ON PATTERN RECOGNITION FOR INTERVAL-VALUED FUZZY SETS

Zhang Xingfang

Department of Mathematics, Liaocheng Education College, Shandong 252000
P. R. China

Abstract: In this paper, pattren recognition problem for interval-valued fuzzy sets (IVFS's) has been discussed. First, two methods of ranking interval numbers are introduced, then based on this, membership principle for IVFS's has been presented. Second, two concrete examples of pattern recognition for IVFS's are given. Finally, in order to meter the reliability of conclusion for recognition proposed in this paper, the concept of confidence has been given.

Keywords: Interval-valued fuzzy set; pattern recognition; membership principle; confidence; relational interval numbers.

Throughout this paper, R will denote real numbers set, and agree on

$[R] = \{ [a,b] : a \leqslant b,\ a,b \in R \}$,

$[I] = \{ [a,b] : a \leqslant b,\ a,b \in [0,1] \}$.

Definition 1. Let $[a,b] \in [R]$, and

$m = \min\{ |x| : x \in [a,b] \}$, $M = \max\{ |x| : x \in [a,b] \}$.

Then $[m,M] \in [R]$ is called the absolute value of $[a,b]$, and written $[m,M] = |[a,b]|$.

Definition 2. Let $[a,b], [c,d] \in [R]$. Then

(1) $[c, d]$ is called greater than $[a, b]$, written $[c, d] > [a, b]$, or $[a, b] < [c, d]$, if $c > b$.

(2) $[c, d]$ is called pre-greater than $[a, b]$, written $[c, d] \nearrow [a, b]$, if $c \leqslant b$ and $c + d > a + b$.

(3) $[c, d]$ is called pre-equal to $[a, b]$, written $[a, b] \approx [c, d]$, if $c + d = a + b$.

Definition 3. Let $[a_i, b_i] \in [R]$, $i = 1, 2, \ldots, n$. Then these interval numbers are called relational, if

$[a_i, b_i] \cap [a_j, b_j] \neq \varphi$, $i \neq j$, $1 \leqslant i, j \leqslant n$.

Remark 1. Relational interval numbers can be ranked according to pre-greater relation $\nearrow$ and pre-equal relation $\approx$. For example, given five interval numbers

$[1, 4]$, $[2, 3]$, $[3, 4]$, $[1, 3]$, $[0, 3]$,

we have

$[3, 4] \nearrow [2, 3] \approx [1, 4] \nearrow [1, 3] \nearrow [0, 3]$.

Definition 4. Let X be a non-empty crisp set. A mapping from X into $[I]$ is called a interval-valued fuzzy set [1] on X . The collection of all IVFS's on X is denoted by $IF(X)$.

Definition 5 (membership principle of IVFS's). Let $A_i \in IF(X)$, $i = 1, 2, \ldots, n$, $x \in X$.

(1) x doesn't relatively belong to A_j, if there exists $i \in \{1, 2, \ldots, n\}$ such that $A_i(x) > A_j(x)$ ($i \neq j$).

(2) x relatively belong to A_i, if $A_i(x) > A_j(x)$, $j = 1, 2, \ldots, i-1, i+1, \ldots, n$. Otherwise, we can only know that x doesn't belong to A_{j1}, A_{j2}, $\ldots$, A_{jm},

$1 \leqslant m < n-1$. Therefore the others interval numbers are relational, we might as well assume that

$A_{i1}(x) \nearrow A_{i2}(x) \nearrow \ldots \nearrow A_{ik}(x)$, $2 \leqslant k+m \leqslant n$.

Then we say that: x 1-th pre-belong to A_{i1}, x 2-th pre-belong to A_{i2}, ..., x k-th pre-belong to A_{ik}.

Example 1. Army A and army B are standing facing each other. Each side commanding officers want to master the military movements for the other side. For example, army B consists of 3 formations. Now army A reconnoitres which one of the three formations is excellentest in weapon equipment, and which one is strongest in control of the air, and which one is bravest in battle . In view of the finite military information, we can only set up three IVFS's . Let $X=\{x_1, x_2, x_3\}$ be the universe, where x_1 is weapon equipment, x_2 is ability of air battle , x_3 is expression in battle.

$A_i \in IF(X)$, i=1, 2, 3, denotes the mililtary qualities of i-th formation.

