MODELING OF A GOAL ORIENTED RULE BASED KNOWLEDGE SYSTEM

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INTRODUCTION

In this paper we restrict our attention to the system which is known as goal oriented knowledge system based on rules and using a model of inexact reasoning. The activity of the system is based on the propagation of knowledge through a set of rules. The system uses computational method of a fuzzy model of inexact reasoning to calculate the effect of multiple rules on the same action. Such formalization allows to create a variety of models of decision making which can serve as a basis for formulation of more efficace knowledge systems.

FORMALIZED MODEL

We shall use the formalism based on the paper [5] and described in [12]. We shall repeat necessary definitions. The reader may consult the mentioned papers for more details.

In our context an entity is an object of the real world. We shall indicate of the characteristics of an entity in the knowledge system as a triple

$$(sp_i,\ pm_j,\ hp_{jk}) \tag{1}$$

where $sp_i \in SP$ is a group of parameters; pm_j is a parameter, $pm_j \in PM$; $hp_{jk} \in pm_j$ is a value of a parameter.

The sets SP, PM, pm can be characterized in several manners. In this way some characteristics of the knowledge system can be controlled. An elementary proposition is a proposition in which each triple occurs at most once, with the weight

$$v(sp_i,\ pm_j,\ hp_{jk}), \quad v \in \langle -1,\ 1\rangle \tag{2}$$

A proposition in the normal conjuctive form composed from elementary propositions is called a conjuction

$V = E_1 \wedge E_2 \wedge \cdots \wedge E_k$, where E_i are elementary propositions with weights $v(E_i)$. Weight of proposition V is

$$v(V) = \mathrm{CONJ}(v(E_1), \dots, v(E_k)) \tag{3}$$

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where CONJ is a combining function of the conjuction.

$$\text{(4)} \qquad \text{If} \quad V = \neg U. \text{ Then} \quad v(V) = \mathrm{NEG}(v(U)) = -v(U)$$

A rule *pr* is an expression of the form

$$\text{(5)} \qquad pr\colon v(P) \to (P, T) = f(pr, v(P))$$

where P is the premise (antecedent) of the rule of the form of a conjuction; T is the conclusion (consequent) of the rule of the form of an elementary proposition; $v(P, T)$ is a contribution to the weight of T; f is combining function-contribution of the rule if the weight of the premise P is $v(P)$. The argument *pr* is identified with $v(P, T)$ if P is known to be true.

A sequence $\{v(P_i) \to v(P_i, T_i),\ i = 1, \dots, m\}$ is a loop if T_m occurs in P_1. A system of rules is a non-empty linearly ordered set $PR = \{pr_1, \dots, pr_m\}$ of rules, such that there are no loops consisting of elements of PR. pm_i is an internal parameter, $pm_i \in IP$, if it occurs both in the premise of a rule and in the premise of another rule; pm_i is an axiomatic parameter, $pm_i \in AP$, if it occurs in the premise of a rule but is not in the conclusion of any rule; pm_i is a goal parameter if it occurs only in the conclusion of some rule.

Let now $PR_T = \{pr_1^{(T)}, \dots, pr_m^{(T)}\} \neq \emptyset$, $pr_j\colon v(P_j) \to v(P_j, T)$, are all rules in the system of rules PR, with the conclusion $T(sp, pm_i, hp_{ik})$, $pm_i \in IP$. Then the global weight of the proposition T is

$$\text{(6)} \qquad v(T) = \mathrm{GLOB}(v(P_i, T), \dots, v(P_m, T))$$

GLOB is the fourth combining function and in the case of its asociativity and commutativity can be computed as follows (throught succesive application of the rules $\{pr_1^{(T)}, \dots, pr_m^{(T)}\}$):

$$\begin{aligned} & v_1(T) = v(P_1, T) \\ & \quad \vdots \\ \text{(7)} \qquad & v_i(T) = \mathrm{GLOB}(v_{i-1}(T), v(P_i, T)) \quad (i = 2, \dots, m) \\ & \quad \vdots \\ & v_m(T) = v(T) \end{aligned}$$

where

$$\text{(8)} \qquad v(P_i) = \mathrm{CONJ}(v(L_1^{(i)}), \dots, v(L_n^{(i)}))$$

For L holds

$$\begin{matrix}\text{(9)} \\ \text{(10)}\end{matrix} \qquad v(L_k^{(i)}) = \begin{cases} v(sp, pm_k, hp), & \text{if} \quad pm_k \in AP \\ v(T_k^{(i)}), & \text{if} \quad pm_k \in IP \end{cases}$$

