j) $I(A \cap B) = I(A) \cap I(B)$;

(k) $I(A \circledast B) \supset I(A) \circledast I(B)$;

(l) $\overline{I(\bar{A})} = C(A)$.

THEOREM 7: For every IFS A and for every $\alpha, \beta \in [0, 1]$,

(a) if $0 \le \alpha + \beta \le 1$:

$CF_{\alpha,\beta}(A) \subset F_{\alpha,\beta}CA$;

(b) if $0 \le \alpha + \beta \le 1$:

$IF_{\alpha,\beta}(A) \supset F_{\alpha,\beta}IA$;

(c) $G_{\alpha,\beta}(CA) = CG_{\alpha,\beta}(A)$;

(d) $G_{\alpha,\beta}(IA) = IG_{\alpha,\beta}(A)$.

Finally, we shall note that if A is an ordinary IFS, then A is an IFS2 too, because from $\mu_A(x) + \gamma_A(x) \le 1$ follows that $\mu_A(x)^2 + \gamma_A(x)^2 \le \mu_A(x) + \gamma_A(x) \le 1$. The opposite is not always valid. For example, $\langle x, 0.9, 0.4 \rangle$ can be an IFS2-element, but it cannot be an IFS-element.

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