$A_1=[0.5, 0.6]/x_1+[0.4, 0.44]/x_2+[0.6, 0.7]/x_3$,

$A_2=[0.7, 0.8]/x_1+[0.45, 0.5]/x_2+[0.59, 0.71]/x_3$,

$A_3=[0.6, 0.65]/x_1+[0.35, 0.47]/x_2+[0.5, 0.6]/x_3$.

(1) It is clear that

$A_2(x_1) > A_1(x_1), A_3(x_1)$.

Hence, from the membership principle of IVFS's we see that x_1 relatively belong to A_2. This shows that 2- th formation is excellentest in weapon equipment.

(2) It follows from

$A_2(x_2) = [0.45, 0.5] > [0.4, 0.44] = A_1(x_2)$

that x_2 doesn't relatively belong to A_1 .

$A_2(x_2)$ and $A_3(x_2)$ are relational , and $A_2(x_2) \nearrow A_3(x_2)$. Hence, according to the membership principle of IVFS's we see that x_2 1-th pre-belong to A_2, and 2-th pre-belong to A_3. This implies that there exists two

possibility of 2-th and 3-th formation are strongest of ability of air battle, however, the possibility of 2-th formation is greater than 3-th that.

(3) Obviously, $A_1(x_3)$, $A_2(x_3)$ and $A_3(x_3)$ are relational, and $A_1(x_3) \approx A_2(x_3) \nearrow A_3(x_3)$. Hence, x_3 1-th pre-belong to A_1 or A_2, and x_3 2-th pre-belong to A_3. This implies that there are three possibility of 1-th and

2-th and 3-th formation are bravest in battle, however, the possibility of 1-th and 2-th formation are greater than 3-th that.

From example 1 we see that if x_0 belong to some A_i, then the conclusion is absolutely raliable. If x_0 1-th pre-belong to A_i, then the conclusion is not absolutely reliable, that is, it is not completely confidenceable. The reason is that, in this case, there are intervals which are relational with $A_i(x_0)$, so the possibility of x_0 belong to these IVFS's whose some value intervals are relational with $A_i(x_0)$ can't be removed. In order to meter the accuracy of conclusion for x_0 1-th pre-belong to some IVFS, we now introduce the concept of confidence.

Definition 6. Assume that $x \in X$ 1-th pre-belong to $A \in IF(X)$, and $A(x) = [a, b]$, and $[a_1, b_1]$, $[a_2, b_2]$, ..., $[a_k, b_k]$, $k \geqslant 2$, are relational with $A(x)$. We call

$$\alpha = \min\{ 1-((b_1-a) / (b-a_1)), \ldots, 1-((b_k-a) / (b-a_k)) \}$$

the confidence of x 1-th pre-belong to A, written $C(x \in A) = \alpha$. If x belong to A, then $C(x \in A) = 1$.

The confidence has the following properties.

Property 1. $0 \leqslant \alpha \leqslant 1$.

Property 2. If x 1-th pre-belong to A_1, 2-th pre-belong to A_2, and suppose that $A_1(x) = [a_1, b_1]$, $A_2(x)=[a_2, b_2]$, $b_2=a_1$. Then $C(x \in A_1) = 1$. If x 1-th pre-belong to A_i, i=1,, 2, . . . , k, $k \geqslant 2$, then
$C(x \in A_i) = 0$, i = 1, 2, . . . , k.

In example 1, the confidence of 2- th formation is excellentest in weapon equipment is 1. The confidence of 2-th formation is strongest in ability of air battle is
$\alpha = 1-((0.47-0.45)/(0.5-0.35)) = 13/15$.
The confidence of 1-th formation is bravest in expression in battle is
$\min\{1-((0.6-0.6)/(0.7-0.5)),\ 1-((0.71-0.6)/(0.7-0.59))\} = 0$.
The confidence of 2-th formation is bravest in expressin in battle is
$\min\{1-((0.7-0.59)/(0.71-0.6)),\ 1-((0.6-0.59)/(0.71-0.5))\} = 0$.