Let $g_T = g_T(\text{GLOB}, f, \text{CONJ})$, then

$$g_T = g_T(v(L_1^{(1)}), \cdot, v(L_{n1}^{(1)}), v(L_1^{(i)}), \ldots, v(L_{nm}^{(m)})). \tag{11}$$

Repeating the preceeding weights ealuation principle (VP) for $v(T)$ holds

$$v(T) = g_T^*(v(A_1^{(T)}, \ldots, v(A_k^{(T)})). \tag{12}$$

A_i is an elementary proposition, where $pm_i \in AP$. Now we are ready to present the following definitions: Knowledge system is a quadruple

$$KS = (PR, F, VP, pm_c), \tag{13}$$

where PR is a system of rules; F is the set of combining functions, $F = \{f, \text{CONJ}, \text{GLOB}, \text{NEG}\}$; VP is the weights evaluation principle (equations (6)–(12)); pm_c is the goal parameter. Let KS be a knowledge system; $v_1, \ldots, v_m$ are all known weights of parameter's values such that $pm \in AP$; $T_1, \ldots, T_k$ are all elementary propositions of pm_c. Then $v(T_i)$ is the goal of the KS if there is a function such that

$$v(T_i) = g^*(v_1, \ldots, v_n)\,. \tag{14}$$

If for given $K \in (0, 1\rangle$ and for $|\,v_1\,| = |\,v_2\,| = \ldots |\,v_n\,| = 1$ holds

$$v(T_j) \geq K \; for \; a \; j \in \{1, \ldots, k\} \tag{15}$$

we shall call $v(T_j)$ a decision of the KS.

2. Weights evaluation algorithm

On the basis of the preceeding described philosophy an algorithm can be developed. We shall assume for purpose of this paper that entities are represented as a couple (pm, hp); rules with the same premises we can integrate into a common rule $pr\colon v(P) \to v(P,T) = (v(P, T_1), \ldots, v(P, T_r))$, where $P = P_1 \wedge \cdots \wedge P_n$ is an elementary premise of the form of the elementary proposition; T_i is elementary conclusion of the form of the elementary proposition; $f(pr^{(i)}, v(P)) = f(K_{pr}(i), v(P))$, where the constant $K_{pr}(i)$ is characteristic of the i-th elementary conclusion of the rule pr.

We shal use "Pidgin Algol" (see for example [1]) to describe proposed algorithm. Input: The system of rules $pr_1, \ldots pr_n \in PR$ is defined throught the sets PO, HP, PT, HT, PP, TT, K, where:

PO(i, j)	is the parameter in the i-th el. premise of the j-th rule
HP(i, j)	is the parameter's value in the i-th el. premise of the j-th rule
PT(i, j)	is the parameter in the i-th el. conclusion of the j-th rule
HT(i, j)	is the parameter's value in the i-th el. conclusion of the j-th rule
PP(j)	is number of elementary premises of the j-th rule
TT(j)	is number of elementary conclusions of the j-th rule

N(i, j) is the constant characterizing the contribution of the j-th rule for i-th el. conclusion

The parameters $pm_1, \ldots, pm_k \in AP$, $pm_{k+1}, \ldots, pm_n \in IP$ are defined throught the sets $V, AZ, Z, NZ, NH, TOT, OT$, where:

V(i, j) is the weight of the i-th value of the j-th parameter

Z(i,j,k) is the order number the i-th rule for the j-th parameter and for its k-th value

$$\mathrm{AZ(i)}=\begin{cases} 0 & \text{if the i-th rule was not used in the consultation} \\ 1 & \text{if the i-th rule was used in the consultation} \end{cases}$$

NZ(i, j) is number of rules of the i-th value of the j-th par.

TOT(i) is a question related to the weights of values of the i-th axiomatic parameter

$$\mathrm{OT(i)}=\begin{cases} 1 & \text{if the question TOT(i) was asked by the system} \\ 0 & \text{if the question TOT(i) was not asked by the system} \end{cases}$$

pm_c is the goal parameter; f, GLOB, CONJ, NEG are combining functions.

Output: The weights of values of goal parameter pm_c

$$V(i, c), \quad i = 1, \ldots, NH(c)$$

Algorithm:

```
BEGIN
  FOR j ← 1 UNTIL m DO
  BEGIN
    IF j ≤ k THEN OT (j) ← 0. ELSE CONTINUE
    FOR i ← 1 UNTIL NH(j) DO V (i,j) ← 0.
  END
  FOR i ← 1 UNTIL n DO AZ(i) ← 0.
  FOR i ← 1 UNTIL NH(c) DO
  BEGIN
    FOR j ← 1 UNTIL NZ(i,c) DO
    IF AZ(Z(j, c, i) = 0. THEN RULE (Z(j, c, i))
    ELSE CONTINUE
  END
  FOR i ← 1 UNTIL NH(c) DO WRITE V(i, c)
END
```

Procedure RULE is used to evaluate the current rule:

```
PROCEDURE RULE (j):
BEGIN
  AZ(j) ← 1
  FOR i ← 1 UNTIL PP(j) DO
    IF PO(i,j) > k
    THEN FOR r ← 1 UNTIL NZ (HP(i, j), PO(i, j)) DO
            IF AZ(Z(r, PO(i, j), HP(i, j))) = ∅
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                THEN RULE (Z(r, PO(i, j), HP(i, j))
                ELSE CONTINUE
5           ELSE IF OT (PO(i, j)) = ∅
                THEN QUESTION (PO(i, j))
                ELSE CONTINUE
6           A← FP(j)
7           FOR i ← 1 UNTIL TT(j) DO
              V(HT(i, j), PT(i, j)) ← FT(V(HT(i, j), PT(i, j)), A, K(i,j))
        END
```

Procedure FP is used to compute the weight of the premise of the currrent rule:

```
PROCEDURE FP(j):
BEGIN
  A← V(HP(1, j), PO(1, j)
  IF PP(j) > 1
  THEN FOR i ← 2 UNTIL PP(j) DO
         A← CONJ(A, V(HP(i, j), PO(i, j)))
  ELSE CONTINUE
  RETURN A
END
```

Procedure FT is used to compute the contribution of the current rule:

```
PROCEDURE FT(V, A, K):
RETURN GLOB(V, f, (K, A))
```

Procedure QUESTION is used to ascertain the weights of the values of axiomatic parameter:

```
PROCEDURE QUESTION (j):
BEGIN
  WRITE TOT(j)
     FOR i ← 1 UNTIL NH(j) DO
        BEGIN
          READ v(i)
          V(i, j) ← v(i)
        END
  OT(j) ← 1
END
```

If n is number of rules in PR, n = card PR, then can be shown that algorithm has time complexity $t(n) \sim 0(n)$.

The algorithm has been used in knowledge system described in [9].

3. Combining functions

In previous parts we have introduced four combining functions, namely CONJ, f, GLOB and NEG. Note, that the same combining functions are used also by

Hájek in [5] (our f correspond to the disjunction of statements. Throughout this paper we will suppose that De Morgan rules hold, i. e. DISJ(v(E_j), ... , v(E_k))= NEG[CONJ[NEG(v(E_1)), ... ,NEG(v(E_k))]]. It is e. g. the case of MYCIN [13].

The general properties of combining functions have to correspond to the real decision making. The analogy of real decision modeling by means of weights and combining functions (in knowledge systems) on the one side, and by means of fuzzy sets theory on the other, induces the possibility of the interpretation of the weights as the values of membership functions of some fuzzy sets [10]. In this interpretation, the induced combining functions correspond to the basic fuzzy operations and principles. Thus a fuzzy model of decision making, used as the mathematical basis of evaluation of uncertain knowledge in a knowledge system, allows to use the tools and results of the fuzzy sets theory and better depicts the real decision making.

Introduced combining functions correspond to the next basic fuzzy (and logical) connectives or principles:

```
NEG    . . . . complementation   . . . negation
CONJ   . . . . intersection      . . . conjuction
DISJ   . . . . union             . . . disjunction
GLOB   . . . . union             . . . disjunction
f      . . . . intersection      . . . modus ponens
```

In the proposed algorithm we deal with the associativity and commutativity of CONJ (DISJ) and GLOB. In general, these two properties are not a necessary condition either for combining functions, or for fuzzy sets operations. A non-associative model is described e. g. in [9, 10]. For the sake of simplicity we will suppose the associativity and commutativity beeing fulfilled.

The results of the subjective logic theory lead to the definition of a weight as a difference of two measures, namely measure of confirmation (measure of belief MB in [13]) and measure of exclusion (measure of disbelief MD in [13]). Both these measures may be simultaneously modeled by means of the fuzzy sets theory.

The fuzzy sets theory works with membership values of some ordered lattice L. The classical approach is $L = [0,1]$. If we work on another closed interval, we can use a suitable isomorphism. So for the interval $[-1,1]$ it may be the isomorphism $h, h\colon [-1, 1] \to [0, 1]$, $h(x) = (1+x)/2$, $h^{-1}(x) = 2x - 1$.