Example 2. In order to judge the looks and way rape seeding are growing, we fall these rape seeding into three categories -- robust A_1 , and

emaciated A_2, and spindling A_3, according to four factors: number of green leaves x_1, and height of seeding x_2, and length of radicle x_3, and thickness of radicle x_4. We have known that each category rape seeding A_i obey normal distribution for every factor x_j:

$$A_{ij} = \begin{cases} \exp(-(x_j-\bar{x}_{ij})/\sigma_{ij}), & |x_j-\bar{x}_{ij}| < \sigma_{ij} \\ 0 & , \text{ otherwise } x_j \end{cases}$$

$$= \begin{cases} 1-\left|\left((x_j-\bar{x}_{ij})/\sigma_{ij}\right)^2\right|, & \bar{x}_{ij}-\sigma_{ij}<x_j<\bar{x}_{ij}+\sigma_{ij} \\ 0, & \text{otherwise } x_j \end{cases}$$

where $\bar{x}_{ij}$ and σ_{ij} are given by the following table:

	A_1		A_2		A_3	
	$\bar{x}_{ij}$	σ_{ij}	$\bar{x}_{ij}$	σ_{ij}	$\bar{x}_{ij}$	σ_{ij}
x_1	6	0.2	4	0.2	5	0.9
x_2	[5.9, 6.1]	0.2	[4, 4.2]	0.4	[9, 9.1]	0.4
x_3	[1.8, 1.9]	0.5	[2.3, 2.4]	0.4	[3, 3.1]	0.1
x_4	[0.6, 0.61]	0.3	[0.5, 0.6]	0.7	[0.5, 0.55]	0.9

The membership grade of each rape seeding $X=(x_1, x_2, x_3, x_4)$ belong to A_i may be computed by

$$A_i(x)=[\min\{\underline{A}_{i1}(x_1), \underline{A}_{i2}(x_2), \underline{A}_{i3}(x_3), \underline{A}_{i4}(x_4)\}, \min\{\bar{A}_{i1}(x_1), \bar{A}_{i2}(x_2), \bar{A}_{i3}(x_3), \bar{A}_{i4}(x_4)\}], \quad (*)$$

where, if $A_{ij}(x_j)=[a, b]$, then $\underline{A}_{ij}(x_j)=a$, $\bar{A}_{ij}(x_j)=b$.

Now we will judge the rape seeding $x=(6, 6, 2, 0.7)$ to belong to which category. From the definition of A_{ij} we have

$A_{11}(6)=1-|((6-6)/0.2)^2|=1-0=1$,

$A_{12}(6)=1-|((6-[5.9, 6.1])/0.2)^2|=[0.75, 1]$,

$A_{13}(2)=1-|((2--[1.8, 1.9])/0.5)^2|=[0.84, 0.96]$,

$A_{14}(0.7)=1-|((0.7-[0.6, 0.61]/0.3)^2|=[0.89, 0.91]$,

$A_{21}(6)=0$ (because $6>4+0.2$),

$A_{31}(6)=0$ (because $6>5+0.9$).

$A_1(x)=[\min\{1, 0.75, 0.84, 0.89\}, \min\{1, 1, 0.96, 0.91\}]=[0.75, 0.91]$.

From $A_{21}(6)=A_{31}(6)=0$ we get

$\overline{A}_{21}(6)=\underline{A}_{21}(6)=\overline{A}_{31}(6)=\underline{A}_{31}(6)=0$.

Hence $A_2(x)=A_3(x)=0$. Therefore $A_1(x)>A_2(x)$, $A_1(x)>A_3(x)$.

This shows that $x=(6, 6, 2, 0.7)$ is robust seeding, and $C(x\in A_1)=1$.

Now we judge the rape seeding $x=(6, 9.1, 3.1, 0.4)$ to belong to which category. From formula (*) we have

$A_1(x)=A_2(x)=A_3(x)=0$.

Hence we are unable to judge the rape seeding to belong to which category.

Now we switch over to using the following formula:

$A_i(x)=(A_{i1}(x_1)+A_{i2}(x_2)+A_{i3}(x_3)+A_{i4}(x_4))/4$.

Then

$A_1(x)=[0.39, 0.44]$, $A_2(x)=[0.23, 0.26]$, $A_3(x)=[0.42, 0.5]$.

From $A_1(x)\not> A_2(x)$ we see that x doesn't belong to A_2.

$A_1(x)$ and $A_2(x)$ are relational and $A_3(x)\not> A_1(x)$. Hence x 1-th pre-belong to A_3, and x 2-th pre-belong to A_1. This shows that it is possible that both the rape seeding is spindling and robust seeding. However the possibility which is spindling seeding is greater than that which is robust seeding. The confidence of the rape seeding is spindling seeding is

$C(x\in A_3)=1-((0.44-0.42)/(0.5-0.39))=9/11$.

References

[1] M.B. Gorzalczany, A method of inference in approximate reasoning based on interval-valued fuzzy sets, Fuzzy Sets and Systems, 21 (1987), 1-17.