A fuzzy complementation is described by a system of decreasing automorphisms of the value domain N_x. For the unit interval we have usually $N_x \equiv n$, $n(x) = 1 - x$. For the interval $[-1, 1]$ we get $n^*(x) = h^{-1}(n(h(x))) = h^{-1}((1-x)/2) = 2.(1-x)/2 - 1 = -x$. In our model we put (using the notation of [13]) MB($\neg$E) = MD(E), MD($\neg$ E) = MB(E), what corresponds to NEG(v(E)) = v($\neg$E) = MB($\neg$E) - MD($\neg$E) = MD(E) - MB(E) = - v(E), i.e. to the n^*.

As far as CONJ (DISJ) is concerned, in most of known knowledge systems CONJ corresponds to the minimum and DISJ to the maximum (see e. g. [5]). Of course, it is possible to use any other appropriate fuzzy intersection and union representation depending on the domain of current knowledge system (see e. g. [10]).

The properties of the combining function GLOB were studied in several papers, e. g. [4, 5, 6]. We may suppose that GLOB is strict increasing in all components. The corresponding fuzzy union (we describe simultaneously the case of the fuzzy intersection, too) can be defined pointwise for fuzzy sets through an operator $d(x, y)$

(or $k(x,y)$), see e. g. [2, 3, 7].

Theorem (Dombi [[3]). *$d(x,y)$ is a fuzzy union operator, associative, commutative, continuous, strict monotone on the* $]0, 1[$*interval and monotone on* $[0, 1]$ *interval in both components iff there exists a monotonously increasing isomorphism* $h\colon [0, 1[\to [0,\infty[$, *such that*

$$d(x,y) = h^{-1}(h(x) + h(y)) .$$

If the fuzzy intersection operator $k(x,y)$ is induced of $d(x,y)$ by means of a complementation operator $n(x)$ and De Morgan rules, then

$$k(x,y) = g^{-1}(g(x) + g(y)) ,$$

where $g(x) = h(n(x))$.

Example. Let $h(x) = -\ell n(1-x)$. Then $h^{-1}(x) = 1-e^{-x}$ and $d(x,y) = h^{-1}(\ell n(1-x) - \ell n(1-y)) = 1-(1-x)(1-y) = x+y-xy$. This fuzzy union operator induces the GLOB function used e. g. in [13].
If $n(x) = 1-x$, we have $g(x) = -\ell nx$ and $k(x,y) = xy$.
If $(n(x) = (1-x)/(1+x)$ we have $k(x,y) = 2xy/(1+x+y-xy)$.

Dombi's theorem implies that any GLOB function (on $[0, 1[$) Is isomorphic to the addition (on R^+). For the GLOB function on $]-1,1[$we use the odd extension of the isomorphism h on $]-1, 1[$, i.e. $h(x) = h(-x)$ for $x \in]-1, 0[$. The same results (without fuzzy sets theory) were obtained by Hájek in [5].

The last combining function f corresponds to the modus ponens principle. On the other side it may be modeled by a fuzzy intersection operator. These two facts lead to the next limits for the function f (see e. g. [8]):

$$\max\{0,\ x+y-1\} \le f(x,y) \le \min\{x,\ y\} \quad x \ge 0,\ y > 0 .$$

Here x is the weight of the consequent in a rule if its antecedent is known to be true, y is the weight of the antecedent.
If $y \le 0$, we define $f(x,y) = 0$.
In the case $x < 0$, we suppose that f is odd in the first component, i. e. $f(x,y) = -f(-x,y)$.

In known knowledge systems the product $f(x,y) = xy$ (for $x \ge 0$, $y > 0$ is often used (see e. g. [9, 13]).

Note, that the combining functions CONJ and GLOB may not commute. Consequently, the strict preserving of the weight's evaluation order described in the exhibited algorithm is necessary.

Conclusion

We have introduced a formalized model of a goal oriented rule-based knowledge system. Our approach allows to define the notion of a knowledge system, its goals and decisions, but especially allows to create a formalized principle for uncertain knowledge evaluation. These facts lead to the elaboration of an evaluation algorithm. The presented algorithm is relatively simple and utilisable in various situations, where the classical algorithms are not efficient. The work of our algorithm is based on single combining functions. The realization of the algorithm in different domains demands the appropriate variation of the combining functions. The fuzzy modeling makes the problem clearer and offers a wide scale of potential utilisable combining functions.

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